SDLV131 - Simulation of a gauge of deformation by the order OBSERVATION

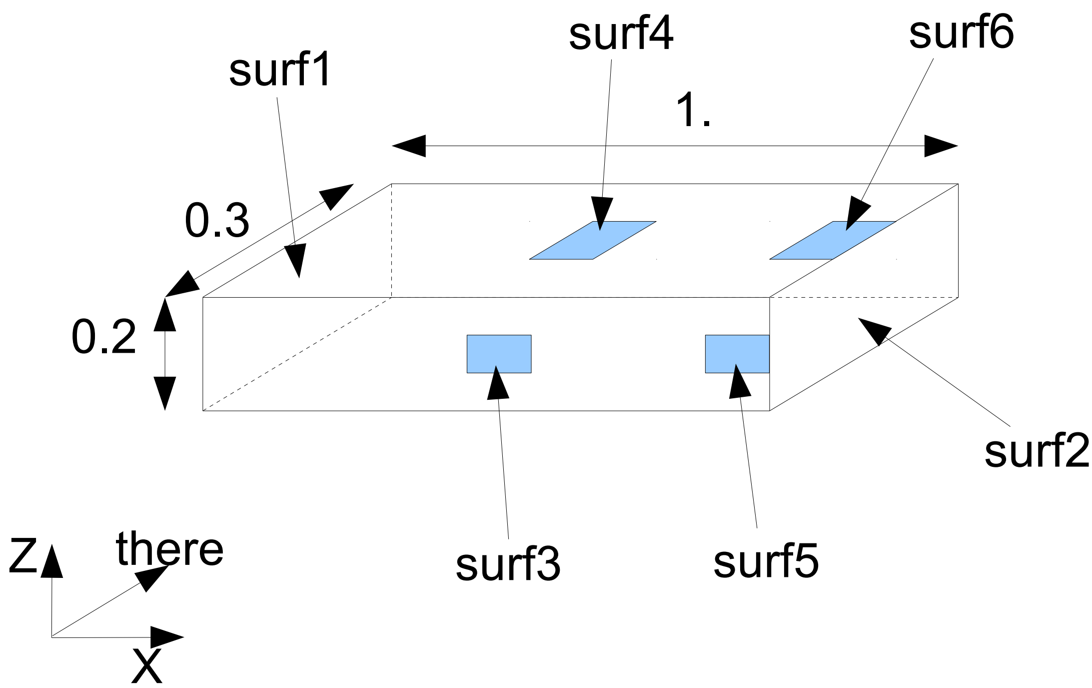
Summary:

This test validates the operation of the calculation of the median value of a field of deformation on an entity given by the user. The field of deformation thus is estimated that a gauge of deformation would have measured. One carries out this calculation via the macro-order `OBSERVATION`. The treated case is a beam in simple traction modelled by voluminal elements.

This case test also validates the observation of mixed fields: only one call to the order **`OBSERVATION`** for the statement of fields of different nature (**`DEPL`**, **`QUICKLY`**,…).

1 Problem of reference

1.1 Geometry



1.2 Properties of material

Young modulus: $E=2.1\,10^{11}\,N/m^2$
Poisson's ratio: $\nu=0.3$
Density: $\rho=7800.\,kg/m^3$

1.3 Boundary conditions and loadings

A horizontal displacement is imposed $u_x=0.0$ on the face $surf1$.
A displacement is imposed $u_y=0.0$ on the nodes which are on the central line of the higher face and on the central line of the lower face.
A displacement is imposed $u_z=0.0$ on the nodes which are on the central lines of the side faces.

A surface force is applied F_B on the face $surf2$ according to the direction x ,

$F_B=1000.\,N/m^2$,

These boundary conditions make it possible to obtain a behavior of the beam in simple traction.

1.4 Initial conditions

Without object

2 Reference solution

2.1 Method of calculating

The deformation is estimated starting from the relative lengthening of the beam.

The lengthening of a beam length L following a longitudinal force F is written:

$$\Delta L=\frac{F L}{E S}$$

In our case, one applies a force per unit of area F_B at the loose lead of the beam, thus the relative lengthening of the beam is put in the following form:

$$\frac{\Delta L}{L}=\epsilon_{xx}=\frac{F_B}{E}$$

For this case test, one calculation results resulting from a static calculation, a harmonic calculation, a transitory calculation and of a modal calculation.

For the static case, one obtains:

$$\varepsilon_{xx}=\frac{F_B}{E} \quad \text{and} \quad \varepsilon_{yy}=\varepsilon_{zz}=-\nu\,\varepsilon_{xx}$$

For the dynamic cases, the system is governed by the following equation:

$$M\frac{\partial^2 u}{\partial t^2}+K\,u=F_{ext}$$

For a beam in traction and compression, if one considers a model which contains one element, the matrices of mass M and of rigidity K put themselves in the following form:

$$M=\frac{\rho S L}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix} \qquad K=\frac{ES}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} \qquad \text{with:} \quad u=\begin{bmatrix}u_1\\u_2\end{bmatrix}$$

u_1 and u_2 are displacements of the nodes of the element.

