SHLL100 - Harmonic answer of a bar by dynamic under-structuring

Summary:

The scope of application of this test relates to the dynamics of the structures, and more particularly the harmonic calculation of answer per dynamic under-structuring.

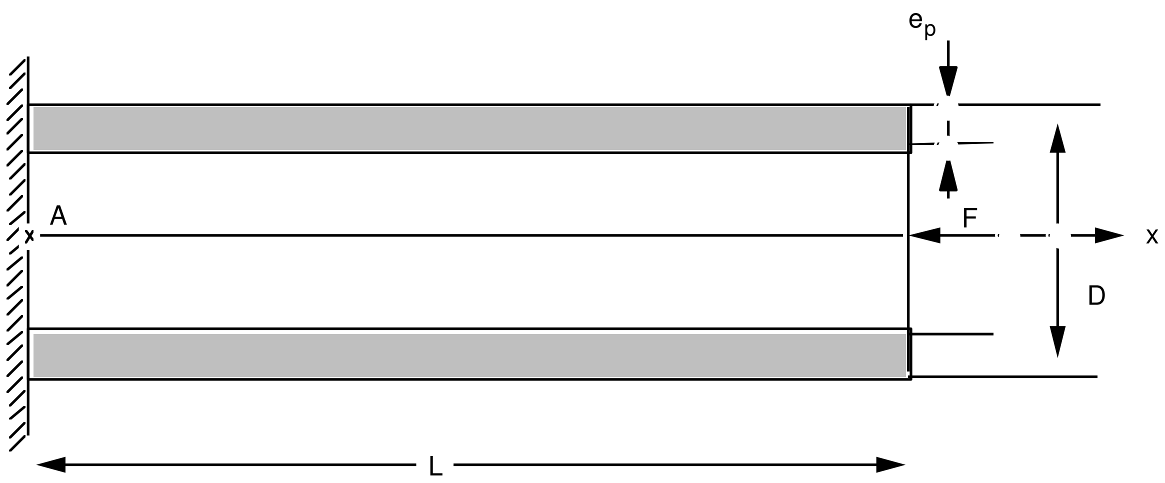
It is a question of calculating the harmonic answer in traction and compression of a fixed beam - free modelled by elements of the type “bars”. The modelled structure is deadened (damping of Rayleigh by elements).

The results of reference result from a direct harmonic calculation. This test thus makes it possible to validate the computational tools of harmonic answer per under-structuring established in *Code_Aster* and more particularly:

- the catch in depreciation account by element,
- the calculation of the second member including the harmonic loading,
- restitution of the harmonic answer on a grid skeleton, including the fields of displacement, speed and acceleration.

1 Problem of reference

1.1 Geometry



$L=1\,m$

$D=0{,}2\,m$ - Circular section

1.2 Material properties

$E=1.10^{10}\,Pa$

$\nu=0.3$

$\rho=1.10^{4}\,kg/m^{3}$

Damping of Rayleigh per element: $\alpha_e=0.1$ $\beta_e=0.1$

1.3 Boundary conditions and loadings

Embedding in the end A : $u(0)=n(0)=w(0)=0$.

For any point $M(x)$: $n(0)=w(0)=0$.

Harmonic loading in time, at the loose lead:

- orientation: according to x ,
- amplitude: $100\ N$,
- frequency: $100\ Hz$.

1.4 Initial conditions

Without object for a harmonic calculation of answer.

2 Reference solution

2.1 Method of calculating used for the reference solution

There exists an analytical solution detailed in the reference [bib2].

Let us use the following notations:

E : Young modulus
L : length of the bar
A : section of the bar
N : normal effort directed according to the axis X
α, β : damping coefficients of Rayleigh
Ω : frequency of excitation

and let us pose

$$r=\sqrt{\frac{1+\beta^2/\Omega^2}{1+\alpha^2/\Omega^2}}$$

$$k=p+iq=\Omega\sqrt{\frac{p}{2E}}\left[\sqrt{r-\frac{1-\alpha\beta}{1+\alpha^2\Omega^2}}+i\sqrt{r+\frac{1-\alpha\beta}{1+\alpha^2\Omega^2}}\right]$$

Displacement in a point $M(x)$ unspecified is given by:

$$V(x)=\frac{N}{EA}\frac{A}{(p+iq)(1+i\Omega\alpha)}\frac{sh\, px\cos qx+i\, ch\, px\sin qx}{ch\, L\cos qL+i\, sh\, pL\sin qL}$$

	Displacement (m)	**Speed** (m/s)	**Acceleration** (m/s^2)
Real part	$-7.00\ 10^{-11}$	$-3.18\ 10^{-6}$	$2.76\ 10^{-5}$
Imaginary part	$5.07\ 10^{-9}$	$-4.40\ 10^{-8}$	$-2.00\ 10^{-3}$

2.2 Results of reference

Fields of displacement, speed and acceleration of the loose lead of the bar.

