

SSLL103 - Elastic buckling of an angle

Summary:

A right beam (corner with equal wings) biarticulée is subjected to a normal effort (excentré or not) or to one bending moment.

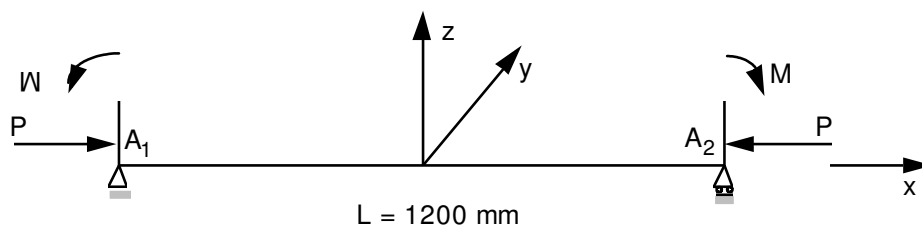
One seeks the critical loads of elastic buckling.

- linear elastic mechanics,
- buckling of a beam,
- offsetting of the center of torsion,
- interest of the test: calculation of the geometrical matrix of rigidity of the elements `POU_D_TG` and `POU_D_T`,
- 2 modelings.

An uncertainty persists on the number of modes of buckling of the reference solution [§5].

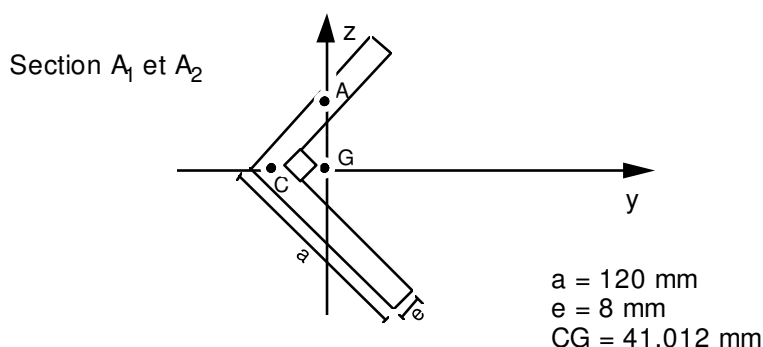
1 Problem of reference

1.1 Geometry



Caractéristiques de la section

$$\begin{aligned} A &= 1856 \text{ mm}^2 \\ I_y &= 4167339 \text{ mm}^4 \\ I_z &= 1045547 \text{ mm}^4 \\ J &= 39595 \text{ mm}^4 \\ I_{\omega} &= 44398819 \text{ mm}^4 \\ I_{yr2} &= 84948392 \text{ mm}^4 \\ y_c &= -41.012 \text{ mm} \\ z_c &= 0 \end{aligned}$$



1.2 Material properties

Young modulus: $E = 2.10E-5 \text{ MPa}$

Poisson's ratio: $\nu = 0.3$

1.3 Boundary conditions and loadings

$$A1 : DX = DY = DZ = DRX = 0$$

$$A2 : DY = DZ = DRX = 0$$

Loading

- case 1: axial load P in G
- case 2: axial load P in C
- case 3: axial load P in A
- case 4: bending moment M

1.4 Remarks

For cases 2 and 3, one applies in $A2$ an effort in G , then one superimposes in $A1$ and $A2$ one bending moment (according to oz for case 2, following oy for case 3) to offset the effort in C (or in A).

2 Reference solution

2.1 Method of calculating used for the reference solution

With taking into account of warping, the calculations done by V. Of City De Goyet [bib1] give: that is to say:

$$I_y = \int_A z^2 dA \quad I_y = \int_A y^2 dA \quad I_{yr^2} = \int_A y(y^2 + z^2) dA \quad I_{yr^2} = \int_A z(y^2 + z^2) dA$$

$$P_{cry} = \frac{\pi^2 E I_z}{L^2} \quad P_{crz} = \frac{\pi^2 E I_y}{L^2} \quad P_{crx} = \left(\frac{GJ + \pi^2 E I_\omega}{L^2} \right) A r_a$$

$$A r_c = \frac{(I_y + I_z)}{A} + y_c^2 + z_c^2 + y_c \left(\frac{I_{yz}}{I_z} - 2 y_c \right) + z_c \left(\frac{I_{xz}}{I_z} - 2 z_c \right)$$

$$A r_a = \frac{(I_y + I_z)}{A} + y_c^2 + z_c^2 + y_a \left(\frac{I_{yz}}{I_z} - 2 y_c \right) + z_a \left(\frac{I_{xz}}{I_z} - 2 z_c \right)$$

with:

