Manuel de Validation
V2.04 booklet: Linear dynamics of the voluminal structures
V2.04.111 document

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

SDLV111 - Homogenisation of a network beams in an incompressible fluid

Summary:

Test in modal analysis, being used to validate the elements of modeling 3D_FAISCEAU : hexahedron with 8 nodes or hexahedron with 20 nodes. These elements represent the homogenized medium of a network of beams bathing in an incompressible fluid, initially at rest.

One tests the Eigen frequencies of the beams of the medium homogenized without or with fluid.
One calculates the mass, the position of the centre of gravity as well as inertias in this point for the cases to have or without fluid.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

1 Problem of reference

1.1 Geometry

One considers a periodic network of 4×4 beams [fig 1.1-a]. The period of the field is εY. The figure [fig 1.1-b] represents an enlarging of $1/\varepsilon$ period. Each beam is right of square section.

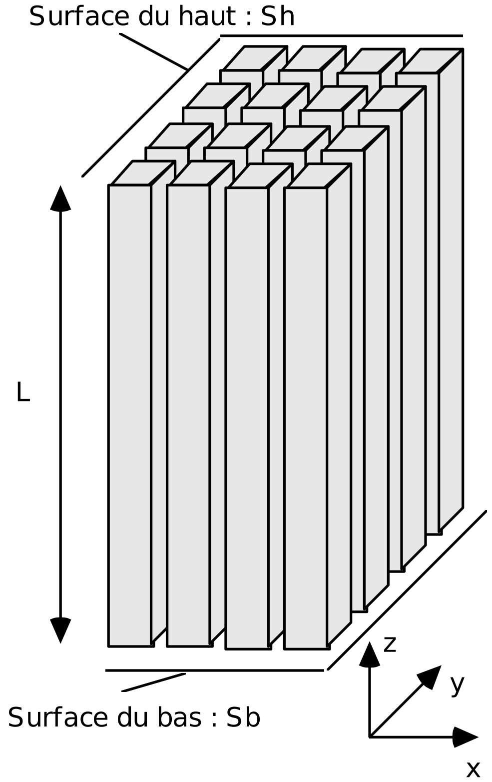

Figure 1.1-a: Geometry of the heterogeneous medium - Beams without fluid

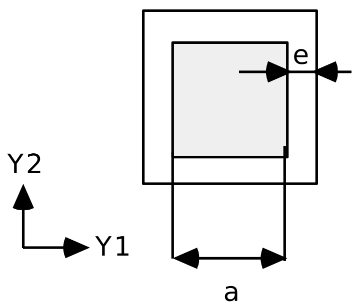

Figure 1.1-b: Cell of reference Y **- Enlarging of** $\frac{1}{\varepsilon}=10$

- Characteristics of the period:
    - Dimensions:

$\varepsilon Y=(0.21\,m, 0.21\,m)$

$a=1.5\,m$

$e=0.3\,m$

Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

- Characteristics of each beam:
  - Section:
    $A=(\varepsilon\times a)^2=(0.1\times1.5)^2=0.0225\,m^2$
  - Length:
    $L=4.1\,m$
  - Moment of inertia of inflection:
    $I_x=I_y=(\varepsilon\times a)^4/12\,m^4$

1.2 Material properties

Isotropic linear elastic material:

$E=10^9\,Pa$
$NU=0.3$

Densities:

Beam:

$Rho=7641\,kg/m^3$

Fluid:

$Rho=0\,kg/m^3$ (case without fluid)
$Rho=1000\,kg/m^3$ (case with fluid)

1.3 Correct terms

The correct terms are calculated on the cell of reference Y [fig 1.1-b].

B_T= $0.79\,m^2$
B_N= $0.79\,m^2$
B_TN= $0\,m^2$
A_FLUI= $2.16\,m^2$
A_CELL= $2.25\,m^2$
COEF_ECHELLE=10

1.4 Boundary conditions and loadings

Case without fluid:

Surface of bottom Sb : embedding
All the degrees of freedom are blocked.

