SDLS503 - Vibrations of inflection of a beam sandwich

Summary:

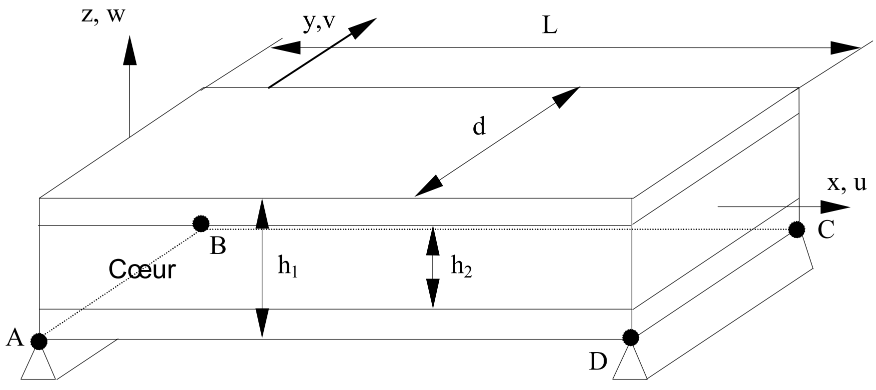
This test represents a calculation in modal analysis of a sandwich beam simply supported. This test makes it possible to validate:

- modeling finite elements DKT with meshs QUAD4 and TRIA3,
- modeling finite elements DST with meshs QUAD4 and TRIA3,
- the taking into account of rigidity in transverse shearing,
- the taking into composite material account.

The frequencies and the modes obtained are compared with an analytical reference solution. It should be noted that modeling DKT, whose formulation does not take into account transverse shearing, is not adapted to model this CAS-test. Thus, the errors are very important for modelings A and B. the results got with DST are satisfactory.

1 Problem of reference

1.1 Geometry



1.2 Properties of material

Coatings:	$E_x = 4.10^{10} Pa$	$G_{xz} = 4.10^{9} Pa$	$\nu_{xz} = 0.3$	$\rho_1 = 2000\, kg/m^3$
Heart:	$E_x = 4.10^{7} Pa$	$G_{xz} = 1.5.10^{7} Pa$	$\nu_{xz} = 0.3$	$\rho_2 = 50\, kg/m^3$

Coefficient of shearing K : $1/K = 110.8$

The Poisson's ratios are identical: $\nu_{xz} = \nu_{xy} = \nu_{yz}$

1.3 Boundary conditions and loadings

The beam rests simply on with dimensions ones AB and CD .

1.4 Initial conditions

Without object

2 Reference solution

2.1 Method of calculating used for the reference solution

Calculation is carried out starting from the relations of dynamic balance and behavior [bib2] pointed out Ci - afterwards:

$$\frac{\partial M_x}{\partial X}+T_y=\langle\rho\, I\rangle\frac{\partial^2\theta}{\partial t^2} \qquad \frac{\partial T_y}{\partial X}=\langle\rho\, S\rangle\frac{\partial^2 v}{\partial t^2}$$

$$M_z=\langle E\, I\rangle\frac{\partial\theta_z}{\partial X} \qquad T_y=K\langle G\, S\rangle\frac{\partial v}{\partial X}-\theta_z$$

These relations make it possible to write the equation of the movement of dynamic inflection transverse $v(x,t)$. One obtains the equation at the Eigen frequencies after having associated the boundary conditions.
The equation at the Eigen frequencies is written:

$$\sin(X_2)=0 \quad \text{with} \quad X_2=\left[\bar{\omega}^2\frac{(1+a)}{2}+\sqrt{\bar{\omega}^2\left(\bar{\omega}^2\left(\frac{1-a}{2}\right)^2+\frac{1}{r^2}\right)}\right]^{1/2}$$

and

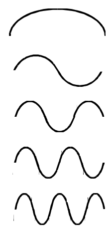
$$\bar{\omega}^2=\frac{\langle\rho\, I\rangle\omega^2 l^2}{\langle E\, I\rangle} \;;\; \bar{r}^2=\frac{\langle\rho\, I\rangle}{\langle\rho\, S\rangle l^2} \;;\; a=\frac{\langle\rho\, S\rangle\langle E\, I\rangle}{K\langle\rho\, I\rangle\langle G\, S\rangle}$$

The solutions of the equation at the Eigen frequencies are written then: $X_2=n\pi \quad (n=1,2,3,\ldots)$

2.2 Results of reference

the first 5 frequencies and clean modes of inflection associated.

