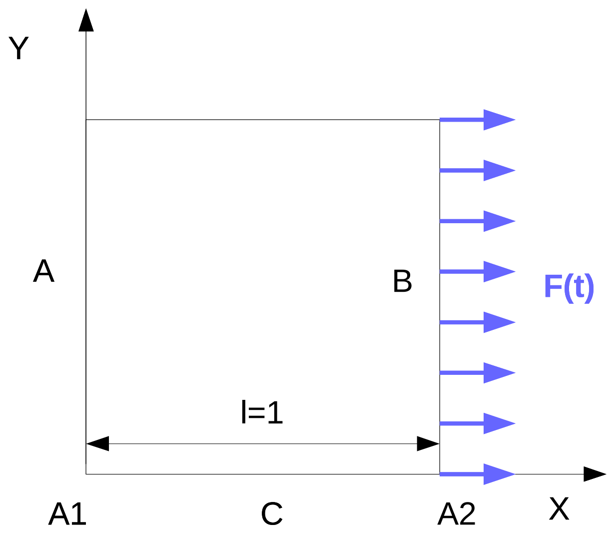
SDLS115 – Comparison with the analytical solution of a plate in traction

Summary:

This test validates the basic operation of modeling DKT for a transitory calculation with a diagram clarifies digital integration by the operator DYNA_NON_LINE. The plate is subjected to the boundary conditions corresponding to a simple traction, making it possible to find the analytically calculated answer.

1 Problem of reference

1.1 Geometry



Square plate :
Length: $l=1.0\,m$
Thickness: $e=0.1\,m$

1.2 Properties of material

Young modulus, $E=4.388\,10^{10}\,N/m^2$
Poisson's ratio, $\nu=0.0$
Density, $\rho=2500.0\,kg/m^3$

1.3 Boundary conditions and loadings

Boundary conditions:

Localisation	**Blocked components**
A1	DX, DY, DZ, DRX, DRY MARTINI, DRZ
With	DX, DZ, DRX, DRY MARTINI, DRZ
C	DZ, DRX, DRY MARTINI, DRZ

Loadings:

One applies the linear force to with dimensions B in the direction x , which depends on time like,

$$F(t)=Q_0\,E\,K\,e\cos(Kl)\sin(\omega t) \quad ,$$

where the following parameters are used:

- Q_0 ($=10^{-4}m$) - amplitude of the loading
- E – Young modulus defined above (in N/m^2)

- e – the thickness defined above (in m)
- l – the dimension of the plate defined above (in m)
- K ($=\frac{\pi}{81}$) the number of wave of the analytical solution (in m^{-1})
- ω – frequency (time 2π), related to the number of wave K , $K=\omega/c$, c

being the celerity of the waves in the structure, $c=\sqrt{\frac{E}{\rho}}$

The introduced parameter setting makes it possible to apply the loading right to obtain the analytical solution, simply given by the parameters Q_0 and K , and then by other parameters of dimensions and properties structural material.

1.4 Initial conditions

At the beginning displacements are worth zero everywhere and speeds obey spatial following,

$v_0(x, y)=\omega Q_0 \sin(K.x)$

2 Reference solution

2.1 Method of calculating

One deals here with a problem of structure (quasi) - unidimensional subjected to a force of edge, $F(t)$, where the analytical solution can be written like,

$u(x,t)=Q_0\cos(Kx)\sin(\omega t)$

In order to obtain this solution one must apply the force and the initial conditions specified above. The parameters are also commented on there.

2.2 Sizes and results of reference

It is displacement x with the node $A2$ and at the moment, $t_{max}=0.0012\,s$, which must be equal to

$u(x_{A2,t})=Q_0\cos(K\,x_{A2})\sin(\omega\,t_{max})$

the value being calculated in the data file starting from the selected values of the parameters.

2.3 Uncertainties on the solution

Exact solution.

2.4 Bibliographical references

S. Timoshenko, *Theory of the vibrations*, 1939

3 Modeling A

3.1 Characteristics of modeling

Modeling: **DKT**

3.2 Characteristics of the grid

Nodes: 16
Meshs: 9 QUAD4

3.3 Sizes tested and results

Three calculations are carried out with the operator DYNA_NON_LINE , the first with an explicit diagram, the second with an implicit scheme (HHT) slightly deadened and the third with an implicit scheme (HHT) strongly deadened.

