

SSLL102 - Fixed beam subjected to efforts unit

Summary:

This test allows a simple checking of calculations of right beams and hull 1D in linear mechanics of the structures static. The model is linear.

Modelings A, B, C, D, F, G, I and J make it possible to test the various types of elements of right beams in *Code_Aster*. For each modeling, one calculates simultaneously 3 beams of different sections: rectangle, circle, angle.

Modeling A makes it possible of more than test the change of reference mark: the beam is directed according to the trisecting one with the total reference mark.

Modeling E tests the loading distributed on voluminal edges of elements.

Modeling F corresponds to a loading distributed varying linearly with modeling `POU_D_E`.

Modeling G corresponds to a loading distributed varying linearly with modeling `POU_D_TG`.

Modeling H makes it possible to test a loading distributed varying linearly with modeling `TUYAU_3M`.

Modeling I takes again the loading of modeling A for `POU_D_EM`.

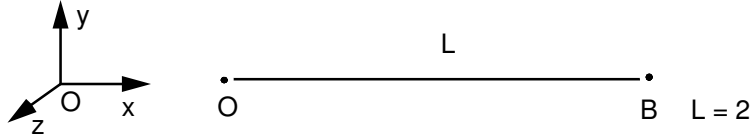
Modeling J corresponds has a loading distributed varying linearly with modeling `POU_D_EM`.

The values tested are the generalized displacements, efforts and the constraints.

1 Problem of reference

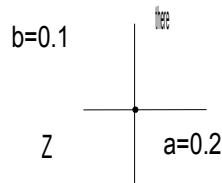
1.1 Geometry

Right beam length L , of direction x . Dimensions are expressed in meters, [m].

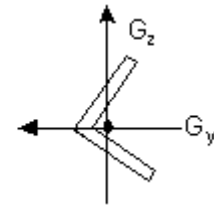
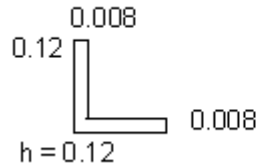


One calculates simultaneously 3 types of different cross sections:

1 rectangular section



1 corner section with equal wings



1 circular section

$R=0.1$

1.2 Material properties

Young modulus: $E = 2.10^{11} Pa$

Poisson's ratio: $\nu = 0.3$

1.3 Boundary conditions and loadings

Embedding in O

6 unit loadings in B :

$$\begin{array}{ll} F_x = 1 & M_x = 1 \\ F_y = 1 & M_y = 1 \\ F_z = 1 & M_z = 1 \end{array}$$

1 loading combined inflection plus traction: $F_x = 1$; $M_y = 1$; $M_z = 1$;

1 loading combined efforts cutting-edges plus torsion: $F_y = 1$ $F_z = 1$ $M_x = 1$

1 loading distributed linear: $F_y = 1000.x$ circular section (modelings F, G, I) (with simple support in A and B in this case)

1.4 Notation of the characteristics of cross sections

The geometrical characteristics of the cross sections are noted:

A	surface of the section
I_y, I_z	geometrical moments of inertia compared to the main axes of inertia of the section
JX	constant of torsion
ay, az	coefficients of shearing in the directions G_y and G_z
$A'_y = \frac{A}{ay}$ and $A'_z = \frac{A}{az}$	equivalent reduced surfaces
e_y, e_z	eccentricity of the center of torsion
JG	constant of warping

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution [bib1] and [bib2]: displacements in B

Simple traction	$u_x = \frac{F_x L}{E S}$		
Pure bending	$u_y = \frac{F_y L^3 (4 + \phi_y)}{12 E I_z}$	$\theta_z = \frac{L^2 F_y}{2 E I_z}$	$\phi_y = \frac{12 E I_y}{L^2 G A'_y}$
Pure bending	$u_z = \frac{F_z L^3 (4 + \phi_z)}{12 E I_y}$	$\theta_y = \frac{-L^2 F_z}{2 E I_y}$	$\phi_z = \frac{12 E I_z}{L^2 G A'_z}$
Torsion		$\theta_x = \frac{M_x L}{G J_x}$	
Pure inflection	$u_z = -\frac{M_y L^2}{2 E I_y}$	$\theta_y = \frac{M_y L}{E I_y}$	
Pure inflection	$u_y = \frac{M_z L^2}{2 E I_z}$	$\theta_z = \frac{M_z L}{E I_z}$	

Notice 1:

For the corner section, as the center of shearing is not confused with the centre of gravity ($e_y \neq 0$), it is necessary to add the torque: $M_x = F_z e_y$ with the loading $F_z = 1$.

