SDLS01 - Thin, free or embedded square plate at the edge

Summary:

The scope of application of this case test relates to the dynamics of the structures, and more particularly modal calculation and the harmonic calculation of answer.

For modal calculation, it is a question of calculating the clean modes of inflection of a thin square plate in two configurations:

- Plate embedded on an edge,
- Free plate.

The plate is with a grid in triangular elements to which elements are affected DKT.

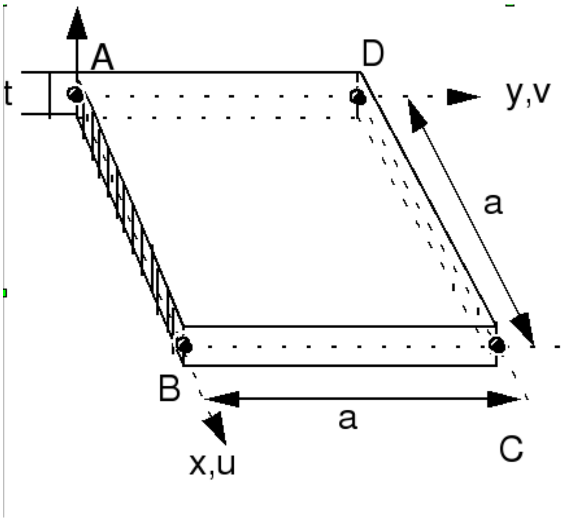
Four different modelings are tested:

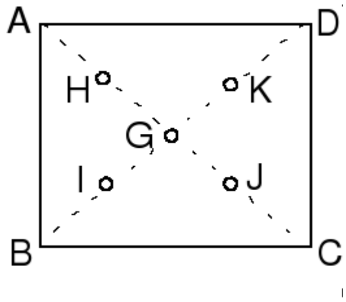
- Modal calculation – Edges of the plate directed according to the axes of the reference mark,
- Modal calculation – unspecified Orientation of the plate and harmonic answer for the embedded plate,
- Modal calculation by classical and cyclic dynamic under-structuring,
- Modal calculation following a condensation of Guyan.

The results of reference of modal calculations result from analytical calculations. They validate on the one hand the tools for creation of the matrices of mass and rigidity, as well as the operators of under - classical and cyclic dynamic structuring implemented in *Code_Aster*. In addition, this case test validates modal calculation following a condensation of Guyan (condensation of the matrix of mass).

1 Problem of reference

1.1 Geometry





Side $a=1\text{m}$
Thickness $t=0.01\text{m}$

Coordinates of the points (in m):

	A	B	C	D	G	H	I	J	K
x	0.	1.	1.	0.	0.5	0.25	0.75	0.75	0.25
y	0.	0.	1.	1.	0.5	0.25	0.25	0.75	0.75
z	0.	0.	0.	0.	0.	0.	0.	0.	0.

1.2 Material properties

$$E=2.1\,10^{11}\,Pa \qquad \nu=0.3 \qquad \rho=7800\,kg/m^3$$

1.3 Boundary conditions and loadings

Case 1: dimensioned AB embedded
for any point P such as $y_P=0$:

$$u=v=w=0.$$
$$\theta_x=\theta_y=\theta_z=0.$$

Case 2: free plate

1.4 Initial conditions

Without object for the modal analysis

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is that given in card SDLS01/89 of the guide VPCS which presents the method of calculating in the following way:

The formulation of M.V. BARTON, for a plate of with dimensions a, led to:

$$f_i = \frac{1}{2\pi a^2} \lambda_i^2 \sqrt{\frac{Et^2}{12\rho(1-\nu^2)}} \qquad i=1,2,\ldots$$

with, for a Poisson's ratio $\nu = 0.3$:

1°: Plate embedded on a side

i	λ_i^2
1	3,492
2	8,525
3	21.43
4	27.33
5	31.11
6	54.44

2°: Free plate

i	λ_i^2
1 to 6	0.
7	13.49
8	19.79
9	24.43
10	35.02
11	35.02

(6 modes of solid body at worthless frequency).

