SDLS505 - Buckling of a cylindrical envelope under external pressure

Summary:

This test represents a calculation of stability of a thin cylindrical envelope stopped at its ends subjected to an external pressure and an axial pressure. One calculates the critical loads leading to the elastic buckling of Euler. The geometrical matrix of rigidity used in the resolution of the problem to the eigenvalues is that which is due to the initial constraints.

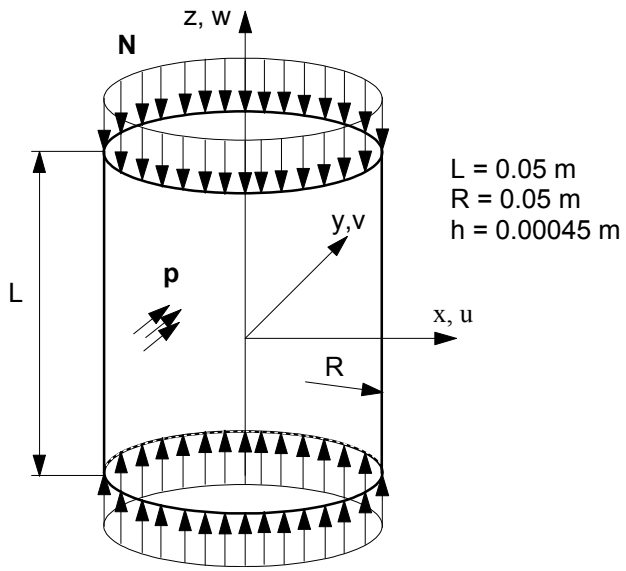
It makes it possible to validate modeling finite elements:

- COQUE_3D with the meshs TRIA7 and QUAD9.
- DKT with the meshs TRIA3 and QUAD4.

The critical load and the clean mode obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



The symmetry of the problem makes it possible to model a half rolls length $L/2$, stopped with the one of its ends, with conditions of symmetry specific to the lower edge.

1.2 Properties of material

The properties of material constituting the plate are:

$E=2.10^{11}Pa$ Young modulus
$\nu=0.3$ Poisson's ratio

1.3 Boundary conditions and loadings

- Loading:
  - pressure uniformly distributed of $p_{cr}=1.523\,10^{6}Pa$ on the cylindrical part. This pressure corresponds to the value of the critical load,
  - effort distributed on the contour of the bottom $N=0.5\times R\cdot p_{cr}=3.8075\,10^{4}N/m$.

1.4 Initial conditions

Without object

2 Reference solution

2.1 Method of calculating used for the reference solution

The critical pressure is given in [bib1] or [bib2] by the following expression:

$$P_{cr}=\frac{Eh}{R}\frac{1}{\left(n^2+\frac{b^2}{2}\right)}\left[\frac{1}{\left(\frac{n^2}{b^2}+1\right)^2}+\frac{h^2}{12R^2(1-\nu^2)}(n^2+b^2)^2\right]$$

with $b=\frac{\pi R}{L}$

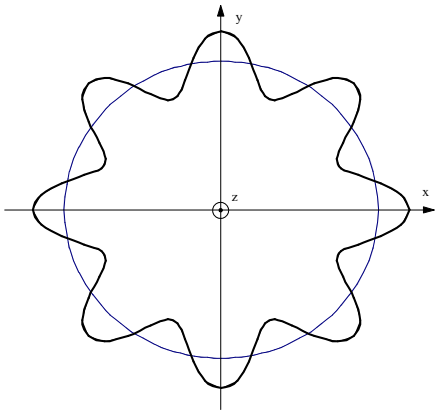
n represent the number of circumferential modes

This formula is valid if $N=0.5\,R\,p_{cr}$.

2.2 Results of reference

Pour les caractéristiques géométriques données, la pression critique est minimum pour un nombre n=8 et vaut p_{cr} = 1.523 10^6Pa.

Le nombre n est obtenu à partir d'un abaque.



