SDLS03 - Thin rectangular plate simply pressed on the edges

Summary:

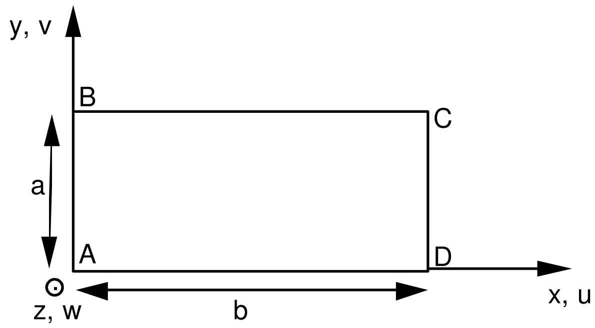
This three-dimensional problem consists in seeking the frequencies of vibration of a mechanical structure of type plates. Two different configurations make it possible to test the modes of vibration in the plan of the plate (behavior out of membrane) with elastic supports on two opposed edges, and the modes of vibration in inflection of a plate supported on its contour.

This test of mechanics of the structures corresponds to a dynamic analysis of a surface model having a linear behavior. It comprises two modelings (grid in triangle or quadrangle).

This problem makes it possible to test the elements of transverse membrane and flexbeam and the calculation of the frequencies of vibration by the method of Lanczos or the method of Bathe and Wilson.

1 Problem of reference

1.1 Geometry



That is to say a plate whose characteristics are the following ones:

length: $a=1.5\,m$ width: $b=1\,m$ thickness: $t=0.01\,m$

The points characteristic of the plate have as coordinates:

	A	B	C	D
x	0.	0.	1.	1.
y	0.	1.5	1.5	0.
z	0.	0.	0.	0.

1.2 Properties of materials

The parameters characterizing the properties of material are:

$E=2.1\,10^{11}\ Pa$ $\nu=0.3$ $\rho=7800\,kg/m^3$

1.3 Boundary conditions and loadings

1.3.1 Problem of inflection

The plate is in simple support on all its sides: for any point P edge one a: $w=0$.

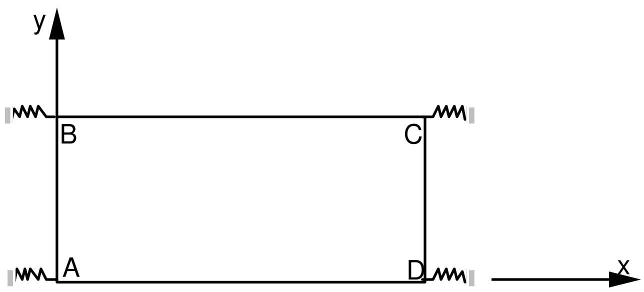
1.3.2 Problem of membrane

For all the points of the plate, one blocks displacement in z and three degrees of rotation, i.e.:

$$w=0. \quad \theta_x=\theta_y=\theta_z=0.$$

On the sides AD and BC one blocks displacement in y : for $y=0.$ or $y=a$ one has $v=0$.

At the points A, B, C, D , springs of stiffness are attached k . The axis of these springs is the direction x .



The digital value of k is the following one: $k=25\,N/m$.

2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Problem of inflection

The reference solution of the problem of inflection is that given in card SDLS03/89 of the guide VPCS which presents the method of calculating in the following way.

The formulation of M.V. BARTON for a rectangular plate, posed on his four sides leads for the modes of inflection to:

$$f_{ij}=\frac{\pi}{2}\left[\left[\frac{i}{a}\right]^2+\left[\frac{j}{b}\right]^2\right]\sqrt{\frac{E\,t^2}{12\,\rho\left(1-\nu^2\right)}}$$

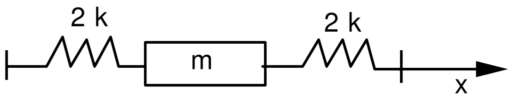
with:

i = number of half-length of wave according to y (dimension a),

j = number of half-length of wave according to x (dimension b).

2.1.2 Problem of membrane

With the problem dealt out of membrane is equivalent for the research of the first frequency of vibration to the following unidimensional problem:



where:

k is the stiffness of the springs,

m is the mass of the plate.

The sought frequency is thus: $f=\frac{1}{2\pi}\sqrt{\frac{4\mathrm{k}}{m}}$

2.2 Results of reference

For the problem of inflection, one calculates the first six frequencies of vibration and for calculation out of membrane, one calculates only the first frequency.

2.3 Uncertainty on the solution

The solutions being analytical, there is no uncertainty.

2.4 Bibliographical references

1) M.V. BARTON “Vibrations of rectangular and skew cantilever punts” - Newspaper of Applied Mechanics, vol. 18, p. 129-134 (1951).

