SDLV121 - Wave planes shearing in an elastic column

Summary

One tests the application of a loading in transient in the form of a plane wave thanks to the elastic paraxial elements of order 0, in 3D and 2D. One applies this loading to an elastic solid mass occupying a half space and which one models a column. This column is supposed to be infinite in its lower part and levels in her upper part on the level of surface of the free half space left. One observes the propagation of the incidental wave, his reflection on the free surface of the solid mass and his absorption by the paraxial elements at the lower end of the column.

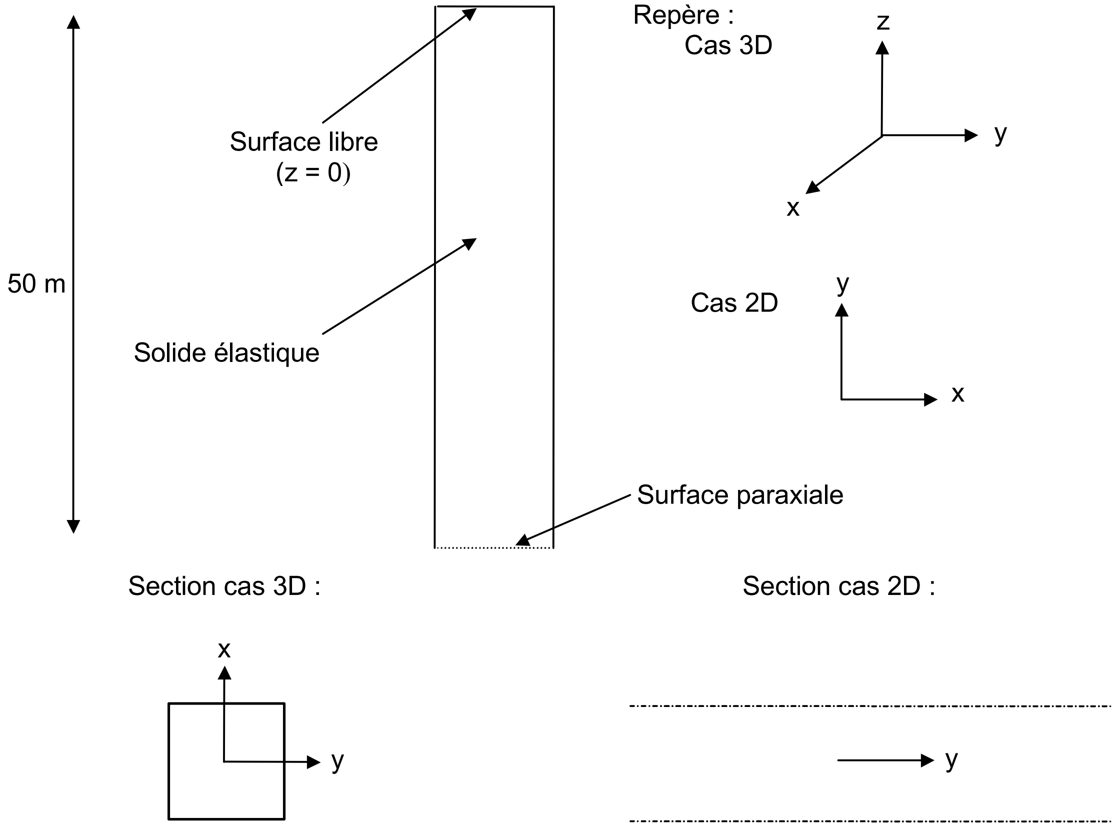
One tests successively the two direct transitory operators of *Code_Aster*, namely `DYNA_VIBRA` and `DYNA_NON_LINE`.

1 Problem of reference

1.1 Geometry

The system considered in the case 3D is that of a homogeneous elastic ground occupying the half space $z<0$. The plan $z=0$ is left free. One models of this ground a vertical column, presumedly infinite in his lower part and levelling at free surface at his higher end. One places the paraxial elements on lower surface, to translate the infinite character of the column and to apply the loading by plane wave. In the case 2D, the principle is identical, with a very broad column which one models only one vertical section (see diagram).

