SDLS07 - Clean modes of an envelope spherical thin

Summary:

This test from guide VPCS makes it possible to validate the algorithms of the operator of search for eigenvalues CALC_MODES with the matrices of rigidity and mass corresponding to following modelings:

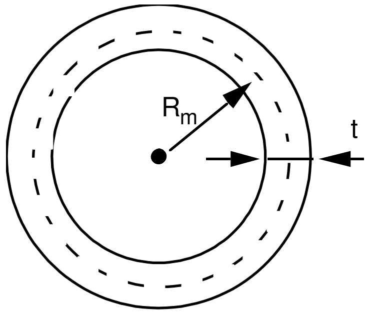
1) Three-dimensional hulls: finite elements DKT (grid of one 1/8 of sphere),
2) Finite elements $2D$ axisymmetric TRIA6 and QUAD8 (grid of a section),
3) Three-dimensional hulls: isoparametric axisymmetric finite elements SEG3 (linear grid of the section),
4) Three-dimensional hulls COQUE_3D : finite element MEC3QU9H (grid 1/8 of sphere),
5) Three-dimensional hulls COQUE_3D : finite element MEC3TR7H.

The got results are compared with the analytical solution (HAYEK) and reveal for the first six modes of the lower deviations than:

- $0{,}45\%$ for the axisymmetric elements continuous mediums,
- $0{,}20\%$ for the elements of hull DKT,
- $0{,}17\%$ for the elements of axisymmetric hulls isoparametric,
- $0{,}17\%$ for the elements COQUE_3D.

1 Problem of reference

1.1 Geometry



It is about a thin sphere, of average radius $R_m = 2.5\,m$, and thickness $t = 0.1\,m$.

1.2 Properties of materials

The material is homogeneous, isotropic, elastic linear. The elastic coefficients are: $E = 200\,000\ MPa$ and $\nu = 0.3$.

The density is constant and is worth: $\rho = 7800\,kg/m^3$.

1.3 Boundary conditions and loadings

The structure is free in space.

2 Reference solution

2.1 Method of calculating used for the reference solution

For the thin spheres ($i.t \ll R$ with i , order of the mode), the clean modes with radial and tangential displacement establish by a theory of membrane are given by [bib1] and [bib2]:

$$f_i = \frac{\lambda_i}{2\pi R}\sqrt{\frac{E}{\rho(1-\nu^2)}}$$

with $\lambda_i = \frac{1}{\sqrt{2}}\sqrt{b \pm \sqrt{b^2 - 4(1-\nu^2)(i^2+i-2)}}$ and $b = i^2 + i + 1 + 3\nu$

The theory presented by Hayek makes it possible to introduce a correction of the effect of inflection (approximation of the general theory of Wilkinson) which leads to values of λ_i function of

$a = t^2/12 R^2$
$b = i(i+1)$

and solution of:

$$\lambda_i^4 - \lambda_i^2[1+3\nu - a(1-\nu) + b(1+a\nu+ab)] + ab[b^2 - 4\mathrm{b} + 5 - \nu^2] + (1-\nu^2)[b - 2(1+a)] = 0$$

2.2 Results of reference

Eigen frequencies:

I	Eigen frequencies
2	237.25
3	282.85
4	305.24
5	324.17
6	346.76
7	376.68
8	416.
9	465.75
10	526.20

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

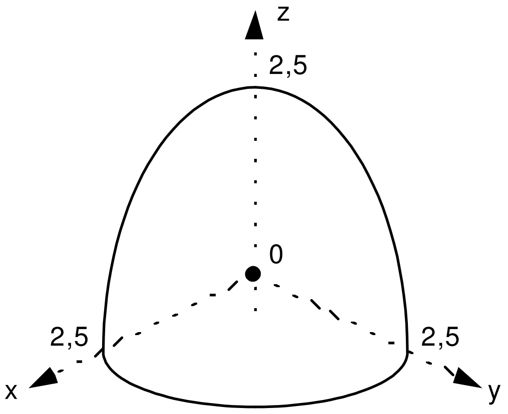
1) Card VPCS SDLS 07/89 in the Guide of Validation of the Software packages of Calculation of Structures/SFM AFNOR TECHNIQUE 1990.

2) S. HAYEK: “Vibrations of has spherical Shell in acoustic medium”, Newspaper of the Acoustical Society of America, vol. 40,2,1996, p. 342-348

3 Modeling A

3.1 Characteristics of modeling

Hulls DKT



(3 plans de symétrie)

The discretized geometry is represented above. Elements DKT are plane facets with 3 nodes. The number of the nodes on the meridian line and the equator is: 34.

The boundary conditions applied to the three borders correspond to the conditions of symmetry (displacements and blocked rotations).