3 Modeling A

3.1 Characteristics of modeling

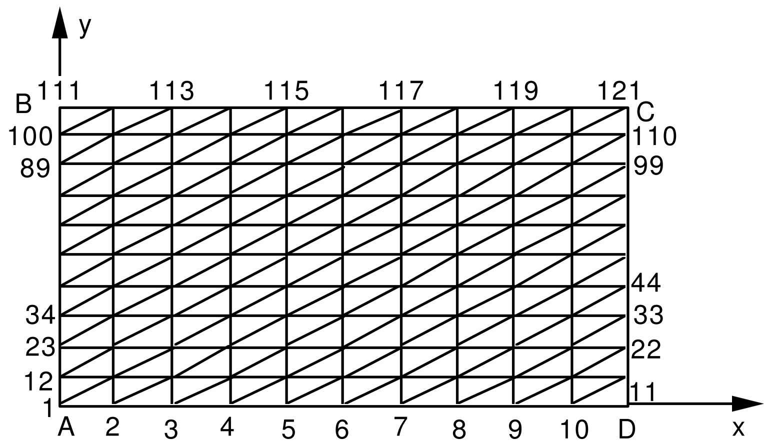
The plate in 200 meshs was cut out TRIA3. Two modelings for the plate are used: DKT and DST.

For the problem of inflection, the boundary conditions are the following ones:

- in all the nodes of the edge: $DZ=0$

For the problem of membrane, the boundary conditions are:

- in all the nodes of the grid: $DZ=0$ $DRX=DRY=DRZ=0$,
- in all the nodes on the sides AB and BC : $DY=0$
- at the points A, B, C, D one adds discrete elements of rigidity (direction x).



3.2 Characteristics of the grid

Many nodes: 121

Many meshs and types: 200 TRIA3

The points characteristic of the grid are the following:

Not A = $N1$	Not C = $N121$
Not B = $N111$	Not D = $N11$

3.3 Sizes tested and results

For the modes of inflection:

Number mode	Frequencies			
	Reference	***Aster* DKT**	**% difference**	**% tolerance**
4	35.63	35.46	– 0,477	0.5
5	68.51	67.82	– 1,003	1.1
6	109.62	108.67	– 0,867	0.9
7	123.32	121.90	– 1,150	1.2
8	142.51	139.99	– 1,761	1.8
9	197.32	191.70	– 2,846	2.9

***Aster* DST**	**% difference**	**% tolerance**
35.45	– 0,492	0.5
67.80	– 1,030	1.1
108.62	– 0,910	1.
121.84	– 1,199	1.3
139.92	– 1,815	1.9
191.57	– 2,912	3.

For the problem out of membrane:

Reference	***Aster* DKT**	**% difference**	**% tolerance**
0.14714	0.147136	– 0,002	0.1

	***Aster* DST**	**% difference**	**% tolerance**
	0.147136	– 0,001	0.1

3.4 Remarks

For the problem in inflection, the modal position of the first mode found in the band $(5.,200.)$ is the fourth, because there are three modes of solid body at frequency zero:

- modes of translation u and v in the plan,
- mode of rotation around the axis z .

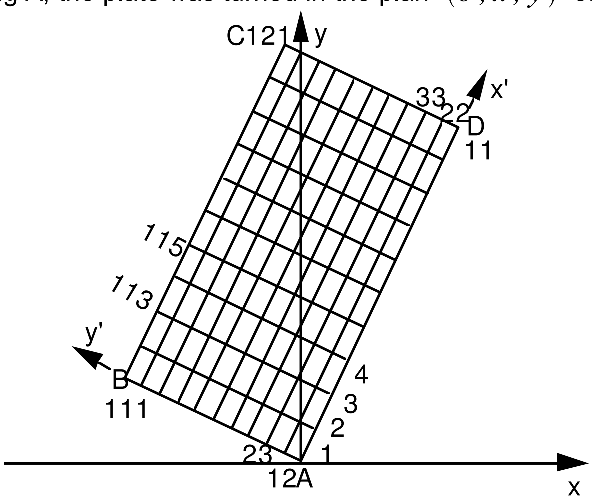
4 Modeling B

4.1 Characteristics of modeling

The plate in 100 meshs was cut out `QUAD4`.

Three modelings for the plate are used: `Q4G`, `DKT` (DKQ), `DST` (DSQ).

Compared to modeling A, the plate was turned in the plan (o, x, y) of an angle of $60°$



For the problem of inflection, the boundary conditions are the following ones:

- in all the nodes of the edge: $DZ=0$

For the problem of membrane, the boundary conditions are:

- in all the nodes of the grid: $DZ=0$ $DRX=DRY=DRZ=0$,
- with the node A , displacement is blocked DY in the reference mark (A, x', y') ,
- at the points A, B, C, D one adds discrete elements of rigidity (direction x').