Moreover, the direction of vibration is the axis of *there* in the case 3D. It is the axis x in the case 2D.



1.2 Properties of materials

Elastic solid mass: floor covering

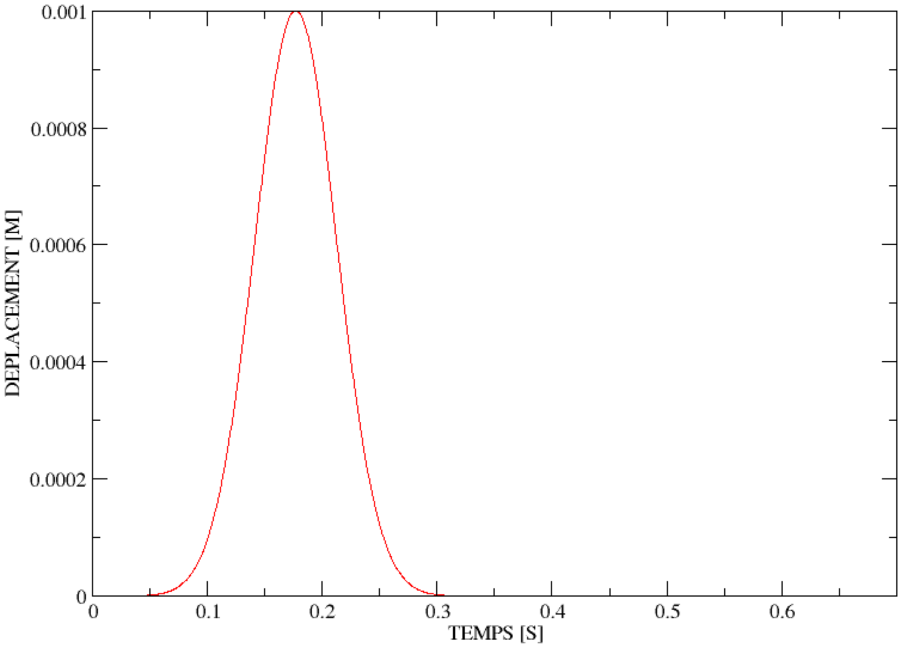
Density:	$1900\,kg.m^{-3}$
Young modulus:	$4{,}44.10^{8}\,Pa$
Poisson's ratio:	$0{,}2$

1.3 Boundary conditions and loadings

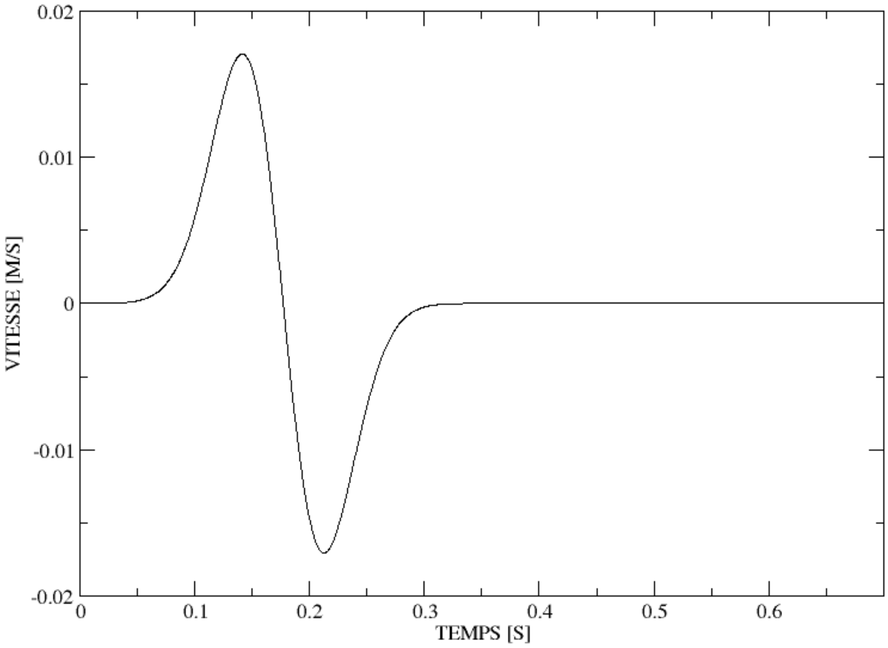
One is interested in the movement 1D of the column under the exiting action of a wave planes vertical. To identify this movement, one forces all the nodes of the same horizontal section to have same displacement.
In this configuration, the loading by plane wave comprises the following characteristics:

- Direction: $(0.,0.,1.)$
- Type of wave: SH

- Signal: function speed data below (which is used as entry with calculation), and displacement corresponding:



The maximum in displacement of the wave front in the column is of $1\,mm$ with $t_m = 0{,}177\,s$.



1.4 Initial conditions

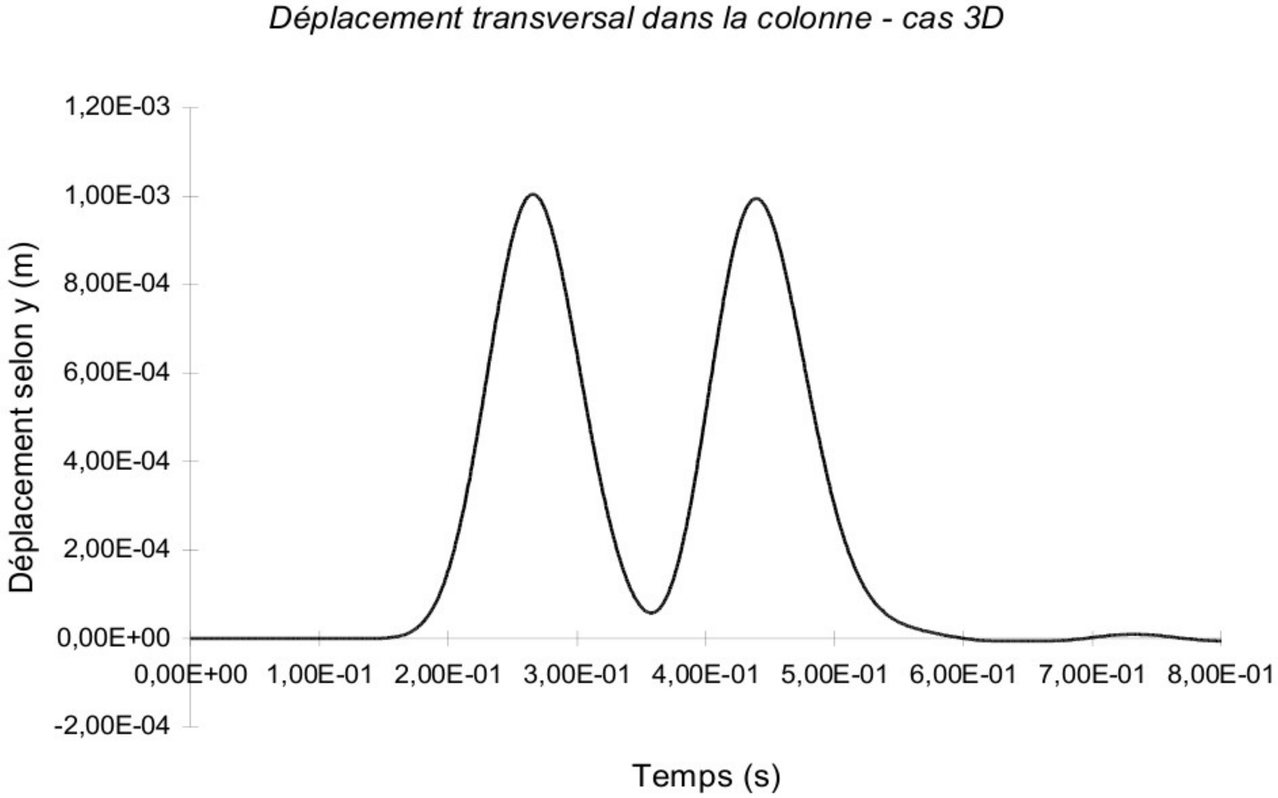
Displacement is null in all the column at the initial moment.