This modifies displacement:

$$u_z = \frac{F_z L^3 (4 + \phi_z)}{12 E I_y} + \theta_x e_y \quad \theta_x = \frac{M_x L}{G J_x}$$

In the same way, the loading $M_x = 1$ involve a displacement $u_z = \theta_x e_y$.

Loading distributed linear:

$$u_y(x) = \frac{p x}{360 L E I} (3x^4 - 10L^2 x^2 + 7L^4) \quad u_y^{max} = \frac{0.00652 p L^4}{E I}$$

$en x = 0.519 L$

Notice 2:

With regard to modeling A, the beam is carried by the vector $e_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. The other vectors of

the local reference mark are: $e_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ and $e_3 = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$

The components of the vector displacement in the total reference mark are obtained by:

$$u_G = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{pmatrix} u_{local}$$

Generalized efforts and constraints in O :

$$N(O) = F_x \quad \sigma_{xx} = \frac{N}{S}$$

$$M_z(O) = T_y L \quad T_y = F_y \quad \sigma_{xx}(y) = \frac{M_z y}{I_z} \quad \sigma_{xy} = \frac{T_y}{k_y S}$$

$$M_y(O) = -T_z L \quad T_z(O) = F_z \quad \sigma_{xx}(y) = \frac{-M_y z}{I_y} \quad \sigma_{xz} = \frac{T_z}{k_z S}$$

$$M_x(0) = M_x(B) \quad \sigma_{xy} = \sigma_{xz} = \frac{M_x R_T}{J_x}$$

$$M_y(0) = M_y(B) \quad \sigma_{xx}(z) = \frac{M_y z}{I_y}$$

$$M_z(0) = M_z(B) \quad \sigma_{xx}(y) = \frac{M_z y}{I_z}$$

Loading distributed linear:

$$M_z(x) = \frac{-1000}{6} (L^2 x - x^3) \quad V_y(x) = \frac{1000 L^2}{6} - \frac{1000 x^2}{2} \quad \sigma_{xx}^{max} = \frac{M_z^{max} R}{I_z}$$

$$en x = \frac{L\sqrt{3}}{3}$$

2.2 Results of reference

Displacement of the point B ,
Efforts generalized at the point O ,
Constraints of the point O .

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

- 1.J.L. BATOZ, G. DHATT: "Modeling of the structures by finite elements" - Volume 2 ED. HERMES.
- 2.N.D. PIKLEY: "Formulated for Stress, Stain & Structural Matrices" ED. John Wiley & Sounds.

3 Modeling A

3.1 Characteristics of modeling

2 elements POU_D_E $k_y = k_z = 1$ $\phi = 0$ by type of section

S1 : Rectangular section modelled by SECTION: 'GENERAL'

$$A = 0.02 \quad I_y = 0.1666E-4 \quad I_z = 0.6666E-4 \quad J_x = 0.45776E-4$$

$$R_y = 0.1 \quad R_z = 0.05 \quad R_T = 0.0892632$$

Point de calcul des contraintes

S2 : Corner section

$$A = 1.856E-3 \quad I_y = 4.167339E-4 \quad I_z = 1.045547E-4$$

$$J_x = 0.39595E-8 \quad e_y = 41.012E-3 \quad e_z = 0.0$$

S3 : Rectangular section modelled by SECTION: RECTANGLE

$$H_y = 0.2 \quad H_z = 0.1$$

S4 : Section CIRCLE $R = 0.1$

$$I_y = I_z = \frac{\pi R^4}{4} = \frac{\pi}{4} 10^{-4}$$

3.2 Characteristics of the grid

4x2 elements POU_D_E. The beam is directed according to the vector (1,1,1).

3.3 Sizes tested and results

Loading case	Beam	Identification	Reference
$F_x = 1$	S1=S3	$u_x(B)$	2.887E-10
		$\theta_{xx}(0)$	50.
	S2	$u_x(B)$	3.11E-9
	S4	$u_x(B)$	1.838E-10
		σ_{xx}	31.83
	$F_y = 1$	S1=S3	$u_y(B)$
$\theta_z(B)$			1.225E-7
$\sigma_{xx}(0)$			3000
S2		$u_y(B)$	9.017E-8
S4		$\sigma_{xx}(0)$	2546.479
$F_z = 1$		S1=S3	$u_z(B)$
	$\theta_y(B)$		-4.243E-7
	$\sigma_{xx}(0)$		6000
	$\sigma_{xz}(0)$		50
	S2		$u_z(B)$
	S4	$\theta_y(B)$	1.553E-5
		$\theta_x(B)$	1.555E-5
		$u_z(B)$	1.386E-7
		$\theta_y(B)$	9th-8