4.2 Characteristics of the grid

Many nodes: 121 Many meshs and types: 100 `QUAD4`

The points characteristic of the grid are the following:

Not A = $N1$ Not B = $N111$ Not C = $N121$ Not D = $N11$

4.3 Sizes tested and results

For the modes of inflection:

Number mode	Frequencies			
	Reference	***Aster* DKQ**	**% difference**	**% tolerance**
4	35.63	35,359	– 0,760	0.8
5	68.51	67,491	– 1,427	1.5
6	109.62	108,563	– 0,964	1.
7	123.32	121,144	– 1,765	1.8
8	142.51	138,402	– 2,882	2.9
9	197.32	188,500	– 4,470	4.5
		***Aster* DSQ**		
4	35.63	35,351	– 0,782	0.8
5	68.51	67,464	– 1,527	1.6
6	109.62	108,494	– 1,027	1.1
7	123.32	121,060	– 1,832	1.9
8	142.51	138,291	– 2,961	3.
9	197.32	188,298	– 4,572	4.6
		***Aster* Q4G**		
4	35.63	36,011	1,068	1.1
5	68.51	70,795	3,336	3.5
6	109.62	114,593	4,536	4.6
7	123.32	134,899	9.39	9.4
8	142.51	142,941	4,513	4.6
9	197.32	212,045	7,463	7.5

For the problem out of membrane:

	Reference	***Aster***	**% difference**	**% tolerance**
DKQ	0.14714	0.14713	– 0,003	0.1
DSQ	0.14714	0.14714	– 0,002	0.1
Q4G	0.14714	0.14714	0.	0.1

4.4 Remarks

For the problem in inflection, the modal position of the first mode found in the band $(5.,200.)$ is the fourth, because there are three modes of solid body at frequency zero:

- modes of translation u and v in the plan,
- mode of rotation around the axis z .

5 Modeling D

5.1 Characteristics of modeling

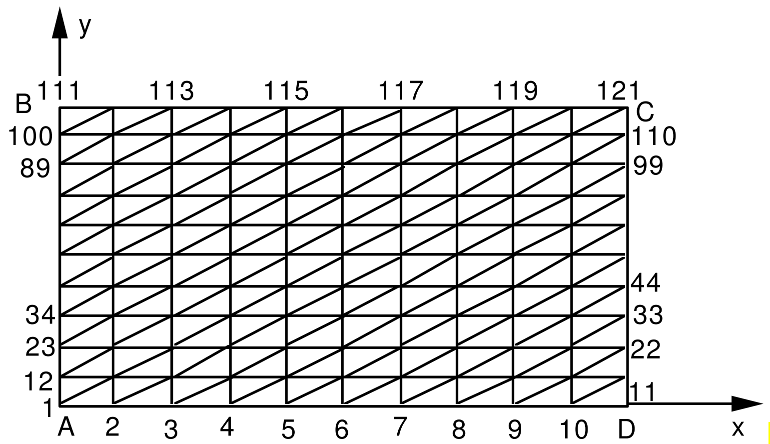
The plate in 200 meshs was cut out `TRIA3`. a modeling for the plate is used: `Q4G`.

For the problem of inflection, the boundary conditions are the following ones:

- in all the nodes of the edge: $DZ=0$

For the problem of membrane, the boundary conditions are:

- in all the nodes of the grid: $DZ=0 \quad DRX=DRY=DRZ=0$,
- in all the nodes on the sides AB and BC : $DY=0$
- at the points A, B, C, D one adds discrete elements of rigidity (direction x).



5.2 Characteristics of the grid

Many nodes: 121

Many meshs and types: 200 `TRIA3`

The points characteristic of the grid are the following:

Not A = $N1$	Not C = $N121$
Not B = $N111$	Not D = $N11$

5.3 Sizes tested and results

For the modes of inflection:

Number mode	Identification	Type of Reference	Reference	% tolerance
4	Frequencies	'ANALYTICAL'	35.63	1.5
5	Frequencies	'ANALYTICAL'	68.51	3.5
6	Frequencies	'ANALYTICAL'	109.62	3.0
7	Frequencies	'ANALYTICAL'	123.32	7.0
8	Frequencies	'ANALYTICAL'	142.51	8.5
9	Frequencies	'ANALYTICAL'	197.32	10.0

6 Summary of the results

The precision, for the modes of inflection, remains acceptable on the first six modes. Let us note however that the precision is less good than in the case of the free plate in space (test SDLS01 [V2.03.001]).

It is noticed that the treatment of the modes of solid body is suitable.

For the test out of membrane, the results are very satisfactory.