This reference solution applies to the thin sections such as: $t/a < 0.1$
Coefficients λ_i are establish by development limited on the modal deformations of a network of cross beams (embed-free beam and free-free beam).

2.2 Results of reference

Case 1: the first 6 clean modes

Case 2: the first 11 clean modes

2.3 Uncertainty on the solution

Semi-analytical solution.

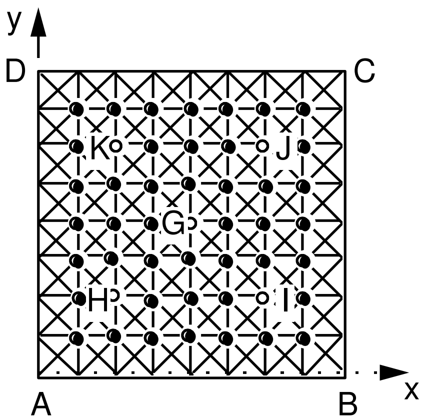
2.4 Bibliographical references

[1] Mr. V. BARTON Vibrations of rectangular and skew cantilever punts. – Newspaper of Applied Mechanics, flight 18, p. 129-134 (1951)

3 Modeling A

3.1 Characteristics of modeling

Modeling DKT



Names of the Points nodes:

$A=N1$	$B=N78$	$C=N145$	$D=N80$	
$G=N65$	$H=N17$	$I=N73$	$J=N121$	$K=N71$

Limiting conditions:

Case 1 in all the nodes on the side AB :

```
DDL_IMPO= _F (GROUP_NO= AB DX =0. , DY =0. , DZ =0. , DRX =0. , DRY
MARTINI =0. , DRZ =0.)
```

Cases 2 none

3.2 Characteristics of the grid

Many nodes: 145

Many meshs and types: 256 TRIA3

3.3 Sizes tested and results

Clean mode	Frequency (Hz) Reference	Aster	% difference	Tolerance
	1°: Plate embedded on a side			
1	8.7266	8.6718	– 0.63	
2	21.3042	21.2904	– 0.06	
3	53.5542	53.0992	– 0.85	$1.\ 10^{-2}$
4	68.2984	67.9269	– 0.54	
5	77.7448	77.4294	– 0.40	
6	136.0471	135.7635	– 0.21	

	Aster $epot=ecin$
1	$1.4796\ 10^4$
2	$1.7331\ 10^4$
3	$4.3802\ 10^4$

4	3.7367 10^4
5	5.4956 10^4
6	1.3483 10^5

2°: Free plate

7	33.7119	33.6839	– 0.08	
8	49.4558	48.9362	– 1.05	
9	61.0513	60.5849	– 0.76	1.1 10^{-2}
10	87.5160	87.0993	– 0.48	
11	87.5160	87.0993	– 0.48	

	Aster $epot=ecin$
7	2.2396 10^4
8	4.7270 10^4
9	7.2453 10^4
10	1.4974 10^5
11	1.4974 10^5

The kinetic energy is calculated `ECIN_ELEM` element `DKT` (connected to the point A one of with dimensions is on AD) problem 1 (“plate embedded on with dimensions”):

Option	**Component**	**Reference (`NON_REGRESSION`)**	***Aster***	**% difference**
`ECIN_ELEM`	`TOTAL`	0.011448	0.0114476	3.5 10^{-4}
`ECIN_ELEM`	`INFLECTION`	2968.79	2968.7918	6.1 10^{-5}

3.4 Remarks

`CALC_MODES` `OPTION= 'BANDAGES'`
Case 1: `FREQ = (8. , 140.)`
Case 2: `FREQ = (32. , 90.)`

Contents of the file results:

1°: the first 6 Eigen frequencies, clean vectors and modal parameters and kinetic energy deformation energy of the 6 modes.