		$\sigma_{xx}(0)$	2546.479
		$\sigma_{xz}(0)$	31,831
$M_x = 1$	$S1 = S3$	$\theta_x(B)$	3.279E-7
		$\sigma_{xy} = \sigma_{xz}(0)$	1950.0
	$S2$	$\theta_x(B)$	3.791E-4
		$u_z(B)$	2.199E-5
	$S4$	$\theta_x(B)$	9.556E-8
		$\sigma_{xy} = \sigma_{xz}(0)$	636.62
$M_y = 1$	$S1 = S3$	$u_z(B)$	-4.899E-7
		$\theta_y(B)$	4.243E-7
		$\sigma_{xx}(0)$	3000
	$S2$	$u_z(B)$	-1.959E-8
		$\theta_y(B)$	1.697E-8
	$S4$	$u_z(B)$	-1.04E-7
		$\theta_y(B)$	9.0E-8
		$\sigma_{xx}(0)$	1273.2395
$M_z = 1$	$S1 = S3$	$u_y(B)$	1.061E-7
		$\theta_z(B)$	1.225E-7
		$\sigma_{xx}(0)$	1500.0
	$S2$	$u_y(B)$	6.763E-8
		$\theta_z(B)$	7.809E-8
	$S4$	$u_y(B)$	9.0E-7
		$\sigma_z(B)$	1.04E-7
		$\sigma_{xx}(0)$	1273.2395
$M_y = 1$	$S1 = S3$	$\sigma_{xx} \max(0)$	4550.0
$M_z = 1$		$\sigma_{xx} \left(\frac{a}{2}, \frac{b}{2} \right)$	1550.0
$F_x = 1$			
	$S4$	$\sigma_{xx} \max(0)$	1832.4636
$F_y = 1$	$S1, S3$	$\sigma_{xy}(0)$	2000.0
$F_z = 1$		$\sigma_{xz}(0)$	2000.0
$M_x = 1$		$\sigma_{xx} \max(0)$	9000.0
	$S1, S3$	$\sigma_{xx} \left(\frac{a}{2}, \frac{b}{2} \right)$	-9000.0
	$S4$	$\sigma_{xx} \max(0)$	3601.27
		$\sigma_{xy}(0)$	668,451

4 Modeling B

4.1 Characteristics of modeling

2 elements POU_D_T.

The coefficients of shearing are:

S1 : Rectangular section

$$AY = AZ = 1.2 = \frac{1}{k_y}$$

S2 : Corner section

$$AY = AZ = \frac{1}{0.358}$$

S4 : Section CIRCLE

$$AY = AZ = \frac{10}{9}$$

4.2 Characteristics of the grid

4×2 elements POU_D_T

4.3 Sizes tested and results

One gives only the values which differ from modeling A (because of the taking into account of transverse shearing).

Loading	Section	Identification	Reference
$F_y = 1$	<i>S1</i> = <i>S3</i>	$u_y(B)$	2.0156E-7
		$\sigma_{xy}(0)$	60.
	<i>S2</i>	$u_y(B)$	1.666552E-7
	<i>S4</i>	$u_y(B)$	1.707308E-7
$F_z = 1$	<i>S1</i> , <i>S3</i>	$u_z(B)$	8.0156E-7
		$\sigma_{xz}(0)$	60.
	<i>S2</i>	$u_z(B)$	1.17559754E-6
	<i>S4</i>	$u_z(B)$	1.707308E-7
$F_y = 1$	<i>S4</i>	$\sigma_{xz}(0)$	37.13615
		$\sigma_{xy}(0)$	673.75592
$F_z = 1$	<i>S4</i>	$\sigma_{xz}(0)$	673.75592
		$\sigma_{xy}(0)$	673.75592
$M_x = 1$			

5 Modeling C

5.1 Characteristics of modeling

2 elements POU_D_TG.

Warping is not constrained.

The coefficients of shearing are identical to those of modeling B.

5.2 Characteristics of the grid

4×2 elements POU_D_TG

5.3 Sizes tested and results

Loading	Section	Identification	Reference
$F_y=1$	S1=S3	$u_y(B)$	2.0156E-7
		$\theta_{xy}(0)$	60.
	S2	$u_y(B)$	1.666552E-7
	S4	$u_y(B)$	1.70684E-7
		$\theta_{xy}(0)$	35.367765
$F_z=1$	S1, S3	$u_z(B)$	8.0156E-7
		$\theta_{xz}(0)$	60.
	S2	$u_z(B)$	1.17559754E-6
	S4	$u_z(B)$	1.70684E-7
		$\theta_{xz}(0)$	35.367765

5.4 Notice

Warping is not constrained. The results are thus identical to those of modeling B.

