SDLV123 - Elastodynamic calculation of G in medium infinite for a crack finite length planes

Summary

It is about a problem of breaking process in a medium in a state of plane deformation in transitory elastodynamic mode. A crack length is considered $2a$ diving in a presumedly infinite medium. One imposes a uniform pressure on the lips of the crack, which reaches a stage in a period of time of 1 microsecond (shock). This test makes it possible to calculate the rate of refund of energy G in the course of time, by taking account of the terms of inertia.

The interest of the test is the stability of G according to various crowns and the comparison with an exact analytical solution until time $t=2a/V_C$, where V_C represent the celerity of the longitudinal waves.

This test contains a modeling in plane deformation and a three-dimensional modeling. Boundary conditions absorbing on the borders of the solid make it possible to avoid the returns of wave and thus to simulate an infinite medium.

Variations of the calculation of G on various crowns compared to the reference solution do not exceed 1,4%.

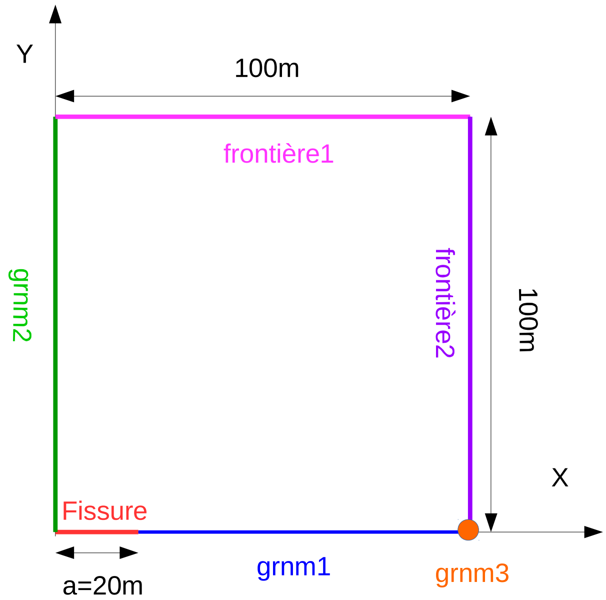
1 Problem of reference

1.1 Geometry and modeling

It is about a plane crack of half-length $a=20\text{m}$.

The presumedly infinite medium is modelled by a square on side of 200m. Plans xz and yz being symmetry planes, one represents only one quarter of the structure (square of $100\,m$ on $100\,m$), and a half-lip of crack length $a=20\,m$.

An absorbing border is present on the groups “frontière1” and “frontière2” to model the infinite medium.



1.2 Properties of materials

Density	$\rho=7500\text{kg}/m^3$
Young modulus:	$E=2.10^{11}\,MPa$
Poisson's ratio:	$\nu=0.3$

1.3 Loadings

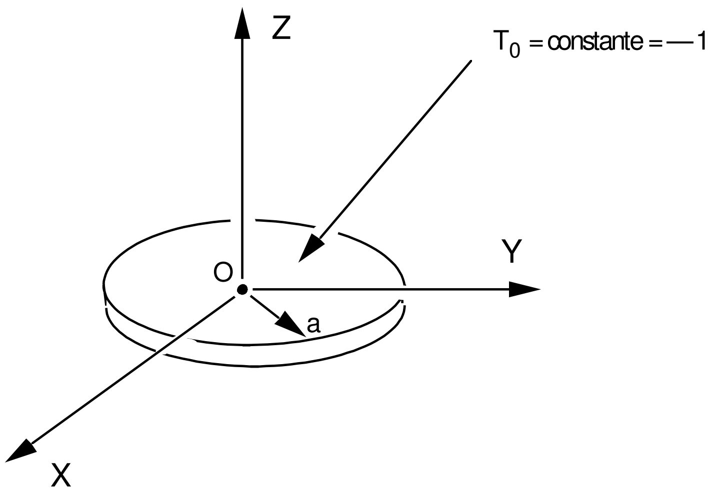
Imposed pressure :

Group of mesh $Fissure$	$PRES=100.\,MPa$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is resulting from THAU and LU [bib1] and from the book of L.B. FREUND [bib2]. The figure which follows represents the infinite medium and not the geometry of the test.



