SDLL403 - Vibrations of a pendulum in rotation

Summary

The scope of application of this test is the modal analysis of the structures. The studied structure is a pendulum in rotation around an axis fixed and plunged in a field of gravity. The pendulum itself is articulated around an axis perpendicular to the axis of rotation and is located at a certain distance from this one.
One is interested in the first six Eigen frequencies.

The interest of this test lies in the following aspects:

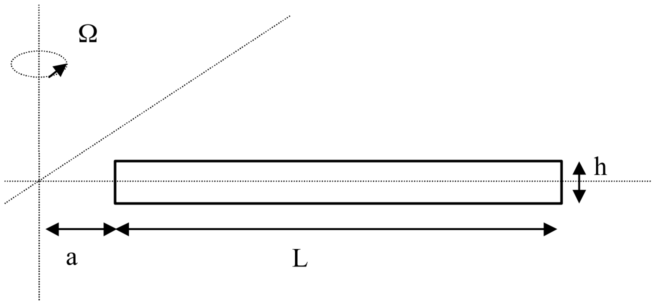
- modal analysis with taking into account of initial constraints (geometrical stiffness)
- modal analysis with taking into account of the centrifugal stiffening
- important relative difference between two successive frequencies of the spectrum

Currently, the taking into account of the centrifugal stiffening is not possible that with voluminal elements. The element used is the element `HEXA20` and one employs the method of Sorensen for the calculation of the Eigen frequencies.

The first Eigen frequency is compared with an analytical reference. The following frequencies are compared with digital values obtained by a software independent of *Code_Aster* and using modelings 'beam' and 'plane constraint'.

1 Problem of reference

1.1 Geometry



Characteristics:

Length of the pendulum	$L=0.6\,m$
Eccentricity	$a=0.1\,m$
Height of the profile	$h=0.01\,m$
Width of the profile	$b=0.004\,m$
Section	$S=bh$
Inertia of inflection	$I_Z=bh^3/12$

1.2 Properties of materials

Young modulus	$E=7.E10\,N/m^2$
Poisson's ratio	$\nu=0.3$
Density	$\rho=2700\,kg/m^3$

1.3 Boundary conditions and loading

The beam is articulated at the point A. The clevis pin is the axis Y. The initial state of stress which makes it possible to carry out the geometrical calculation of the stiffnesses and centrifuges is obtained by imposing a number of revolutions and gravity.

Acceleration of gravity	$g=-9.81\,m/s^2$ (parallel with the axis Z)
Number of revolutions	$\Omega=10\,rad/s$

The static position of balance θ_0 corresponding to loading is calculated by the relation:

$$3\,g\cos\theta_0=\Omega^2(3\mathrm{a}+2\mathrm{L}\cos\theta_0)\sin\theta_0$$

One finds $\theta_0=11.269931365\,°$

Conditions on displacements at the point A are the following ones:

$$u=v=w=0\ ;\ \phi_X=\phi_Z=0$$

One considers moreover than the section passing by A remain rigid.

1.4 Initial conditions

Without object in modal analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

- first Eigen frequency

The facts of the case are selected in such a way that the stiffnesses in inflection and extension are large with respect to the stiffnesses geometrical and centrifugal. Under these conditions, the value of the first Eigen frequency is obtained analytically by considering a rigid pendulum.

By taking as degree of freedom the angle θ between the pendulum and the axis X the equation of the movement is written:

$$2L\ddot{\theta}=3g\cos\theta-\Omega^2(3a+2L\cos\theta)\sin\theta$$

The small oscillations here are considered $\Delta\theta$ pendulum around a static position of balance θ_0. By linearizing the equation of the movement in the vicinity of this position, one obtains the equation with the small disturbances:

$$2L\Delta\ddot{\theta}+\left[3g\sin\theta_0+\Omega^2(3a\cos\theta_0+2L\cos 2\theta_0)\right]\Delta\theta=0$$

One from of deduced the pulsation from the first mode:

$$\omega=\sqrt{\frac{3g}{2L}\sin\theta_0+\Omega^2\left[\frac{3a}{2L}\cos\theta_0+\cos 2\theta_0\right]}$$

This own pulsation can be still written in the form

$$\omega=\sqrt{\frac{K(\sigma)+K(\Omega^2)}{I}}$$

with

$K(\sigma)=\frac{1}{2}\rho S L^2 g\sin\theta_0+\rho S L^2\Omega^2\left[\frac{a}{2}\cos\theta_0+\frac{L}{3}\cos^2\theta_0\right]$ (geometrical stiffness)

$K(\Omega^2)=-\frac{1}{3}\rho S L^3\Omega^2\sin^2\theta_0$ (stiffness centrifuges)

$I=\frac{1}{3}\rho S L^3$ (inertia in rotation)

- other Eigen frequencies

The values of reference of frequencies 2 to 6 are obtained numerically by means of version 7 of software the SAMCEF software. Two different modelings were used: 20 elements of deformable beam to the shearing action and 20×4 elements of membrane with 8 nodes. The results got in both cases are identical if one limits oneself to the first 4 significant figures. Considering the corrections of stiffness are small with respect to the terms of linear stiffness, one can check that frequencies 2 to 6 differ little from the analytical values obtained for a nondeformable beam hurled with the shearing action. In fact, the maximum difference between the digital and analytical values does not exceed 1%.