- Frequency mode 1: $64.476\,Hz$
- Frequency mode 2: $131.918\,Hz$
- Frequency mode 3: $198.734\,Hz$
- Frequency mode 4: $265.383\,Hz$
- Frequency mode 5: $331.963\,Hz$



2.3 Uncertainties on the solution

The reference solution is calculated within the framework of the assumptions of the theory of the beams [bib2]: $\sigma_y=\sigma_z=0$.

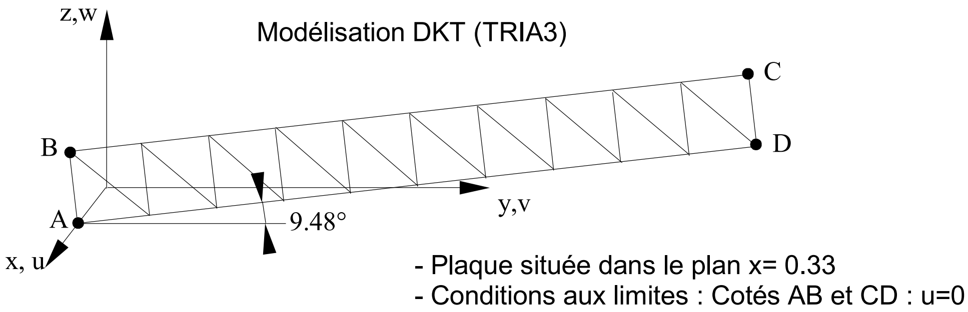
2.4 Bibliographical references

1) VPCS: Software package of composite structural analysis; Examples of validation. Review of the composites and advanced materials, Volume 5 - number except series 1995 - Hermes Edition.

2) CIEAUX J.M.: Dynamic inflection of the composite beams with orthotropic phases; Validity of the quasi-static field, thesis of the university Paul Sabatier Toulouse III, 1988.

3 Modeling A

3.1 Characteristics of modeling



3.2 Characteristics of the grid

Many nodes: 22
Number of meshs and type: 20 `TRIA3`

3.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
Frequency mode 1	64,476	277,449	330.
Frequency mode 2	131,918	1105.83	738.
Frequency mode 3	198,734	2473.80	1.14E3
Frequency mode 4	265,383	4363.97	1.54E3
Frequency mode 5	331,963	6753.904	1.93E3

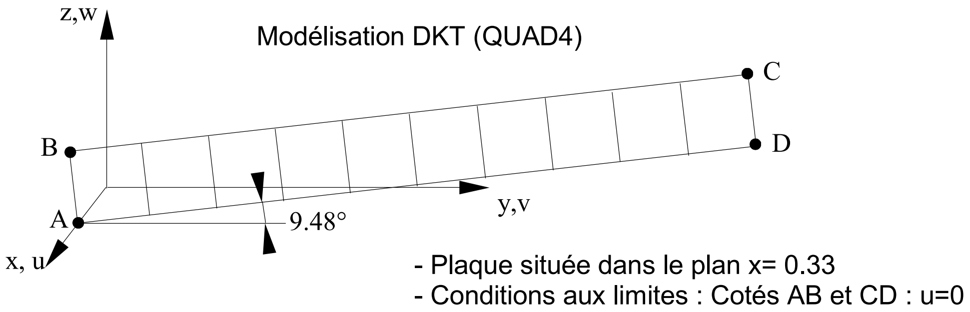
3.4 Remarks

In the table of results, we deferred the frequencies whose modes are identical to the modes of reference.

- the effects of transverse shearing are neglected in modeling `DKT`,
- the Aster results are much higher than the results of reference,
- appearance of a mode of membrane enters modes 2 and 3 and between modes 5 and 6 of reference.