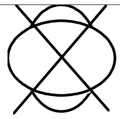
3.2 Characteristics of the grid

Many nodes: 1128

Many meshs and types: 2125 TRIA3

3.3 Sizes tested and results

(Frequencies in Hertz)

Value of parameter l of the reference solution	Reference	*Aster*	% difference
 2	237.25	237.24	– 0,005
		237.24	– 0,003
3	282.85	not obtained [§4.2]	
4	305.24	304.97	– 0,089
		304.99	– 0,080
		305.08	– 0,054
5	324.17	not obtained [§4.2]	
6	346.76	346.11	– 0,186
		346.12	– 0,185
		346.30	– 0,133
		346.38	– 0,108
7	376.68	not obtained [§4.2]	
8	416.00	414.89	– 0,266
		414.92	– 0,259
		415.16	– 0,201
		415.24	– 0,183
		415.33	– 0,161
9	465.75	not obtained [§4.2]	
10	526.20	524.34	– 0,353
		524.43	– 0,337
		524.71	– 0,283
		524.94	– 0,240
		524.97	– 0,234
		525.12	– 0,205

3.4 Remarks

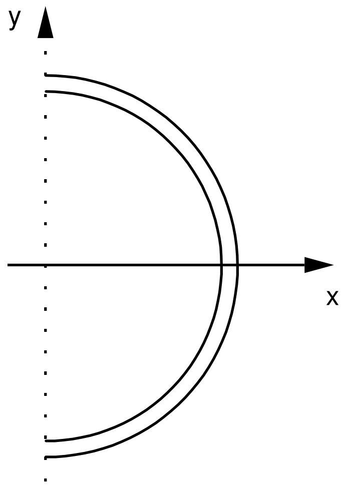
The reference solution does not give the multiplicity of the modes. One observes with calculations of the orders of multiplicity which grow with the value of the frequency.

Modes 3,5,7,9 are not obtained because of boundary conditions chosen for this model, with the three symmetry planes.

4 Modeling B

4.1 Characteristics of modeling

2D axisymmetric



No boundary condition.

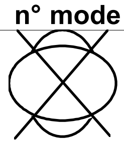
4.2 Characteristics of the grid

Many nodes: 365

Many meshs and types: 40 `QUAD8` and 80 `TRIA6`

4.3 Sizes tested and results

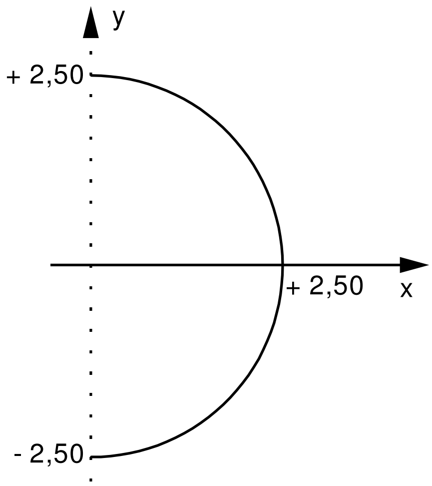
Frequencies in Hertz

Identification n° mode	Reference	*Aster*	% difference
 2	237.25	237.24	0,036
3	282.85	282.78	– 0,023
4	305.24	304.85	– 0,125
5	324.17	323.32	– 0,262
6	346.76	345.22	– 0,443
7	376.68	374.14	– 0,674
8	416.00	412.03	– 0,955
9	465.75	459.75	– 1,286
10	526.20	517.51	– 1,651

5 Modeling C

5.1 Characteristics of modeling

Hulls 1D axisymmetric



No boundary condition.

One chooses the model of Coils-Kirchhoff to describe kinematics. With the element chosen, this kinematics is obtained by penalization: one puts a great value for the coefficient `A_CIS`. In addition, one neglects the correction of metric.

5.2 Characteristics of the grid

Many nodes: 81

Many meshs and types: 40 `SEG3`

5.3 Sizes tested and results

(Frequencies in Hertz)

Identification n° mode	Reference	*Aster* `OPTION=' BANDE'` (method of the simultaneous iterations)	*Aster* `OPTION=' AJUSTE'` (method of the powers opposite)	% difference
2	237.25	237.31	237.32	0.025/0.029
3	282.85	282.77	282.78	– 0.028/-0.025
4	305.24	304.95	304.95	– 0,096
5	324.17	323.68	323.68	– 0,150
6	346.76	346.23	346.23	– 0,154

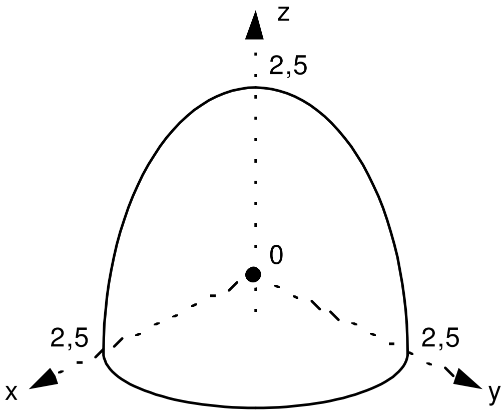
5.4 Remarks

The purpose of this test with this modeling is only to test the matrix of mass. A satisfactory variation being observed on the first six frequencies, one chose not to calculate the following ones.