2.2 Results of reference

The first 5 critical loads are classified by order of increasing module.

Mode	Eigen frequency (Hz)
1	1.75556
2	100.2
3	324.0
4	674.4
5	1150.
6	1748.

2.3 Uncertainty on the solution

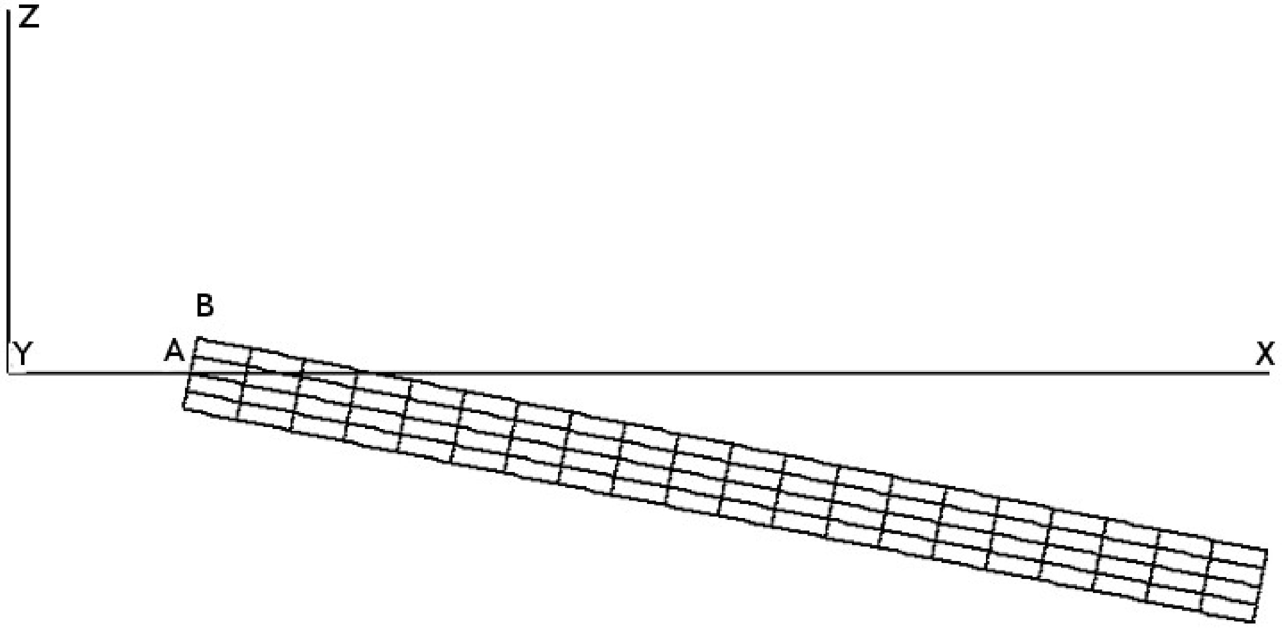
Analytical solution for the first frequency. Digital solution for the others. The estimated tolerance of the digital results is of 1%.

2.4 Bibliographical references

Without object.

3 Modeling A

3.1 Characteristics of modeling



The beam is with a grid by means of elements HEXA20.

Boundary conditions:

At the point A such as $X=0.1$, $Y=0$, $Z=0$:

$$DX=DY=DY=DRX=DRZ=0$$

In addition, all nodes of the section passing by A are rigidly dependent.

3.2 Characteristics of the grid

Many nodes: 1077
Many meshs: 160 HEXA20
8 QUAD8

3.3 Sizes tested and results

Frequencies in Hz

Mode	Reference	*Code_Aster*	Tolerance (%)
1	1.75556	1.75979	0.6
2	100.2	100,272	0.1
3	324.0	324.65	0.3
4	674.4	677.1	0.5
5	1150.	1157.8	0.7
6	1748.	1766.7	1.1

4 Summary of the results

Good agreement with the reference solution (less $1.\%$ of error on all the modes except on the first where the error is of 2.2%).

This test could not be carried out with an element of beam because the calculation of the centrifugal matrix of rigidity is not available for this kind of element. In the same way, as it is not available for the discrete elements, we could not use connection 3D-beam. In order to stage this problem, all nodes of surface containing the point A were bound by a solid connection.