Surface top Sh : embedding
All the degrees of freedom are blocked.

Case with fluid:

Surface of bottom Sb : embedding
All the degrees of freedom are blocked.

Surface top Sh : support plan (bilateral connection)
All rotations are blocked.
Longitudinal displacement DZ is blocked.
All nodes of Sh have same transverse displacement DX and same normal displacement DY .

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

2 Reference solution

2.1 Method of calculating used for the reference solution

Case without fluid:

Let us consider the heterogeneous field described with [§1] in absence of the fluid. It is supposed that the beams respect the assumptions of modeling of a right beam of Euler Bernoulli. Since the boundary conditions applied to the whole of the beams are the same ones as for each one of them, one can bring back the research of the Eigen frequencies of the unit to that of only one beam.

The following problem is thus studied:

That is to say an bi--embedded beam [fig 2.1-a] in the same way characteristic geometrical and material that beams of the heterogeneous medium. One notes A the surface of the section, L its length, and I moment of inertia of inflection.

By the method of rigidity dynamic one shows that such a beam admits double frequencies of the form:

$$f_i=\frac{\lambda_i^2}{2\pi L^2}\left[\frac{EI}{\rho A}\right]^{\frac{1}{2}}$$

$\lambda_i=(2i+1)\pi/2 \quad i=1,2,...$ for the second case of boundary conditions: [fig 2.1-a].

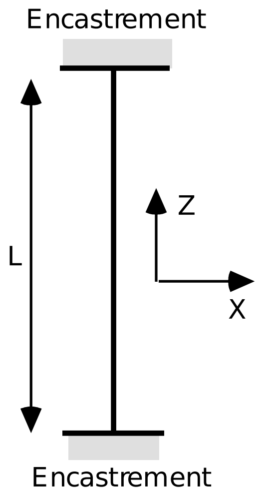

Figure 2.1-a

The field contains N beams independent between them (not from boundary conditions which couples displacements of two different beams), It results from it that the multiplicity of the frequencies is equal to $2\mathrm{N}$ (2 modes of inflections by beams).

For the homogenized medium discretized by the finite elements hexahedron with 8 nodes or hexahedron with 20 nodes, the number N must be replaced by the number of straight lines parallel with the axis of the beams.

Case with fluid:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

The case with fluid is more difficult to solve analytically: no analytical result was found until the drafting of this case test. The results of reference which one established thus come from one digital resolution by finite elements of the complete heterogeneous problem. One used for this fact version 3.6.2 of *Code_Aster*.

Each beam is represented in the grid by its average fibre modelled by `POU_D_E` (right beam of Euler). For all the beams, one binds each node of average fibre to the nodes of side surface, located in the same transverse section as the node in question, by `LIAISON_SOLIDE`. The fluid interface beam is modelled by `FLUI_STRU` who translates the continuity normal speeds to the walls. The fluid, was to be perfect incompressible, one deduced his modeling from that of the compressible true fluid `3D_FLUIDE` by removing the contribution of the pressure.

Boundary conditions imposed on the fields [§1.3], and especially the relation which couples the displacement of all the beams with the level of Sh , reveal two kinds of clean modes of the structure:

Modes of units: all the beams become deformed in the same way and top surfaces it admits a displacement not no one.

Local modes: they correspond to modes of embed-embedded beams. Surfaces top thus admits a null displacement. None of these modes can correspond to an overall mode.

The action of the fluid results in an effect of added mass and thus a lowering of the frequencies compared to the case without fluid. It also causes, in the case of the local modes, to spread out the frequency spectrum associated. In the case without fluid one saw that this spectrum was concentrated in only one frequency of vibration.