All the values tested are taken at the moment $t_{max}=0.0012\,s$.

Calculation 1: explicit

Identification	Reference	Type of reference	Tolerance
Displacement DX in $A2$	3.51957D-05	'ANALYTICAL'	0.5 %
DEGE_ELNO, (M1 Mesh), node $A2$, Comp. EXX	3.33675621777E-05	'ANALYTICAL'	5 %
EFGE_ELNO, (M1 Mesh), node $A2$, Comp. NXX	1.4641686283587E5	'ANALYTICAL'	5 %
ENEL_ELNO, (M1 Mesh), node $A2$, Comp. TOTAL	2.442786887272105	'ANALYTICAL'	10 %
ENEL_ELNO, (M1 Mesh), node $A2$, Comp. MEMBRANE	2.442786887272105	'ANALYTICAL'	10 %
ENEL_ELGA, (M1 Mesh), point 2, comp. TOTAL	2.442786887272105	'ANALYTICAL'	10 %
ENEL_ELGA, (M1 Mesh), point 2, comp. INFLECTION	0.	'ANALYTICAL'	1E-13 (Absolute)

Identification	Value of Référence	Type of reference	Tolerance
Table OBSERVATION, Field SIEF_ELGA, Comp. SIXX, Mesh M4, EVAL_ELGA = 'MIN'	–	'NON_REGRESSION'	–

Identification	Value of Référence	Type of reference	Tolerance
ENER_ELAS, Comp. TOTAL	2.7322	' AUTRE_ASTER '	1E-3 %
ENER_ELAS, Comp. MEMBRANE	2.7322	' AUTRE_ASTER '	1E-3 %

TRAV_EXT, Comp. TRAV_ELAS	2.7322	'AUTRE_ASTER'	1E-3 %

Calculation 2: implicit slightly deadened

Identification	Reference	Type of reference	Tolerance
Displacement DX in $A2$	3.51957D-05	'ANALYTICAL'	0.05 %

Calculation 3 : implicit strongly deadened

Identification	Reference	Type of reference	Tolerance
Displacement DX in $A2$	-	'NON_REGRESSION'	-

4 Modeling B

4.1 Characteristics of modeling

Modeling: **BAR**

In this modeling, the plate square is modelled by a rectangular bar of section of one meter broad and a thickness of 0,1m.

4.2 Boundary conditions and loadings

Boundary conditions:

Localisation	**Blocked components**
A1	DX, DY, DZ
C	DX, DZ

Loadings:

The force is affected in A2, it is a nodal force which must be multiplied by the width of the plate to be equivalent to modeling A.

4.3 Characteristics of the grid

Nodes: 4
Meshs: 3 SEG2

4.4 Sizes tested and results

Three calculations are carried out, the first with DYNA_NON_LINE in explicit diagram and MASS_DIAG=' NON'(this calculation is cut into two in order to test the good transmission of the table of observation), the second with DYNA_NON_LINE in explicit diagram and MASS_DIAG=' OUI' and the third with DYNA_VIBRA without matrix of diagonal mass.

The first calculation:

Identification	**Reference**	**Type of reference**	**Tolerance**
Displacement DX in $A2$, INST = 1,2E-3	3.51957D-05	'ANALYTICAL'	0.05 %
Displacement DX in $A1$, INST = 3rd-4 (via table of observation)	0.	'ANALYTICAL'	0,1 %

Displacement DX in $A1$, INST = 9th-4 (via table of observation)	0.	'ANALYTICAL'	0,1 %

The second calculation:

Identification	Reference	Type of reference	Tolerance
Displacement DX in $A2$, INST = 1,2E-3	3.51957D-05	'ANALYTICAL'	0,5 %

The third calculation:

Identification	Reference	Type of reference	Tolerance
ECIN_ELEM , Mesh $M21$, Comp. TOTAL	9.10387D-02	'ANALYTICAL'	0,5 %

5 Summary of the results

The purpose of this test is principal to check if the combination of modeling DKT and of the operator DYNA_NON_LINE function correctly. The difference between the solution Aster and that of the reference is very weak.