2.3 Uncertainties on the solution

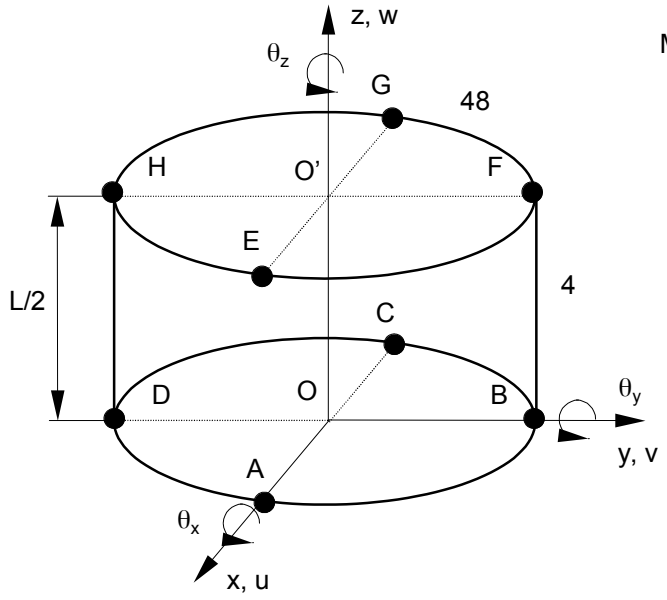
Analytical solution

2.4 Bibliographical references

1) S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, page 500, second edition, DUNOD 1966.
2) BO O. ALMROTH, D.O. BRUSH: Buckling of bars, punts and shells, page 173, Mc Graw-Hill, New York, 1975.

3 Modeling A

3.1 Characteristics of modeling



Modélisation COQUE_3D (TRIA7)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

3.2 Characteristics of the grid

Many nodes: 2464
Many meshs and types: 834 TRIA7

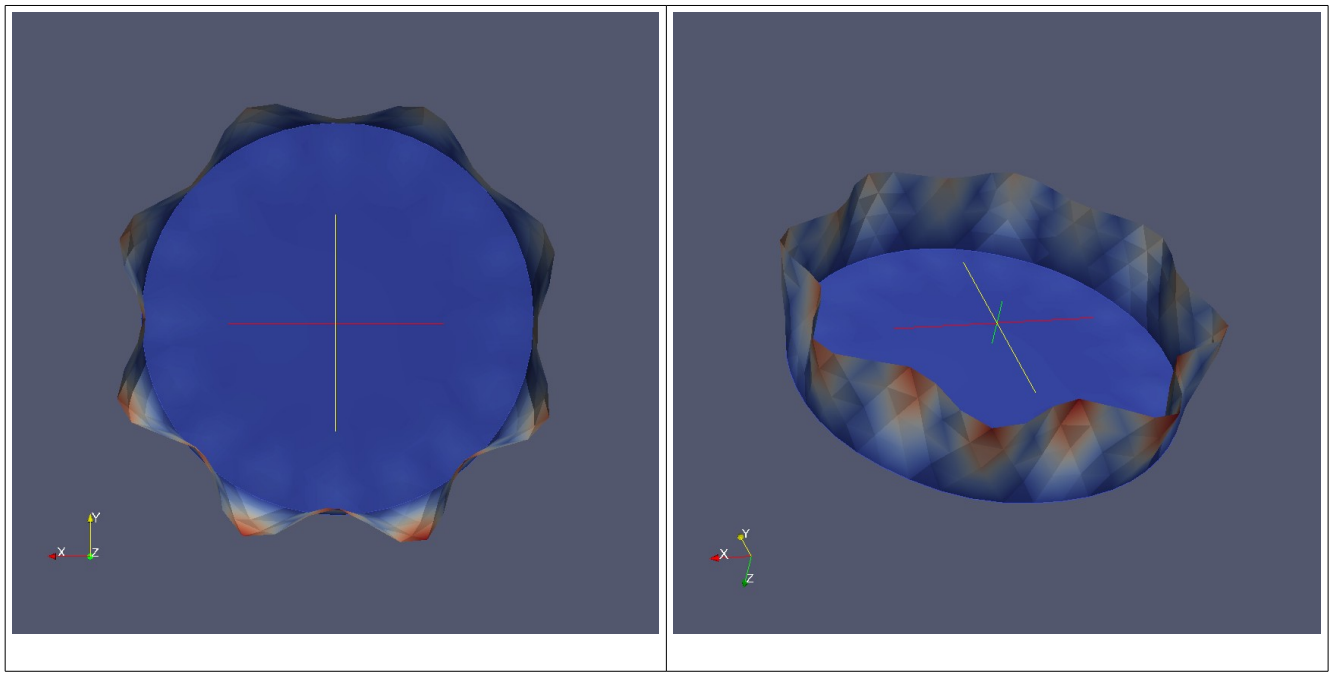
3.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
Pressure criticizes ($n=8$)	– 1,523 10^6 Pa	– 1.6862 10^6 Pa	10,715