6 Modeling D

6.1 Characteristics of modeling

Elements POU_D_TG , constrained torsion

$$JG = \begin{cases} 5.5556E-8 & \text{pour } S_1 \\ 4.439822E-11 & \text{pour } S_2 \end{cases}$$

in 0 $GRX = 0$

6.2 Characteristics of the grid

- 10 elements,
- refinement towards embedding.

6.3 Sizes tested and results

Same results as for modeling C, except those which relate to the effects of warping.

Loading	Section	Identification	Reference
$F_z = 1$	S2	$\theta_x = DRX$	2.62034E-5
		$u_z = DZ$	1.14578E-6
		GRX	1.34652E-5
$M_x = 1$	S1	$u_z = DZ$	5.52E-7
		GRX	2.84E-7
	S2	u_z	2.6203E-5
		θ_x	6.3892E-4
		GRX	3.28324E-4

6.4 Remarks

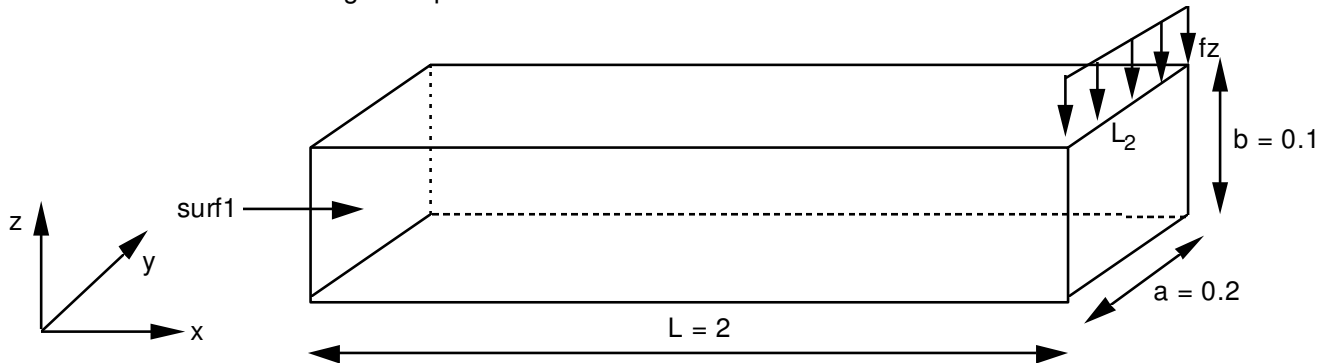
For θ_x the solution is (cf [bib1]):

$$\theta_x = \frac{M_x L}{G J_x} + \frac{M_x (1 - e^{2\alpha L} - 2e^{\alpha L})}{\alpha^3 E J G (1 + e^{2\alpha L})} \quad \alpha^2 = \frac{G J}{E J G}$$

7 Modeling E

7.1 Characteristics of modeling

The beam is with a grid in quadratic solid elements HEXA20.



The beam is embedded on the level of the section *surf1*. It is subjected to a unit shearing action which is modelled by a linear density of load fz applying to the 4 meshes SEG3 constituting the higher edge $L2$.

7.2 Characteristics of the grid

The beam is with a grid with 640 quadratic solid elements HEXA20.
The model comprises 3665 nodes.

7.3 Sizes tested and results

One tests the value of the arrow according to z node medium of the section where one applies the loading (node $N62$).

Identification	Reference	Aster	% difference
dz node $N62$	-8.0E-7	-7.9523E-7	-0,596

7.4 Remarks

The value of reference corresponds to the value given by the Resistance Of Materials.

8 Modeling F

8.1 Characteristics of modeling

The model is composed of 10 elements right beam of Euler. The section is circular full, of ray 0.1m .

8.2 Characteristics of the grid

It consists of 10 elements `POU_D_E`. The length of the beam is $L=6m$

8.3 Sizes tested and results

8.3.1 Interior efforts

	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ(2\sqrt{3})$	-1.3856E+04

8.3.2 Constraint

	Analytical results
$SIXX(2\sqrt{3})$	1.7642E+07

9 Modeling G

9.1 Characteristics of modeling

The model is composed of 10 elements right beam of Timoshenko with warping. The section is circular full, of ray 0.1m .