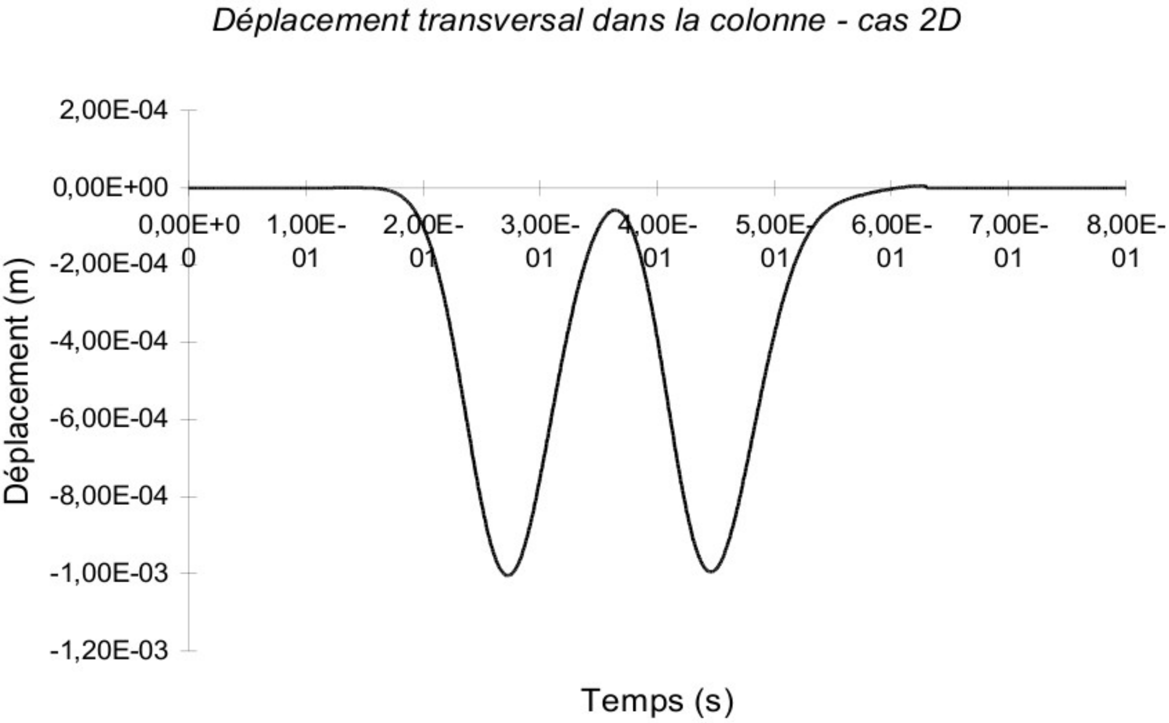
2 Reference solution

The propagation 1D of the signal of the incidental wave in the column is known analytically [bib1]. One can for example determine the moment of passage of the maximum of the incidental wave with middle height, that is to say with a depth of $\frac{H}{2}=25\,m$, and that of the maximum of the wave thought of the same point. The speed of the waves of shearing being of $C_S=281\,m.s^{-1}$ for the ground considered, the time of passage of the wave front with middle height is of $t_{\frac{H}{2}}=\frac{H}{2C_S}=0{,}089\,s$. One can thus expect the maximum of displacement with middle height in the column for time $t_i=t_m+t_{\frac{H}{2}}=0{,}266\,s$. Moreover, at the time of the passage of the reflected wave, the signal will have traversed $50\,m$ moreover, therefore one can expect it for time $t_r=t_m+t_{\frac{3H}{2}}=0{,}444\,s$. The value of the maximum measured at these moments must be of $1\,mm$. These are these analytical values that one will test in calculation.