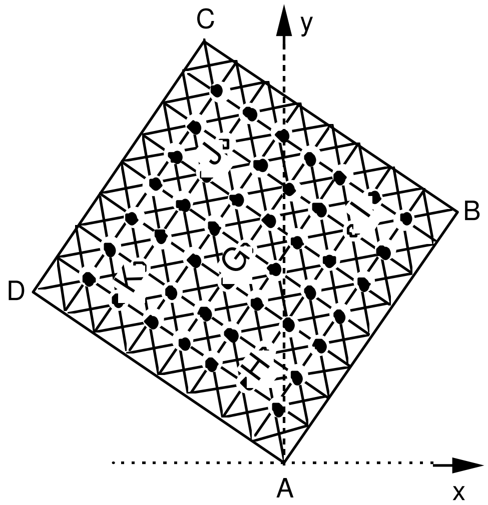
2°: 5 Eigen frequencies, clean vectors and modal parameters ($f>0$) and kinetics deformation energy of the 5 modes.

4 Modeling B

4.1 Characteristics of modeling B

Modeling DKT with grid identical to modeling A.

Rotation of the plate such as the side AB is on the line $3y=4x$



Names of the Points nodes:	$A=N1$	$B=N78$	$C=N145$	$D=N80$	
	$G=N65$	$H=N17$	$I=N73$	$J=N121$	$K=N71$

Limiting conditions:

Case 1 in all the nodes on the side AB :

```
DDL_IMPO= (GROUP_NO= AB DX =0. , DY =0. , DZ =0. , DRX =0. , DRY
MARTINI =0. , DRZ =0.)
```

Case 2: none

Harmonic answer:

Nodal force point C ($N145$): $Fz=-98100$

Material: AMOR_ALPHA : 0.1 AMOR_BETA : 0.1

4.2 Characteristics of the grid

Many nodes: 145

Many meshs and types: 256 TRIA3

4.3 Sizes tested and results

The values of the Eigen frequencies are identical to those of modeling A.

Harmonic answer:

FREQ : $50\,Hz$ NODE : $N145$ MESH : $M255$

Reference	*Aster* 3.03.15	*Aster* 3.05.16	% difference
DEPL 'DZ'	2.90290E-02 5.20606E-02	2.90290E-02 5.20606E-02	0.0
DEPL 'DRX'	2.52920E-02 9.44717E-02	2.52920E-02 9.44717E-02	0.0
QUICKLY 'DZ'	– 1.63553E+01 9.11973E+00	– 1.63553E+01 9.11973E+00	0.0
QUICKLY 'DRX'	– 2.96792E+01 7.94573E+00	– 2.96792E+01 7.94573E+00	0.0
ACCE 'DZ'	– 2.86505E+03 – 5.13817E+03	– 2.86505E+03 – 5.13817E+03	0.0
ACCE 'DRX'	– 2.49622E+03 – 9.32398E+03	– 2.49622E+03 – 9.32398E+03	0.0
'EFGE_ELNO' 'MXX'	1.14053E+01 1.45539E+03	1.14053E+01 1.45539E+03	0.0
'EFGE_ELNO' 'MYY'	1.10224E+01 – 1.31441E+03	1.10224E+01 – 1.31441E+03	0.0
'EFGE_ELNO' 'MXY'	1.03148E+01 3.55382E+02	1.03148E+01 3.55382E+02	0.0
'EFGE_ELNO' 'QX'	3.66163E+02 – 3.77331E+03	3.66163E+02 – 3.77331E+03	0.0
'EFGE_ELNO' 'QY'	– 3.14676E+02 2.06813E+03	– 3.14676E+02 2.06813E+03	0.0
'SIGM_ELNO' 'SIXZ'	5.49245E+04 – 5.65997E+05	5.49245E+04 – 5.65997E+05	0.0
'SIGM_ELNO' 'SIYZ'	– 4.72014E+04 3.10219E+05	– 4.72014E+04 3.10219E+05	0.0

4.4 Remarks

CALC_MODES OPTION= 'BANDAGES'

Case 1: FREQ = (8. , 140.)

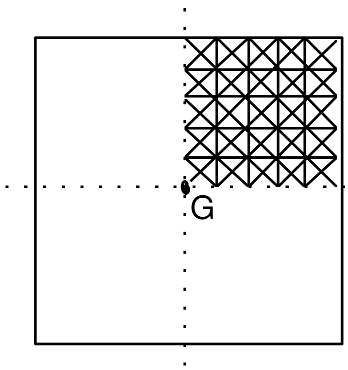
Case 2: FREQ = (32. , 90.)