In harmonic answer of pulsation ω , displacements of the nodes of the element are governed by the following relation:

$$\frac{ES}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}-\omega^2\frac{\rho S L}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}=\begin{bmatrix}F_1\\F_2\end{bmatrix}$$

By exploiting the second-row forward of this relation, and by applying the boundary conditions ($u_1=0$ and $F_2=F=F_B S$), one obtains:

$$u_2 = \frac{F_B}{\frac{E}{L} - \omega^2 \frac{\rho L}{3}} = \Delta L$$

The deformation at the loose lead of the beam is written:

$$\varepsilon_{xx} = \frac{\Delta L}{L} = \frac{F_B}{E - \omega^2 \frac{\rho L^2}{3}} \quad \text{and} \quad \varepsilon_{yy} = \varepsilon_{zz} = -\nu\, \varepsilon_{xx}$$

The reference solution for the transitory solution can be obtained same manner. If a longitudinal force is applied $F(t) = F\,t = F_B S\,t$ with an initial condition of beam in balance (initial displacement no one and worthless initial speed), one obtains:

$$u_2(t) = \frac{3 F_B}{\rho\, L \omega_0^2} \left[t - \frac{\sin(\omega_0 t)}{\omega_0} \right] = \Delta L \quad \text{with:} \quad {\omega_0}^2 = \frac{3E}{\rho L^2}$$

And the deformation at the loose lead of the beam is written:

$$\varepsilon_{xx} = \frac{\Delta L}{L} = \frac{3 F_B}{\rho L^2 \omega_0^2} \left[t - \frac{\sin(\omega_0 t)}{\omega_0} \right] \quad \text{and} \quad \varepsilon_{yy} = \varepsilon_{zz} = -\nu\, \varepsilon_{xx}$$

In the case of modal calculation, one carries out a test of not-regression on the deformations calculated at the point medium of the beam.

One also simulates a rotation of 90 degrees in order to check the change of reference mark in `OBSERVATION`.

2.2 Sizes and results of reference

One tests the value of the average deformation on surfaces: $surf3$, $surf4$, $surf5$ and $surf6$. The got results are then projected on the model “measures” which understands only the nodes $P3$, $P4$, $P5$ and $P6$ associated with surfaces $surf3$, $surf4$, $surf5$ and $surf6$.

For the validation of the static solution, one has chooses: $F_B = 1000.\, N/m^2$

For the validation of the harmonic solution, one chose: $F_B = 1000.(1+2j)\, N/m^2$ and: $\omega = 2\pi\, 200\, rd\, s^{-1}$

For the validation of the transitory solution, one chose: $F_B = 1000.\, t\, N/m^2$ and the solution is tested at the moment $t = 1\, s$

One also tests the values of the fields obtained by mixed observation.

2.3 Uncertainties on the solution

Analytical solution for the static case, the harmonic case and the transitory case.
One suggests a solution of not-regression in the case of the modal deformation.

2.4 Bibliographical reference

[R3.08.01] “exact” Elements of beams (right and curved).

3 Modeling A

3.1 Characteristics of modeling

One calculates the average deformation resulting from a static calculation of answer with MECA_STATIQUE.

One also calls upon the operator **OBSERVATION** for the statements of fields of displacement and deformation.

3.2 Characteristics of the grid

Nodes: 1029
Meshs: 720 HEXA8

3.3 Sizes tested and results

One tests the value of the deformation to the nodes which are in the middle of the beam.

Identification	Reference	*Aster*	Tolerance
ε_{xx} in $P3$	4.7619047619 048D-09	4.7619047619 047D-09	0.1%
ε_{zz} in $P3$	– 1.4285714285 714D-09	– 1.4285714285 714D-09	0.1%
ε_{yy} in $P4$ (after rotation of $90°$)	4.7619047619 048D-09	4.7619047619 047D-09	0.1%
ε_{xx} in $P4$ (after rotation of $90°$)	– 1.4285714285 714D-09	– 1.4285714285 714D-09	0.1%

Tests for the mixed observation:

Identification	Reference	*Aster*	Tolerance
ε_{xx} in $P3$	4.7619047619 048D-09	4.7619047619 047D-09	0.1%
DX in $P5$ (m)	4.7619047619 048D-09	4.5238095238 095D-09	6.0%

4 Modeling B

4.1 Characteristics of modeling

One calculates the average deformation resulting from a harmonic calculation.
One chooses a frequency of excitation equalizes with $200\,Hz$.

One also calls upon the operator **OBSERVATION** for the statements of fields of displacement and deformation.

4.2 Characteristics of the grid

Nodes: 1029
Meshs: 720 HEXA8

4.3 Sizes tested and results

One tests the value of the deformation to the nodes which are at the loose lead of the beam.