9.2 Characteristics of the grid

It consists of 10 elements POU_D_TG. The length of the beam is $L=6m$

9.3 Sizes tested and results

9.3.1 Interior efforts

	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ(2\sqrt{3})$	-1.3856E+04

9.3.2 Displacement (marks with arrows near to the medium of the beam)

	Results Aster (not regression)	Tolerance (%)
$DY(3.115977734)$	3.23499E-03	1.E-4

10 Modeling H

10.1 Characteristics of modeling

The model is composed of 21 elements TUYAU_3M being pressed on meshes SEG4 .
The effort distributed is imposed along the axis y . the inflection thus takes place around z .

10.2 Characteristics of the grid

It consists of 21 meshes SEG3. The length of the pipe is $L=6m$

10.3 Sizes tested and results

10.3.1 Displacements

	Analytical results
$D_y \text{ maxi}$	9.38888E-03

10.3.2 Interior efforts

	Analytical results
$V_y(x=0)$	6.0000E+03
$V_y(x=L=6)$	-1.2000E+04
$MFZ 2\sqrt{3}$	-1.3856E+04

10.3.3 Constraints

It are calculated at the point of X-coordinate $x = \frac{L\sqrt{3}}{3}$ who corresponds to the maximum moment:

$$M_z(x) = \frac{-1000}{9\sqrt{3}} L^3 = -13856.41 \text{ N.m}$$

For the angle 0 on the circumference of the pipe (the origin of the angles being the axis z), the constraints are worthless, and for angle 90, they are maximum:

$$\sigma_{xx}^{max} = \frac{M_z^{max}(R-e/2)}{I_z} = -4.87363E+07 \text{ Pa}$$

	Reference	Tolerance
$\sigma_{xx}(\alpha=0)$	0	0,10%
$\sigma_{xx}(\alpha=90)$	-4.87363E+07	1,00%
MFZ	-1.3856E+04	1.%

11 Modeling I

11.1 Characteristics of modeling

The model is composed of 2 elements `POU_D_EM`.

The loading is similar to that of modeling `A` (torque only)

11.2 Characteristics of the grid

It consists of 2 meshes `SEG2`. The length of the beam is $L = 2\text{ m}$

The beam is directed according to the vector $(1, 1, 1)$.

The section is rectangular, identical to that of modeling `A`.

11.3 Sizes tested and results

11.3.1 Displacement (rotation due to the torque)

	Results Aster (not regression)	Tolerance (%)
$DX = DY = DZ$	3.2792525E-07	1.E-6

12 Modeling J

12.1 Characteristics of modeling

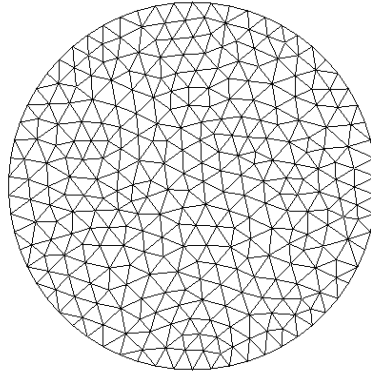
The model is composed of 10 elements `POU_D_EM`.
One applies a force distributed of $6000\text{N}/m$ on all the beam.

12.2 Characteristics of the grid

It consists of 10 meshes `SEG2`. The length of the beam is $L=6m$

The grid of the section consists of:

- 373 nodes
- 62 `SEG2`
- 682 `TRIA3`



12.3 Sizes tested and results

12.3.1 Interior efforts

Interior efforts	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
MFZ	-1.3856E+04

12.3.2 Displacement (marks with arrows near to the medium of the beam)

	Results Aster (not regression)	Tolerance (%)
$DY(3.115977734)$	3.23499E-03	1.E-4

13 Summary of the results

This test makes it possible to check the good performance of the elements simultaneously `POU_D_E`, `POU_D_T` and `POU_D_TG` on 3 types of different sections. The perfect coincidence of the results with the analytical solutions (RDM) is normal, and must always be observed, since the solution is contained in the functions of form of the elements.

Moreover, modeling E makes it possible to test the loading distributed on voluminal edges of elements. The variation with the analytical solution (RDM) is lower than 0.6% .

Modelings F, G, H and J make it possible to test the loading distributed (linear variation) for the elements of beam `POU_D_E`, `POU_D_TG`, `POU_D_EM` and elements of `PIPE`. The variation with the analytical solution (Resistance of Materials) is lower than 0.6% .