2.1 Results of reference

One gives in this paragraph the results got with *Code_Aster* in this configuration. It is checked that they are satisfactory qualitatively and quantitatively.
They concern, for the case 3D, the evolution of displacement in the three directions in a point of the column located at middle height, that is to say with $25\,m$ free surface in the direction z . The measurement of displacement is identical in the case 2D.
Moreover, the direction of vibration is the axis of y in the case 3D. It is the axis x in the case 2D.





It is checked first of all that displacement is null according to x and z in the case 3D and according to y in the case 2D.

It is thus checked that the width of the peaks is not deformed and is well $0{,}3\,s$ at the base.
One also observes at the moments envisaged the presence of the two identical peaks due to the reflection without change of sign on free surface. Their amplitude of $1\,mm$ also find the imposed signal.

The inversion of the sign of displacement in the case 2D is due only to the orientation of the reference mark. Orientation of the positive sign for the signal being given compared to the direction of propagation according to the direction of Fresnel.

2.2 Uncertainties

It is about a digital result of the study. One finds the qualitative and quantitative forecasts. The digital values are related to the precision of calculation.

2.3 Bibliographical references

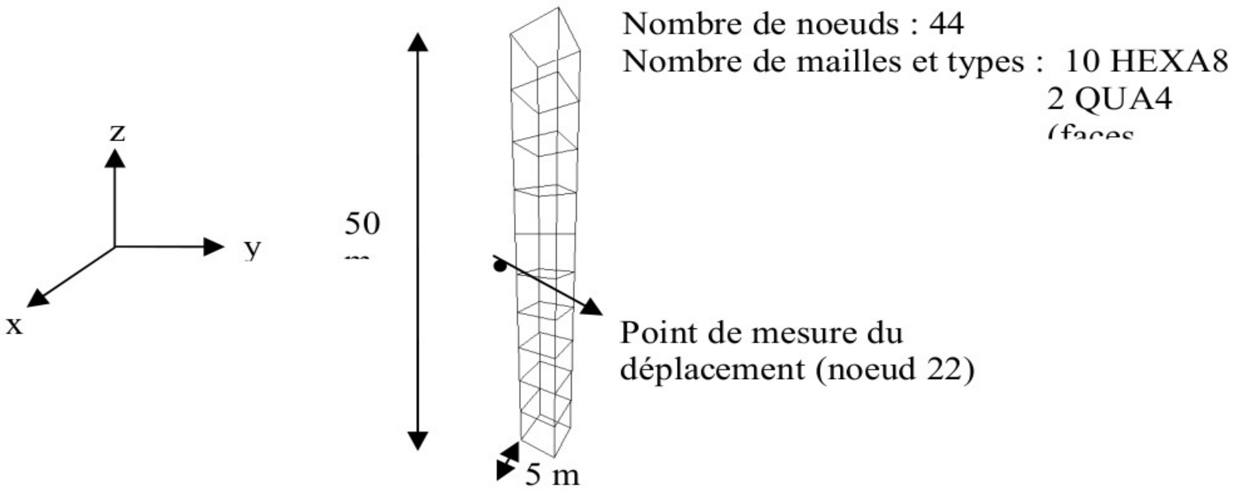
1) H. MODARESSI “digital Modeling of the wave propagation in the elastic porous environments.” Thesis doctor-engineer, Central School of Paris (1987)

3 Modeling A

3.1 Characteristics of modeling

The bar is modelled in `3D`. Elements of border absorbing in `3D_ABSO`.

3.2 Characteristics of the grid



3.3 Values tested

One tests the values of displacement in the three directions with the central node 22 (see grid). For the direction y , one tests the value of the two maximum ones and the return at rest after the passage of the wave. For the two other directions, one tests the nullity of displacement, for example at the moment of the first maximum in y .

- `DYNA_VIBRA` :

Direction	**Moment** (s)	**Results of reference** (**displacement in** m **)**
Y	2.65600E-01	1.E-03
	4.38400E-01	1.E-03
	8.00000E-01	0.
X	2.65600E-01	0.
Z	2.65600E-01	0.