3.4 Remarks

The critical pressure is tested without difficulty.
On the other hand, it is difficult to test the mode of buckling because it is a double mode.

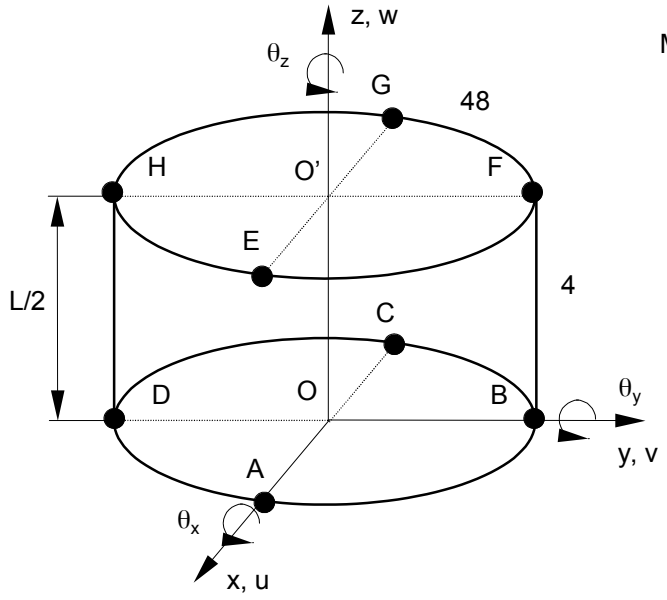
The validation of this mode of buckling is made in the following way:
1) The coordinates of the grid are very slightly modified (of 1.e-8) so that the mode associated with n=8 is the same one, with each execution, on all the machines where the test is carried out. (One die-symmetrizes a little the problem).
2) One validates visually that the modes of buckling 7 and 8 have well a form with 8 lobes (see images below).
3) It is checked (with a test of not-regression) that mode 8 remains the same one during evolutions of the code.



Note:
For 4 modelings of this test, the smoothness of the grid is insufficient for representing the form of the mode well. It is what explains why the mode observed is not rigorously symmetrical by rotation of 45 degrees.
It was checked that symmetry was improved with a finer grid. But, to preserve a fast test, the coarse grid is preserved.

4 Modeling B

4.1 Characteristics of modeling



Modélisation COQUE_3D (QUAD9)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