Identification	Reference	*Aster*	Tolerance
ε_{xx} in $P5$	4.8568623280004D-09 + i9.7137246560009D-09	4.7813760355714D-09 + i9.5627520711429D-09	2.0%
ε_{zz} in $P5$	-1.4570586984001D-09 - i2.9141173968003D-09	-1.4339303380850D-09 - i2.8678606761701D-09	2.0%
ε_{yy} in $P6$ (after rotation of $90°$)	4.8568623280004D-09 + i9.7137246560009D-09	4.7792223250189D-09 + i9.5584446500377D-09	2.0%
ε_{xx} in $P6$ (after rotation of $90°$)	-1.4570586984001D-09 - i2.9141173968003D-09	-1.4339110329539D-09 - i2.8678220659079D-09	2.0%

Tests for the mixed observation:

Identification	Reference	*Aster*	Tolerance
ε_{xx} in $P5$	4.8568623280004E-09 + i9.7137246560009E-09	4.7792696544452E-09 + i9.5585393088903E-09	2.0%

Displacement DX in $P5$ (m)	4.8568623280004E-09 + i9.7137246560009E-09	4.6188418141639E-09 + i9.2376836283278E-09	5.0%
Speed DX in $P5$ (m/s)	-1.2206626407315E-05 + i6.1033132036573E-06	-1.1608415609177E-05 + i5.8042078045883E-06	5.0%

5 Modeling C

5.1 Characteristics of modeling

One calculates the average deformation resulting from a transitory calculation on physical basis. For the resolution of the system, one chooses a temporal discretization equalizes with $0.1\,s$.

One also calls upon the operator **OBSERVATION** for the statements of fields of displacement and deformation.

5.2 Characteristics of the grid

Nodes: 1029
Meshs: 720 HEXA8

5.3 Sizes tested and results

One tests the value of the deformation to the nodes which are at the loose lead of the beam at the moment $t=1\,s$.

Identification	Reference	Aster	Tolerance
ε_{xx} in $P5$	4.7614812131 879D-09	4.7619095908 375D-09	2.0%
ε_{zz} in $P5$	- 1.4284443639 564D-09	- 1.4285727587 481D-09	2.0%
ε_{yy} in $P6$ (after rotation of $90°$)	4.7614812131 879D-09	4.7619090586 422D-09	2.0%
ε_{xx} in $P6$ (after rotation of $90°$)	- 1.4284443639 564D-09	- 1.4285727564 358D-09	2.0%

Tests for the mixed observation:

Identification	Reference	Aster	Tolerance
ε_{xx} in $P5$	4.7614812131 879E-09	4.7619090690 47E-09	2.0%
Displacement DX in $P5$ (m)	4.7614812131 879E-09	4.5238330203 209E-09	6.0%

6 Modeling D

6.1 Characteristics of modeling

One calculates the average deformation resulting from a modal calculation with CALC_MODES.

One also calls upon the operator OBSERVATION for the statements of fields of displacement and deformation.

6.2 Characteristics of the grid

Nodes: 1029
Meshs: 720 HEXA8

6.3 Sizes tested and results

One tests the median value of the deformation of the first longitudinal mode of the beam to the nodes which are in the medium and the loose lead. The values of reference are those obtained with the version 10.1 (test of not-regression). They are given with four significant figures.

Identification	Reference	*Aster*	Tolerance
ϵ_{xx} in $P3$	1,012	1.0120571181683D+00	0.1%
ϵ_{zz} in $P3$	-3.063D-1	-3.0631275158658D-01	0.1%
ϵ_{xx} in $P5$	1.762D-1	1.7617357057031D-01	0.1%
ϵ_{zz} in $P5$	-4.646D-2	-4.6458825653459D-02	0.1%
ϵ_{yy} in $P4$ (after rotation of $90°$)	1,017	1.0168194189613D+00	0.1%
ϵ_{xx} in $P4$ (after rotation of $90°$)	-3.118D-1	-3.1181161388859D-01	0.1%
ϵ_{yy} in $P6$ (after rotation of $90°$)	1.533D-1	1.5327248667577D-01	0.1%
ϵ_{xx} in $P6$ (after rotation of $90°$)	-4.178D-2	-4.1782881091537D-02	0.1%

Tests for the mixed observation:

Identification	Reference	*Aster*	% difference
ε_{xx} in $P3$	1,01	1.0144135640865	0.1%
Displacement DX in $P3$ (m)	0.7570	0.75705679759059	0.1%

7 Summary of the results

The purpose of these tests are to check the good progress of calculation of the median value of the field of deformation using the macro-order OBSERVATION. The reference solution is analytical for the static answer, the harmonic answer and the transitory answer.

This case test also validates the mixed observation of field (**DEPL**, **QUICKLY**,…) with only one call to the operator **OBSERVATION**.

The differences between the solutions obtained with Aster and the analytical solutions are very weak. For the results resulting from a modal calculation, a test of not-regression is proposed.