SHLV301 – Harmonic answer by under-structuring: bi--supported beam

Summary:

The studied structure is a bi--supported beam subjected to a distributed load varying in a harmonic way in the course of time.
This beam is deformable with the shearing action. It is modelled by hexahedral elements of volume to 20 nodes (modeling `3D`).

The harmonic answer is calculated by the method of under-structuring dynamic of Mac-Neal.

The results are compared with values obtained analytically for a model of beam of deformable Timoshenko to the shearing action and taking account of the rotatory inertia of the sections.

1 Problem of reference

1.1 Geometry

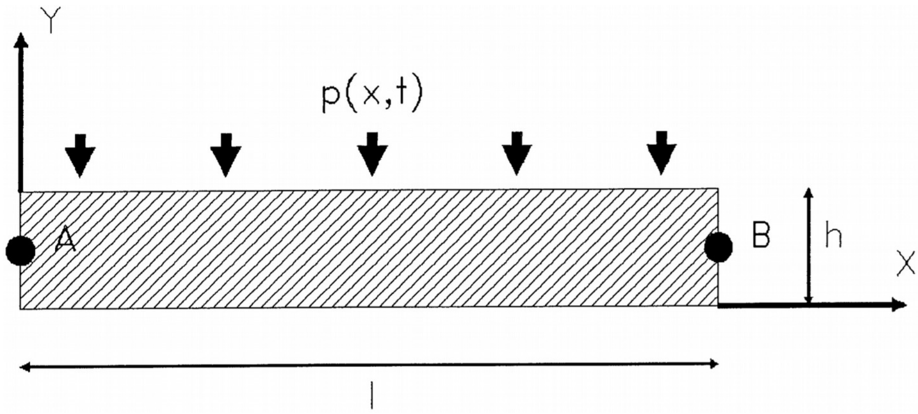

Figure 1.1-1 : Geometry of the problem

Height: $h=0.2\,\mathrm{m}$
Width: $b=0.1\,\mathrm{m}$
Length: $L=2\,\mathrm{m}$
Section: $A=b\times h=0.02\,\mathrm{m}$
Inertia: $I=\frac{b\times h^3}{12}=1.66\times 10^{-5}\,\mathrm{m}$
Coefficient of reduction of section $k'=\frac{5}{6}$

1.2 Properties of material

Young modulus	$E=2.1\times 10^{11}\,Pa$
Poisson's ratio	$\nu=0.3$
Density	$\rho=7800.0\,kg.m^{-3}$
Modulus of rigidity	$G=\frac{E}{2(1+\nu)}=8.076\times 10^{10}\,\mathrm{Pa}$
Damping coefficients of Rayleigh	$\alpha=1.6\times 10^{-5}\,\mathrm{s}$ and $\beta=16\,\mathrm{s}^{-1}$

1.3 Boundary conditions and loadings

One authorizes that the inflection in the plan XY and extension along the axis X. The model being voluminal, the boundary conditions differ somewhat from those which one would impose on a model beam.

Imposed displacement:

In $X=0$, $Y=h/2$	$DX=0$, $DY=0$
In $X=L$, $Y=h/2$	$DX=0$, $DY=0$

In $Z=b/2$	$DZ=0$

To the preceding conditions, one adds the constraint of flatness of the sections in $X=0$ and $X=L$. This constraint can be expressed as follows. Let us indicate by $x^T=(X,Y,Z)$ the vector of the coordinates and by $u^T=(DX,DY,DZ)$ the vector of displacements; the position of a point is located by the vector $x'^T=x^T+u^T=(X',Y',Z')$. Are A, B and C three non-aligned points of the section. An unspecified point P east compels in the condition:

$$\begin{vmatrix} X'_P & Y'_P & Z'_P & 1 \\ X'_A & Y'_A & Z'_A & 1 \\ X'_B & Y'_B & Z'_B & 1 \\ X'_C & Y'_C & Z'_C & 1 \end{vmatrix}=0$$

Imposed force:

In $Y=h$, for all X, loading is defined by:

$$p(x,t)=p(x)\sin(\omega t)$$

with $p(x)=p_0=5.0\times10^4\,\mathrm{N.m^{-1}}$ and $\omega=2000\pi\,\mathrm{rd.s^{-1}}$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is obtained analytically for a beam of Timoshenko, fascinating of account the deformation with the shearing action and the rotatory inertia of the sections.
The solution is developed in series of the clean modes. The theoretical aspects are developed in the reference given in 2.4.