2.3 Uncertainty on the solution

Digital solution.

2.4 Bibliographical references

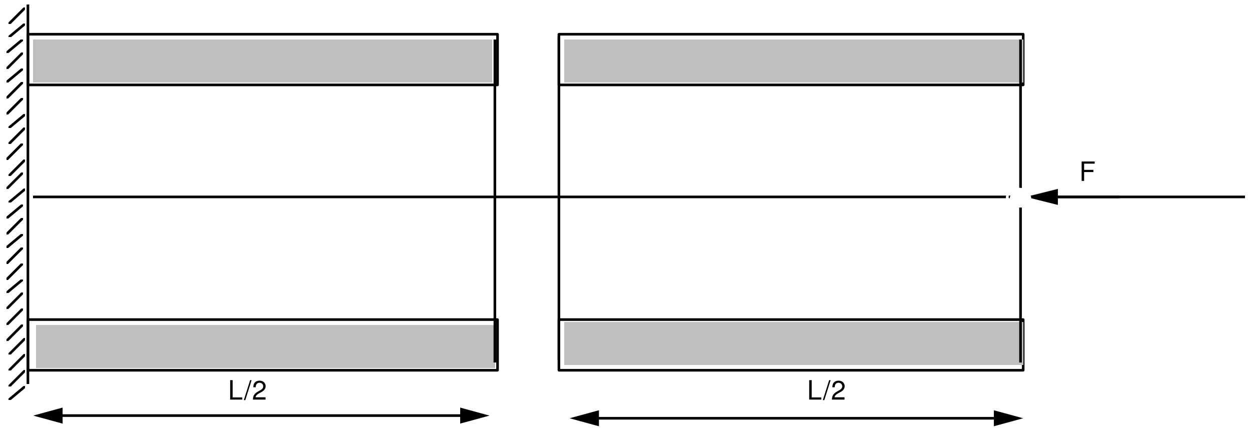
1) T. KERBER "harmonic Under-structuring in *Code_Aster*", EDF Report, HP - 61/93 - 104.

2) G. ROBERT, analytical Solutions in dynamics of the structures, Report Samtech n°121, March 1996.

3) P. RICHARD, Methods of under-structuring in *Code_Aster*, Report interns EDF - DER, HP-61/92-149.

3 Modeling A

3.1 Characteristics of modeling



The bar is cut out in 2 parts of equal size. Each substructure considered is with a grid in segments to which elements are affected “bars”.

The structure is studied using the method of the harmonic under-structuring with interfaces of the HARMONIC type CRAIG-BAMPTON.

The modal base used is made up of 4 clean modes for the substructure of right-hand side, of 5 clean modes for the substructure of left to which the harmonic constrained modes associated are added with the interfaces (calculated with $300\,Hz$. This value of the pulsation does not have any influence on the result, it is arbitrary [bib3]).

3.2 Characteristics of the grid

Many nodes: 5

Many meshs and types: 5 `SEG 2`

3.3 Sizes tested and results

	Displacement (m)	
	Reference	**Tolerance**
Real part	$-7.00\ 10^{-11}$	2.10^{-3}
Imaginary part	$5.07\ 10^{-9}$	2.10^{-3}
	Speed (m/s)	
Real part	$-3.18\ 10^{-6}$	2.10^{-3}
Imaginary part	$-4.40\ 10^{-8}$	2.10^{-3}
	Acceleration (m/s^2)	
Real part	$2.76\ 10^{-5}$	2.10^{-3}
Imaginary part	$-2.00\ 10^{-3}$	2.10^{-3}

4 Summary of the results

The precision on the complex coordinates of the fields of displacement speed and acceleration is lower than $0{,}1\%$.

This test thus validates the operators of harmonic under-structuring.