6 Modeling D

6.1 Characteristics of modeling

Hulls 3D MEC3QU9H



(3 plans de symétrie)

The boundary conditions applied to the three borders correspond to the conditions of symmetry (displacements and blocked rotations).

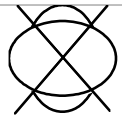
6.2 Characteristics of the grid

Many nodes: 331

Many meshs and types: 75 QUAD9

6.3 Sizes tested and results

(Frequencies in Hertz)

Identification n° mode	**Reference**	***Aster***	**% difference**
 2	237.25	237.25	0
		237.26	0,004
3	282.85	not obtained [§10.2]	
4	305.24	305.18	– 0,019
		305.19	– 0,017
		305.20	– 0,011
5	324.17	not obtained [§10.2]	
6	346.76	346.17	– 0,169
		346.19	– 0,165
		346.25	– 0,147
		346.36	– 0,114
7	376.68	not obtained [§10.2]	
8	416.00	413.81	– 0,525
		413.84	– 0,520
		413.84	– 0,518
		414.02	– 0,476
		414.09	– 0.46
9	465.75	not obtained [§10.2]	
10	526.20	520.57	– 1,071
		520.62	– 1.06
		520.64	– 1,056
		521.28	– 0,935
		521.29	– 0,933
		521.31	– 0,929

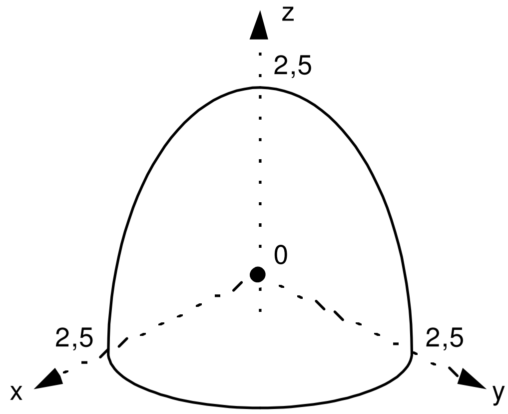
6.4 Remarks

Modes 3,5,7,9 are not obtained because of boundary conditions chosen for this model, with the three symmetry planes.

7 Modeling E

7.1 Characteristics of modeling

Hulls `3D MEC3TR7H`



(3 plans de symétrie)

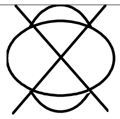
7.2 Characteristics of the grid

Many nodes: 925

Many meshs and types: 294 `TRIA7`

7.3 Sizes tested and results

(Frequencies in Hertz)

Identification n° mode	Reference	*Aster*	% difference
 2	237.25	237.25	– 0,001
		237.25	– 0,001
3	282.85	not obtained [§12.2]	
4	305.24	305.20	– 0,011
		305.22	– 0,008
		305.22	– 0,005
5	324.17	not obtained [§12.2]	
6	346.76	346.32	– 0,126
		346.43	– 0,095
		346.46	– 0,086
		346.58	– 0,051
7	376.68	not obtained [§12.2]	
8	416.00	413.91	– 0,502
		414.33	– 0,402
		414.36	– 0,394
		414.99	– 0,241
		415.14	– 0,206
9	465.75	not obtained [§12.2]	
10	526.20	520.	– 1,176
		521.02	– 0,985
		521.43	– 0,907
		522.32	– 0,738
		523.03	– 0,602
		523.77	– 0,461

7.4 Remarks

Modes 3,5,7,9 are not obtained because of boundary conditions chosen for this model, with the three symmetry planes.

8 Summary of the results

- Modeling hull DKT, here restricted with the modes having 3 symmetries compared to the plans $x=0$, $y=0$, $z=0$, provides the Eigen frequencies with an error lower than 0.4% on the first 20 modes.
- Modeling continuous medium $2D$ axisymmetric provides the Eigen frequencies with an error lower than 2%.
- Modeling COQUE_AXIS (quadratic isoparametric elements) provides the Eigen frequencies with an error lower than 0.2% on the first 5 modes (space discretization identical to the trace of the grid $2D$ axisymmetric).
- Modeling COQUE_3D degenerated (elements of thick hull MEC3QU9H, MEC3TR7H) to provide the Eigen frequencies with an error lower than 1.2% on the first 10 modes.