2.1.1 Base modal

Let us define the following adimensional sizes:

$\lambda_n = k_n L$ wavelengths

$\Omega_n = \dfrac{\rho A L^4}{EI}\omega_n^2$ eigenvalues

$j = \dfrac{I}{A L^2}$ rotatory inertia

$g = \dfrac{EI}{k' A G L^2}$ coefficient of shearing

Each clean mode of many waves k_n is characterized by the following sizes:
Eigen frequencies:

$$\Omega_{1,2} = \frac{\rho A L^4}{EI}\omega_{1,2}^2 = \frac{(g+j)\lambda_n^2 + 1 \pm \sqrt{(g-j)^2\lambda_n^4 + 2(g+j)\lambda_n^2 + 1}}{2 g j}$$

with

$$\lambda_n = n\pi \,, \qquad n = 1, 2, 3, \ldots \,,$$

(indices 1 and 2 correspond to the signs + and – in front of the root).

Generalized masses:

$$\mu_{1,2} = \rho A = \left(k_n - \frac{\omega_{1,2}^2 \rho}{k_n k' G}\right)^2 \rho I \;.$$

Percentages of critical damping:

$$\epsilon_{1,2} = \frac{1}{2}\left(\alpha\omega_{1,2} + \frac{\beta}{\omega_{1,2}}\right)$$

2.1.2 Harmonic answer

The amplitude and the phase of the arrow W are given by

$$W(x) = \sum_{n=1}^{+\infty} p_n \left[\sum_{m=1}^{2} \frac{1}{\mu_m(\omega_m^2 - \omega^2 + 2 i \epsilon_m \omega_m \omega)}\right] \sin(k_n x)$$

with

$$p_n=\frac{2\,p_0}{n\,\pi}\left[1-(-1)^n\right]$$

2.2 Results of reference

Position	Arrow W	
	Amplitude (m)	Phase
$x=\frac{L}{4}$	2.136×10^{-5}	$22.4°$
$x=\frac{L}{2}$	1.342×10^{-5}	$-121.5°$
$x=3\frac{L}{4}$	2.136×10^{-5}	$22.4°$

The sizes actually tested in the CAS-test are the real and imaginary parts which one gives the values below.

Position	Arrow W	
	Real part (m)	Imaginary part (m)
$x=\frac{L}{4}$	$-1.9599467159360556\times 10^{-5}$	$-8.4917893914738073\times 10^{-6}$
$x=\frac{L}{2}$	$-6.9993870268574731\times 10^{-6}$	$-1.1450108350939712\times 10^{-5}$
$x=3\frac{L}{4}$	$-1.9599467159360556\times 10^{-5}$	$-8.4917893914738073\times 10^{-6}$

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

ROBERT G., Solutions analytical into dynamic of the structures, Report Samtech n° 121, Liege, 1996.

3 Modeling A

3.1 Characteristics of modeling A

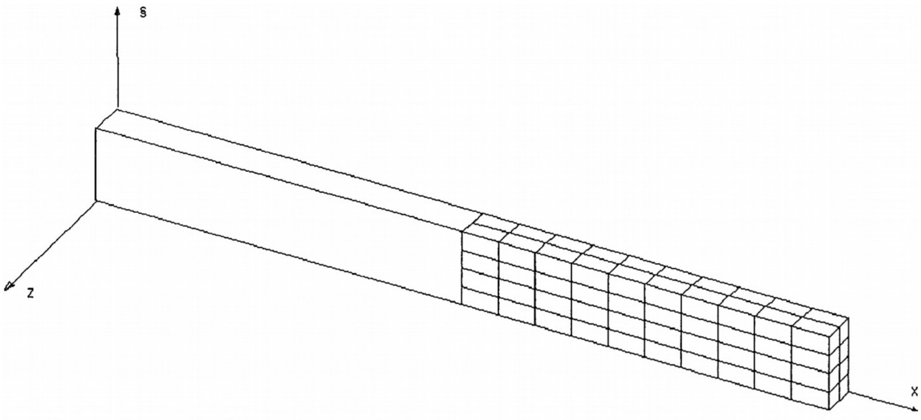

Figure 3.1 Grid of the geometry of the problem.

The beam is divided into two equal parts. Each half is represented by a substructure. Those are generated by the method of Mac-Neal.

3.2 Characteristics of the grid

Many nodes: 557
Many meshs and types: 80 `HEXA20`, 20 `QUAD8`

3.3 Sizes tested and results

Place	Type of size	Value of reference	Type of reference	Tolerance (%)
$X=\frac{L}{4}$ (first half)	DY	$1.95994\times 10^{-5}+8.49179\times 10^{-6} j$	'ANALYTICAL'	5.0
$X=\frac{L}{2}$ (first half)	DY	$-6.999387\times 10^{-6}-1.14501\times 10^{-5} j$	'ANALYTICAL'	5.0
$X=\frac{L}{2}$ (second half)	DY	$-6.999387\times 10^{-6}-1.14501\times 10^{-5} j$	'ANALYTICAL'	5.0
$X=3\frac{L}{4}$ (second half)	DY	$1.95994\times 10^{-5}+8.49179\times 10^{-6} j$	'ANALYTICAL'	5.0

4 Summary of the results

This test makes it possible to validate the dynamic under-structuring with interface of the Mac-Neal type. The solution is compared with an analytical solution. The got results are in concord.