4 Modeling B

4.1 Characteristics of modeling



4.2 Characteristics of the grid

Many nodes: 22
Number of meshs and type: 10 QUAD4

4.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
Frequency mode 1	64,476	277,788	331
Frequency mode 2	131,918	1111.225	742.
Frequency mode 3	198,734	2500.930	1.16E3
Frequency mode 4	265,383	4449.073	1.52E3
Frequency mode 5	331,963	6960.324	2.00E3

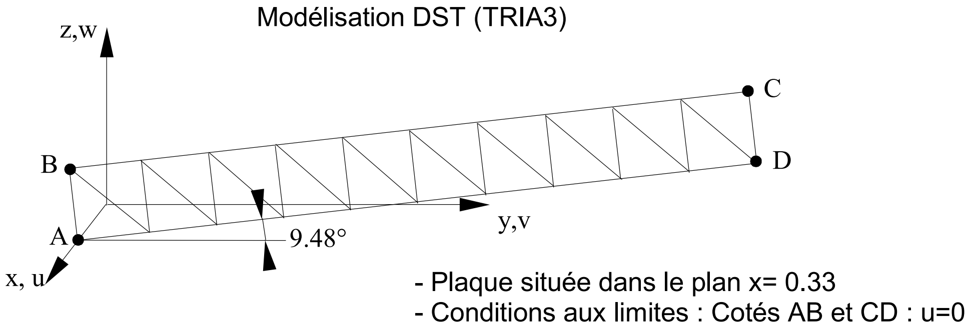
4.4 Remarks

In the table of results, we deferred the frequencies whose modes are identical to the modes of reference.

- the effects of transverse shearing are neglected in modeling DKT,
- the Aster results are much higher than the results of reference,
- appearance of a mode of membrane enters modes 2 and 3 and between modes 5 and 6 of reference.

5 Modeling C

5.1 Characteristics of modeling



5.2 Characteristics of the grid

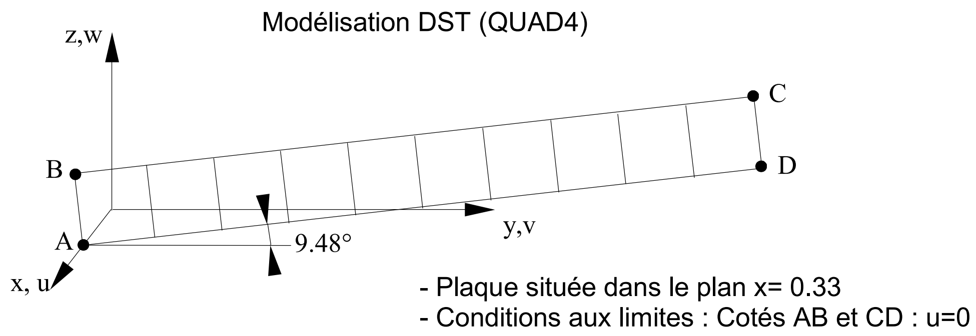
Many nodes: 22
Number of meshs and type: 20 TRIA3

5.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
Frequency mode 1	64,476	64,573	0,150
Frequency mode 2	131,918	133,987	1,568
Frequency mode 3	198,734	206,046	3,679
Frequency mode 4	265,383	282,875	6,591
Frequency mode 5	331,963	365,919	10,229

6 Modeling D

6.1 Characteristics of modeling



6.2 Characteristics of the grid

Many nodes: 22
Number of meshs and type: 10 `QUAD4`

6.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
Frequency mode 1	64,476	64,595	0,184
Frequency mode 2	131,918	131,495	-0,320
Frequency mode 3	198,734	196,861	-0,942
Frequency mode 4	265,383	260,247	-1,935
Frequency mode 5	331,963	320,409	-3,480

7 Summary of the results

Modeling DKT is not adapted to model this CAS-test, the errors are very important. The formulation DKT does not take into account transverse shearing contrary to modeling DST. For this kind of example, where the structure consists of a composite and relatively thick material ($h/L=0.1$), it is preferable to use modeling DST.

Results got with DST are:

- satisfactory for the first 3 frequencies with the mesh TRIA3 and for the first 5 frequencies for the mesh QUAD4 with a better precision for the mesh QUAD4,
- the error of 10% for the 4ième and 5ième frequency with the mesh TRIA3 is significant. A finer grid should make it possible to improve the results by having a better representation of the last modes.