2.2 Results of reference

Value of the Eigen frequencies.
For the mass and inertias in the centre of gravity:

- In absence of fluid, the mass is determined by the product of the volume occupied by the beams and the density of these elements: $MASSE_{solide}=\rho_{poutre}\times Vol_{poutres}$ where $Vol_{poutres}$ is determined by the product amongst beam, of the surface of the section and length of the beam. Starting from the data defined previously, one can calculate $MASSE_{solide}$: one obtains thus $MASSE_{poutres}=7641\times 16\times(1.5/10)^2\times 4.1=11278.116\ kg$.
- The mass being distributed uniformly in volume (because of the position of the beams in volume), the centre of gravity is thus localised in its center, namely at the point of coordinates $(0.42, 0.42, 2.05)$. This result is confirmed analytically by the calculation of the following integrals:

$$X_G=Y_G=\frac{1}{Vol}\int_V x\,.\,\rho\ dV \text{ and } Z_G=\frac{1}{Vol}\int_V z\,.\,\rho\ dV$$

- Inertias in the centre of gravity G are calculated by: . One obtains analytically:

$$I_{xx}(G)=\int_V\left(\left(y-y_G\right)^2+\left(z-z_G\right)^2\right)\rho\,dV\ ;\ I_{yy}(G)=\int_V\left(\left(x-x_G\right)^2+\left(z-z_G\right)^2\right)\rho\,dV\ ;$$

$$I_{zz}(G)=\int_V\left(\left(y-y_G\right)^2+\left(x-x_G\right)^2\right)\rho\,dV$$

One obtains analytically:

$$I_{xx}(G)=I_{YY}(G)=\frac{8\rho}{3}\left(4\times 0.15^2\times 2.05^2+4.1\times 0.15\times\left(0.39^3-0.24^3+0.18^3-0.03^3\right)\right)=16441.61\ ;$$

$$I_{zz}(G)=\frac{16\rho\times 4.1\times 0.15}{3}\left(0.39^3-0.24^3+0.18^3-0.03^3\right)=1285.71$$

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

- In the presence of fluid, the total mass corresponds to the sum of the solid mass and the fluid mass. Knowing total volume, one determines the volume occupied by the fluid:

$$Vol_{fluide}=Vol-Vol_{poutres}=0.84^2\times 4.1-16\times(1.5/10)^2\times 4.1=1.41696\,m^3 \quad ;$$

$$MASSE_{fluide}=1000\times 1.41696=1416.96\,kg \quad ;$$

$$MASSE_{totale}=MASSE_{solide}+MASSE_{fluide}=12695.076\,kg$$

One from of deduced the density from the element (with fluid):

$$\rho=\frac{MASSE_{totale}}{Vol}=4388.265\,kg/m^3$$

- The centre of gravity remains unchanged with or without fluid.

- Inertias in the centre of gravity G are:

$$I_{xx}(G)=I_{YY}(G)=\frac{2\,\rho\times 0.84\left(4.1\times 0.42^3+0.84\times 2.05^3\right)}{3}=18530.155 \quad ;$$

$$I_{zz}(G)=\frac{2\,\rho\times 0.84\times 4.1\times 2\times 0.42^3}{3}=1492.941$$

2.3 Bibliographical references

1) Walter D. Pilkey: "Formulated for Stress, Strain and Structural Matrices", A Wiley-Interscience Publication JOHN WILEY & SOUNDS, Inc. Edition 1994.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

3 Modeling A

3.1 Characteristics of modeling

Modeling `3D_FAISCEAU`

Boundary conditions:

Case without fluid:

```
      DDL_IMPO: (  GROUP_MA: Sb DX: 0  DY: 0  DZ: 0  DRX: 0 DRY MARTINI: 0
DRZ: 0
                   GROUP_MA: HS DX: 0  DY: 0  DZ: 0  DRX: 0 DRY MARTINI: 0
DRZ: 0
                   N1 NODE: PHI: 0                                   )
```

Case with fluid:

```
      DDL_IMPO: (  GROUP_MA: Sb DX: 0  DY: 0  DZ: 0  DRX: 0 DRY MARTINI: 0
DRZ: 0
                   GROUP_MA: HS DZ: 0  DRX: 0  DRY MARTINI: 0 DRZ: 0
                   N1 NODE: PHI: 0                                   )
      LIAISON_UNIF: (  GROUP_MA: HS
                     DDL: 'DX'   )
      LIAISON_UNIF: (  GROUP_MA: HS
                     DDL: 'DY'   )
```

3.2 Characteristics of the grid

Grid of the homogenized medium used, for the two cases: with or without fluid, is represented by [fig 3.2-a].