- `DYNA_NON_LINE` :

Direction	**Moment** (s)	**Calculation with *Code_Aster* (displacement in** m **)**	**Results of reference** (**displacement in** m **)**	**Variations reference - calculation with *Code_Aster* (%)**
Y	2.67200E-01	1.00396E-04	1.E-03	0.40 RELATIVE
	4.40000E-01	9.94928E-04	1.E-03	0.51 RELATIVE
	7.20000E-01	5.1E-6	0.	5.1E-4 ABSOLUTE
X	2.67200E-01	0.	0.	0. ABSOLUTE
Z	2.67200E-01	0.	0.	0. ABSOLUTE

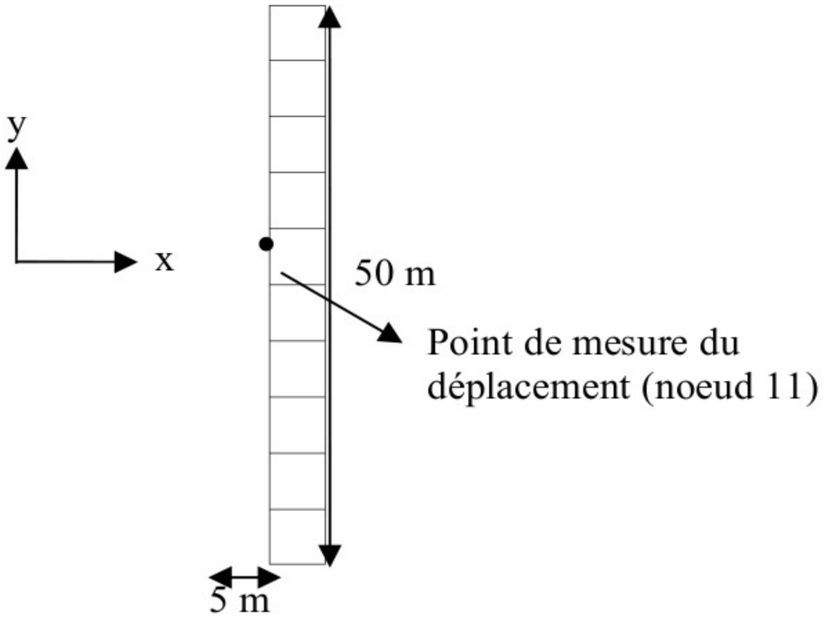
4 Modeling B

4.1 Characteristics of modeling

The bar is modelled in `D_PLAN`. Elements of border absorbing in `D_PLAN_ABSO`.

4.2 Characteristics of the grid

NombRe of nodes: 22
Many meshs and types: 10 QUA4
2 SEG2 (faces of QUA4)



4.3 Values tested

One tests the values of displacement in the three directions with node 11 (see grid). For the direction x , one tests the value of the two maximum ones and the return at rest after the passage of the wave. For the direction y , one tests the nullity of displacement, for example at the moment of the first maximum in y .

- `DYNA_VIBRA :`

Direction	**Moment** (s)	**Results of reference (displacement in** m **)**
X	2.65600E-01	– 1.E-03
	4.38400E-01	– 1.E-03
	8.00000E-01	0.
Y	2.65600E-01	0.

- `DYNA_NON_LINE` :

Direction	**Moment** (s)	**Results of reference (displacement in** m **)**
X	2.65600E-01	– 1.E-03
	4.38400E-01	– 1.E-03
	8.00000E-01	0.
Y	2.65600E-01	0.

5 Modeling C

5.1 Characteristics of modeling

The modélisation is absolutely identical to modeling A. the only difference is that one uses the type of wave now SV for the loading by plane wave. That results in to permute the role of the horizontal directions X and Y.

5.2 Values tested

One tests exactly the same values of displacement at the same moments as for modeling A. the only difference consists of the permutation of the role of the horizontal directions X (which tests the same values as those in the direction Y for modeling A) and Y where one must test a zero value now.

6 Modeling D

6.1 Characteristics of modeling

The bar is modelled in `D_PLAN`. Elements of border absorbing in `D_PLAN_ABSO`.