$$K_I^D(t)=2P\ H(t)\frac{\sqrt{1-2\nu}}{(1-\nu)}\sqrt{\frac{c_d t}{\pi}},\ \ 0<t<2a/c_d$$

The expression of the rate of refund of energy is the following one:

$$G(t)=\frac{K_I^{D^2}(t)}{E}(1-\nu^2)=4P^2\ H(t)\frac{(1-2\nu)(1+\nu)}{(1-\nu)E}\frac{c_d t}{\pi},\ \ 0<t<2a/c_d$$

$$G=\frac{(1-\nu^2)}{E}K_1^2 \quad \text{with} \quad K_1=\frac{\alpha E}{P(1-\nu)}T_0\sqrt{Pa}$$

that is to say: $G=\frac{(1-\nu^2)}{P(1-\nu)^2}\alpha^2 E T_0^2 a$

2.2 Result of reference

The result of reference is thus: $G=5.911510^{-7}N/mm$

t	G_{freund}
2nd-3	5.677e5
4th-3	1.135e6
6th-3	1.703e6

2.3 Bibliographical reference

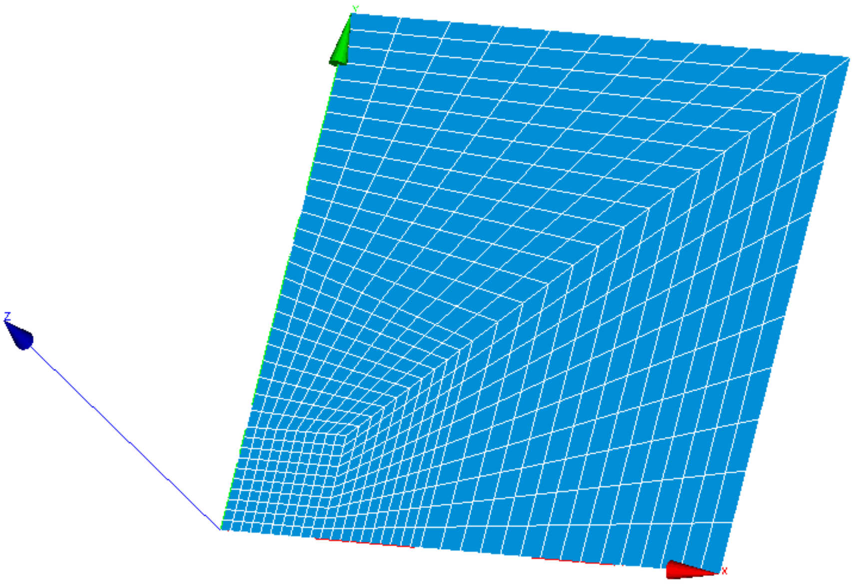
1) Transient stress intensity factors for has finite ace in year elastic solid caused by has dilatational wave, International Newspaper of Solids and Structures 7, THAU and LU (1971)

2) Dynamic Fractures Mechanics L.B FREUND.

3 Modeling A

3.1 Characteristics of modeling

It is about a plane modeling in deformationsS (D_PLAN). The absorbing borders are modelled by elements D_PLAN_ABSO.



3.2 Characteristics of the grid

Many nodes: 646

Many meshs and types: 600 QUAD4, 245 SEG2

Crown 1:	$R_{\mathrm{inf}}=3.$	$R_{\mathrm{sup}}=10.$
Crown 2:	$R_{\mathrm{inf}}=5.$	$R_{\mathrm{sup}}=15.$
Crown 3 :	$R_{\mathrm{inf}}=5.$	$R_{\mathrm{sup}}=18.$

3.3 Boundary conditions

DisplacementS imposedS :

Group of nodes $grnm\,1$	$DY=0.$
Group of nodes $grnm\,2$	$DX=0.$
Group of nodes $grnm\,3$	$DX=0.\quad DY=0.$