It comprises 48 meshs `HEXA8`.

The grid contains 9 straight lines parallel with average fibre of each beam.

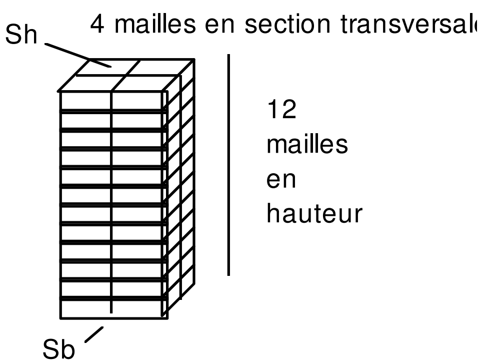

Figure 3.2-a: grid

3.3 Values tested

Case without fluid:

Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Sequence number	Size and unit	Reference
1.2 and 6	frequency (Hz)	3.3333
19 and 20	frequency (Hz)	9.2584

Case with fluid:

Sequence number	Size and unit	Reference
1 and 2	frequency (Hz)	0.6908
19 and 20	frequency (Hz)	3.7871

Size and unit	Reference
Mass (kg)	12695.076
Inertia I_{xx} in G	18530.155
Inertia I_{zz} in G	1492.941

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4 Modeling B

4.1 Characteristics of modeling

Modeling 3D_FAISCEAU

Boundary conditions:

Case with fluid:

```
      DDL_IMPO: (  GROUP_MA: Sb  DX: 0  DY: 0  DZ: 0  DRX: 0 DRY MARTINI: 0
DRZ: 0
                   GROUP_MA: HS  DZ: 0  DRX: 0 DRY MARTINI: 0 DRZ: 0
                   N1 NODE: PHI: 0                                  )
      LIAISON_UNIF: (  GROUP_MA: HS
                     DDL: 'DX'   )
      LIAISON_UNIF: (  GROUP_MA: HS
                     DDL: 'DY'   )
```

4.2 Characteristics of the grid

Grid of the homogenized medium used, for the two cases: with or without fluid, is represented by [fig 5.2-a].

It comprises 48 meshs HEXA20.

The grid contains 9 straight lines parallel with average fibre of each beam.

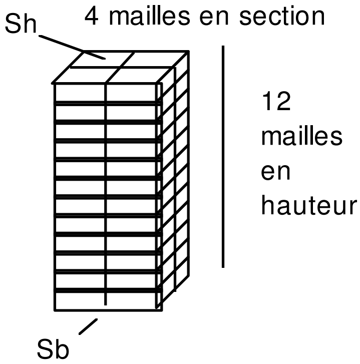

Figure 5.2-a: grid

4.3 Values tested

Case with fluid:

Sequence number	Size and unit	Reference
1 and 2	frequency (Hz)	0.6908
19 and 20	frequency (Hz)	3.7871

Size and unit	Reference
Mass (kg)	12695.076
Inertia I_{xx} in G	18530.155
Inertia I_{zz} in G	1492.941

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

5 Summary of the results

The results show the good modal behavior of the elements of modeling `3D_FAISCEAU` in inflection, absence of the fluid. It show also very a good agreement of the frequencies of the overall modes with calculation *Aster* into heterogeneous, when there is fluid.

For the frequencies of the overall modes, one does not observe differences between a grid `HEXA8` and `HEXA20` (this is not true for the other modes).

Digital results of the masses and inertias obtained by *Code-aster* are very close to the analytical results (error $<6.10^{-6}$).

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2019 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)