6.2 Characteristics of the grid

The grid is the same one as that of modeling B. the only difference comes from the addition of 20 meshs SEG2 from border absorbing on the side faces from the column. From where:
NombRe of nodes: 22
Many meshs and types: 10 QUA4
22 SEG2 (faces of QUA4)

6.3 Boundary conditions and loadings

Compared to modeling B, one replaces the condition of imposition to all the nodes of the same horizontal section of having same displacement by the assignment of condition of border absorbing on the side faces of the column.
Moreover, in this configuration, the loading by plane wave comprises, in addition to the same characteristics as for modeling B, the activation of a reflected wave, of the same intensity than the incidental wave but of opposite sign. This considered wave is activated by the data of the operand `DIST_REFLECHI` in the keyword `ONDE_PLANE` order `AFFE_CHAR_MECA_F`. One informs there the scalar product of the vector direction of the wave by the exit point of the wave, that is to say in our case the dimension of the top of the column.
The loading by plane wave is, in this modeling, affected on all the elements of absorbing border, that is to say at the same time the elements at the base of the column like those on its side faces.

6.4 Values tested

One tests the values of displacement in the three directions with the central node 11 (see grid). For the direction x , one tests the value of the two maximum ones and the return at rest after the passage of the wave. For the direction y , one tests the nullity of displacement, for example at the moment of the first maximum in y .

- `DYNA_VIBRA` :

Direction	**Moment** (s)	**Results of reference (displacement in** m **)**
X	2.65600E-01	-1.E-03
	4.40000E-01	-1.E-03
	8.00000E-01	0.
Y	2.65600E-01	0.

- `DYNA_NON_LINE` :

Direction	**Moment** (s)	**Results of reference (displacement in** m **)**
X	2.65600E-01	– 1.E-03
	4.40000E-01	– 1.E-03
	8,00000E-01	0.
Y	2.65600E-01	0.

7 Modeling E

7.1 Characteristics of modeling

The bar is modelled in `3D` . Elements of border absorbing in `3D_ABSO` .

7.2 Characteristics of the grid

The grid is identical to that of modeling A. the only difference comes from the addition of 40 meshs QUA4 of border absorbing on the side faces of the column. From where:
NombRe of nodes: 44
Many meshs and types: 10 HEXA8
42 QUA4 (faces of HEXA8)

7.3 Boundary conditions and loadings

Compared to modeling A, one replaces the condition of imposition to all the nodes of the same horizontal section of having same displacement by the assignment of condition of border absorbing on the side faces of the column.
Moreover, in this configuration, the loading by plane wave comprises, in addition to the same characteristics as for modeling A, the activation of a reflected wave, of the same intensity than the incidental wave but of opposite sign. This considered wave is activated by the data of the operand `DIST_REFLECHI` in the keyword `ONDE_PLANE` order `AFFE_CHAR_MECA_F`. One informs there the scalar product of the vector direction of the wave by the exit point of the wave, that is to say in our case the dimension of the top of the column.
The loading by plane wave is, in this modeling, affected on all the elements of absorbing border, that is to say at the same time the elements at the base of the column like those on its side faces.

7.4 Values tested

One tests the values of displacement in the three directions with the central node 26. For the direction y , one tests the value of the two maximum ones and the return at rest after the passage of the wave. For the two other directions, one tests the nullity of displacement, for example at the moment of the first maximum in y .

- `DYNA_VIBRA` :

Direction	**Moment** (s)	**Results of reference** (**displacement in** m **)**
Y	2.65600E-01	1.E-03
	4.40000E-01	1.E-03
	8.00000E-01	0.
X	2.65600E-01	0.
Z	2.65600E-01	0.

- `DYNA_NON_LINE` :

Direction	**Moment** (s)	**Results of reference** (**displacement in** m **)**
Y	2.65600E-01	1.E-03
	4.40000E-01	1.E-03
	7,20000E-01	0.
X	2.65600E-01	0.
Z	2.65600E-01	0.

8 Modeling F

8.1 Characteristics of modeling

The bar is modelled in `3D`. Elements of border absorbing in `3D_ABSO`.