Contents of the file results:

1°: the first 6 Eigen frequencies, clean vectors and modal parameters.
2°: the first 11 Eigen frequencies, clean vectors and modal parameters.
3°: displacement DZ DRX with the node $N145$
efforts generalized and forced mesh $M255$

5 Modeling C

5.1 Characteristics of modeling



In the 2 cases, the plate is cut out in 4 parts of equal size. Each substructure considered is with a grid in triangles to which elements of plate are affected `DKT`.

Case 1: Plate embedded on an edge
The structure is studied using the method of under-structuring classical with interfaces of the type CRAIG_BAMPTON. The modal base used for each substructure is made up of 25 clean modes and the constrained modes associated with the interfaces.
Case 2: Free plate
The structure is studied using the method of under-structuring cyclic with interfaces of the type CRAIG_BAMPTON HARMONIC and taken into account of the specificity of the node of the axis (not G). The modal base used for the basic sector is made up of 25 clean modes and the harmonic modes associated with the interfaces.

5.2 Characteristics of the grid

Many nodes: 121
Many meshs and types: 200 `TRIA3`

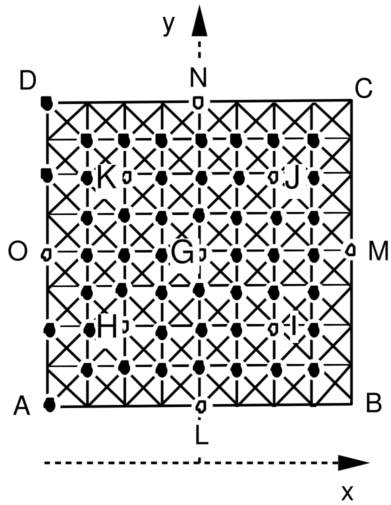
5.3 Sizes tested and results

Order of clean mode I	Frequency (Hz) Reference	*Aster*	% difference	Tolerance
	1°: Plate embedded on a side			
1	8.7266	8.6419	– 0.97	
2	21.3042	21.2253	– 0.37	
3	53.5542	52.9693	– 1.09	$1.25\ 10^{-2}$
4	68.2984	67.5444	– 1.10	
5	77.7448	77.3966	– 0.45	
6	136.0471	134.5785	– 1.08	
	2°: Free plate			
7	33.7119	33.6808	– 0.09	
8	49.4558	48.9785	– 0.96	
9	61.0513	60.6739	– 0.62	$1.\ 10^{-2}$
10	87.5160	87.0662	– 0.51	
11	87.5160	87.0662	– 0.51	

6 Modeling D

6.1 Characteristics of modeling

DKT + under-structuring of GUYAN



Limiting conditions: Free plate

Condensation of the matrices of mass and rigidity on the nodes:

$$(A, B, C, D, G, H, I, J, K, L, M, N, O) \quad .$$

6.2 Characteristics of the grid

Many nodes: 145

Many meshs and types: 256 TRIA3

6.3 Sizes tested and results

Order of clean mode I	Frequency (Hz) Reference	*Aster*	% difference	Tolerance
	2°: Free plate			
7	33.7119	33.8758	– 0.48	
8	49.4558	49.5240	– 0.14	$1.1\ 10^{-2}$
9	61.0513	61.6240	– 0.94	

6.4 Remarks

One seeks to calculate the first 3 nonworthless Eigen frequencies of the problem of the free plate on his edges.

If one condenses the matrices on the only nodes:

$$(A, B, C, D, G, H, I, J, K)$$

The precision of the frequencies is not whereas of 2% .

To get the results wanted with the expected precision (1%), the points should be added (L, M, N, O) .

7 Summary of the results

- Modelings A and b:

Precision on the Eigen frequencies $\leq 1\%$ until the sixth mode of inflection.

- Modeling C:

In under-structuring, the quality of the results could be improved by the use of a finer grid of substructure.

- Modeling D:

To obtain a precision of 1% on the Eigen frequencies, it is necessary to also condense on the 4 nodes mediums of the edges L , M , N and O .