3.4 Results

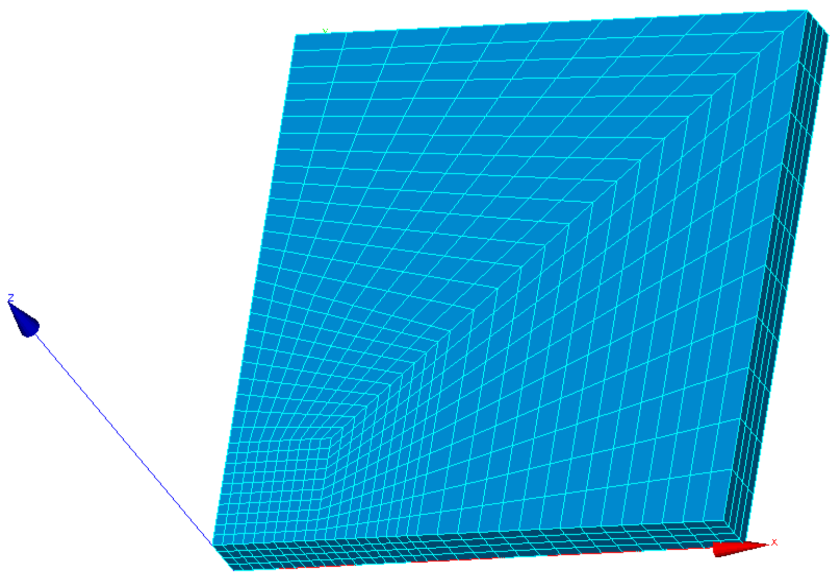
The values tested are those of the rate of refund of energy G on the various crowns of integration.

Identification	Type of reference	Reference	Tolerance $\%$
Moment $t=2\text{e-}3$			
G , crown n°1	'ANALYTICAL'	5.67700E5	1.4
G , crown n°2	'ANALYTICAL'	5.67700E5	1.4
G , crown n°3	'ANALYTICAL'	5.67700E5	1.4
Moment $t=4\text{e-}3$			
G , crown n°1	'ANALYTICAL'	1.135000E6	1.4
G , crown n°2	'ANALYTICAL'	1.135000E6	1.4
G , crown n°3	'ANALYTICAL'	1.135000E6	1.4
Moment $t=6\text{e-}3$			
G , crown n°1	'ANALYTICAL'	1.703000E6	1.4
G , crown n°2	'ANALYTICAL'	1.703000E6	1.4
G , crown n°3	'ANALYTICAL'	1.703000E6	1.4

4 Modeling B

4.1 Characteristics of modeling

It is about a modeling into 3D, the thickness of the modelled square is of 10. The absorbing borders are modelled by elements 3D_ABSO.



4.2 Characteristics of the grid

Many nodes: 3230

Many meshs and types: 1560 QUAD4, 2400 HEXA8, 253 SEG2

Crown 1:	$R_{\text{inf}}=3.$	$R_{\text{sup}}=10.$
Crown 2:	$R_{\text{inf}}=5.$	$R_{\text{sup}}=15.$
Crown 3 :	$R_{\text{inf}}=5.$	$R_{\text{sup}}=18.$

4.3 Boundary conditions

DisplacementS imposedS :

Group of nodes $grnm1$	$DY=0.$
Group of nodes $grnm2$	$DX=0.$
Group of nodes $grnm3$	$DX=0.$ $DY=0.$

One specifies that in 3D, the groups defined in 1.1 concernNT all the thickness according to Z .

Lastly, nodes of the higher face ($Z=10$) as well as the nodes in $Z=0$ groups $grnm1$, $grnm2$ and $frontière1$ are blocked in Z .

4.4 Results

The values tested are identical to those of modeling A.

Identification	Type of reference	Reference	Tolerance %
Moment $t=2e-3$			
G , crown n°1	'ANALYTICAL'	5.67700E5	1.4
G , crown n°2	'ANALYTICAL'	5.67700E5	1.4
G , crown n°3	'ANALYTICAL'	5.67700E5	1.4
Moment $t=4e-3$			
G , crown n°1	'ANALYTICAL'	1.135000E6	1.4
G , crown n°2	'ANALYTICAL'	1.135000E6	1.4
G , crown n°3	'ANALYTICAL'	1.135000E6	1.4
Moment $t=6e-3$			
G , crown n°1	'ANALYTICAL'	1.703000E6	1.4
G , crown n°2	'ANALYTICAL'	1.703000E6	1.4
G , crown n°3	'ANALYTICAL'	1.703000E6	1.4

5 Summary of the results

Invariance of the result compared to the crowns. Correct thermal term.