8.2 Characteristics of the grid

The grid is that of the modeling A passed into quadratic. Elements `3D` are `HEXA20` and elements `3D_ABSO` are `QUAD8`.

8.3 Values tested

One tests the values of displacement in the three directions with the central node 22 (see grid modeling A). For the direction y , one tests the value of the two maximum ones and the return at rest after the passage of the wave. For the two other directions, one tests the nullity of displacement, for example at the moment of the first maximum in y .

Modeling A is used as reference to the values tested by reference ‘`AUTRE_ASTER`’.

- `DYNA_VIBRA :`

Direction	Moment (s)	Value of reference (displacement in m)	Type of reference	Tolerance
X	2.65600E-01	0.	`‘ANALYTICAL’`	0.1 %
Y	2.65600E-01	1.E-03	`‘ANALYTICAL’`	0,7 %
	4.38400E-01	1.E-03	`‘ANALYTICAL’`	4 %
	8.00000E-01	0.	`‘ANALYTICAL’`	0,1 %
Z	2.65600E-01	0.	`‘ANALYTICAL’`	0,1 %

- `DYNA_NON_LINE :`

Direction	Moment (s)	Value of reference (displacement in m)	Type of reference	Tolerance
X	2.67200E-01	-	`‘NON_REGRESSION’`	-
Y	2.67200E-01	9.989051E-04	`‘WithUTRE_ASTER’`	1.1 %
	4.40000E-01	9.914943E-04	`‘WithUTRE_ASTER’`	1.3 %
	7.20000E-01	-	`‘ANALYTICAL’`	-
Z	2.67200E-01	-	`‘ANALYTICAL’`	-

9 Modeling G

9.1 Characteristics of modeling

The bar is modelled in `3D`. Elements of border absorbing in `3D_ABSO`.

9.2 Characteristics of the grid

The grid is that of the modeling A passed in tetrahedral grid quadratic. Elements `3D` are `TETRA10` and elements `3D_ABSO` are `TRIA6`.

9.3 Values tested

One tests the values of displacement in the three directions with the central node 22 (see grid modeling A). For the direction y, one tests the value of the two maximum ones and the return at rest after the passage of the wave. For the two other directions, one tests the nullity of displacement, for example at the moment of the first maximum in y.

Modeling F is used as reference to the values tested by reference '`AUTRE_ASTER`'.

- `DYNA_VIBRA :`

Direction	Moment (s)	Value of reference (displacement in m)	Type of reference	Tolerance
X	2.65600E-01	0.	`'ANALYTICAL'`	0.1 %
Y	2.65600E-01	1.E-03	`'ANALYTICAL'`	1 %
	4.38400E-01	1.E-03	`'ANALYTICAL'`	8 %
	8.00000E-01	0.	`'ANALYTICAL'`	0,1 %
Z	2.65600E-01	0.	`'ANALYTICAL'`	0,1 %

- `DYNA_NON_LINE :`

Direction	Moment (s)	Value of reference (displacement in m)	Type of reference	Tolerance
X	2.67200E-01	-	`'NON_REGRESSION'`	-
Y	2.67200E-01	1.009252145125E-03	`'WithUTRE_ASTER'`	0.7 %
	4.40000E-01	9.794603508685E-04	`'WithUTRE_ASTER'`	3.6 %
	7.20000E-01	-	`'ANALYTICAL'`	-
Z	2.67200E-01	-	`'ANALYTICAL'`	-

10 Summary of the results

One finds by calculation with seven modelings, quantitatively, values of maximum of displacement equal to the maximum amplitude of the signal and values of the corresponding moments and qualitatively, the return at rest after the passage of the considered wave.

Results got with the operators `DYNA_VIBRA` and `DYNA_NON_LINE` are very close. The difference comes from obtaining to each step in time from the state from balance from the efforts from the second member with the operator `DYNA_NON_LINE`, which explains why its results are in general a little bit better even with a step of larger time. This difference remains however tiny because the step of time used with `DYNA_VIBRA` is sufficiently small.

One notices finally that the results with quadratic grids are a little less good, especially with the tetrahedral grid.