4.2 Characteristics of the grid

Many nodes: 1802
Many meshs and types: 472 QUAD9

4.3 Sizes tested and results

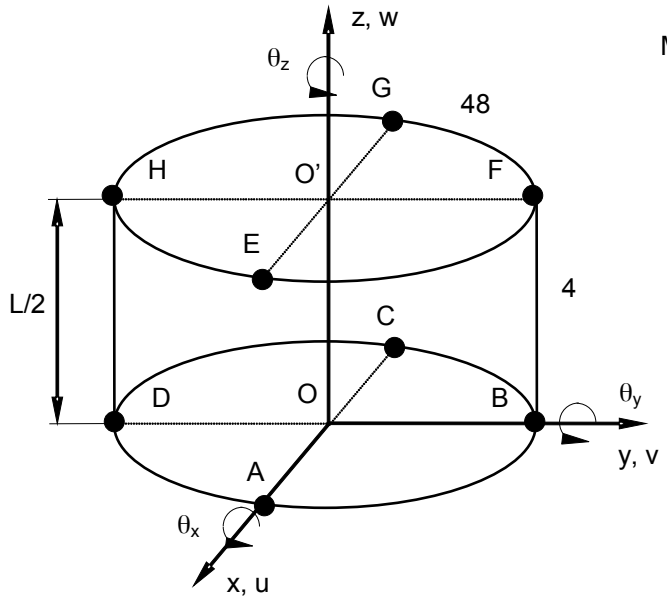
Identification	Reference	*Aster*	% difference
Pressure criticizes ($n=8$)	– 1,523 10^6 Pa	1.5576 10^6 Pa	2,272

4.4 Remarks

The mode of buckling is tested in the same way that for modeling A.

5 Modeling C

5.1 Characteristics of modeling



Modélisation DKT (TRIA3)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

5.2 Characteristics of the grid

Many nodes: 418
Many meshs and types: 786 TRIA3

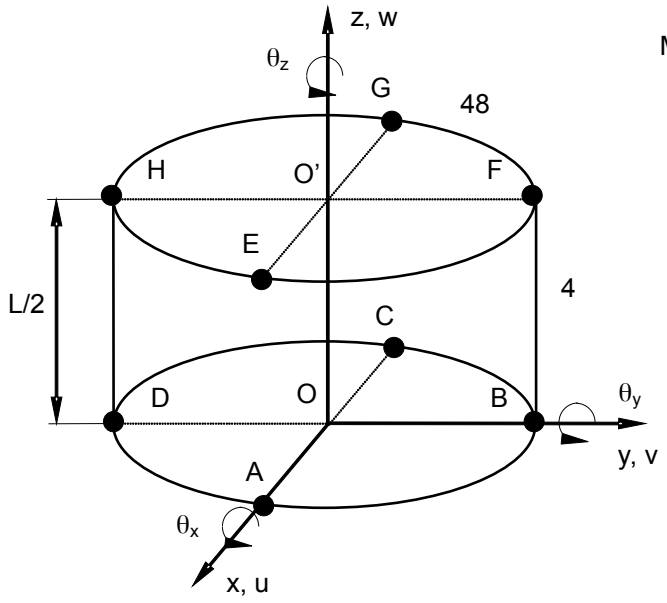
5.3 Sizes tested and results

Identification	Type of reference	Reference	% Tolerance
Pressure criticizes ($n=8$)	'ANALYTICAL'	– 1,523 10^6 Pa	12.5

The mode of buckling is tested in the same way that for modeling A.

6 Modeling D

6.1 Characteristics of modeling



Modélisation DKT (QUAD4)

Conditions de symétrie :
- Contour ABCD : w = 0

Conditions limites :
- Point O' : u = v =0
- Point E : v = 0

6.2 Characteristics of the grid

Many nodes: 449
Many meshs and types: 424 QUAD4

6.3 Sizes tested and results

Identification	Type of reference	Reference	% Tolerance
Pressure criticizes ($n=8$)	'ANALYTICAL'	– 1,523 10^6 Pa	12.5

The mode of buckling is tested in the same way that for modeling A.

7 Summary of the results

Modeling COQUE_3D

The got results are satisfactory. Uncertainties on the critical pressure do not exceed:

- 10.71% with meshs TRIA7,
- 2.3% with meshs QUAD9.

The modal deformation obtained corresponds well to the expected circumferential mode: $n=8$ for two modelings.

It is noted that modeling COQUE_3D with meshs QUAD9 is more precise than modeling COQUE_3D with meshs TRIA7.

Modeling DKT

The got results are satisfactory for elements of plates. Uncertainties on the critical pressure do not exceed 12.5% some is the mesh (QUAD4, TRIA3).

The modal deformation obtained corresponds well to the expected circumferential mode: $n=8$ for two modelings.

This test made it possible to test modelings COQUE_3D and DKT in linear buckling of Euler of a mean structure subjected to an external pressure.