(y_a, z_a) : coordinates of the point of load application
 (y_c, z_c) : coordinates of the center of torsion

Case 1,2,3:

One obtains 3 critical loads by solving the equation of the 3° degree in P :

$$A r_a (P_{cry} - P)(P_{crz} - P)(P_{crx} - P) - P^2 (P_{crz} - P)(z_c - z_a)^2 - P^2 (P_{cry} - P)(y_c - y_a)^2 = 0$$

Case 4:

Critical moment M_{cr} (around the axis y) is worth:

$$M_{cr} = \pm \left(\left(GJ + \frac{\pi^2 E I_\omega}{L^2} \right) P_{cry} \right)^{1/2}$$

By neglecting warping: the analytical solution of reference is given in [bib2] [bib3].

2.2 Results of reference

Values of the critical loads corresponding to the first modes of buckling for the various loading cases.

2.3 Uncertainty on the solution

Analytical solution. The values of reference are obtained using *NAG* (routine *COSAGF*, $EPS = 10^{-8}$).

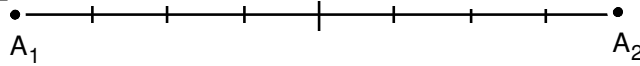
2.4 Bibliographical references

1. V. OF TOWN OF GOYET "Analyzes static nonlinear by the finite element method of the formed space structures by beams with nonsymmetrical section" - Doctorate University of Liege, MSM, academic year (1988-1989).
2. P. PENSERINI "elastic Instability of the beams with open mean profile: theoretical and digital aspects" Notes EDF/DER/HM77/112.
3. J. CHERRY TREE "Propagation of two cases tests of modeling of the calculation of the beams in elastic buckling in *Code_Aster*" HM77/184

3 Modeling A

3.1 Characteristics of modeling

8 elements POU_D_TG



3.2 Characteristics of the grid

Many nodes: 9

Many meshes and types: 8 SEG2

3.3 Sizes tested and results

Identification	Reference
Case 1	
mode 1	- 6.92531E+05
mode 2	- 1.50487E+06
mode 3	- 1.00589E+07
Case 2	
mode 1	- 1.50487E+06
mode 2	- 5.99812E+06
mode 3	1.47904E+06
Case 3	
mode 1	- 5.72260E+05
mode 2	- 2.45950E+06
mode 3	- 1.85673E+07
Case 4	
mode 1	7.00631E+07

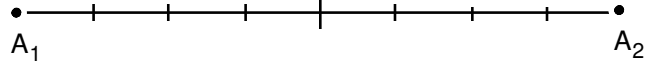
3.4 Remarks

The precision is excellent with 8 elements in the length.

4 Modeling B

4.1 Characteristics of modeling

8 elements POU_D_T



4.2 Characteristics of the grid

Many nodes: 9

Many meshes and types: 8 SEG2

4.3 Sizes tested and results

Identification	Reference
Case 1	
mode 1	- 6.796E+05
mode 2	- 1.505E+06
mode 3	- 1.0055E+07
Case 2	
mode 1	- 1.505E+06
mode 2	- 5.998E+06
Case 3	
mode 1	- 5.638E+05
mode 2	- 2.453E+06
mode 3	- 1.8525E+07
Case 4	
mode 1	6.9376E+07

4.4 Remarks

The precision is rather good with 8 elements in the length. The solution differs a little that obtained with warping (modeling A).

5 Summary of the results

The analytical solution gives us 3 modes of buckling of which the critical loads are roots of an equation of the 3° degree.

Y-a it of other critical loads inserted between the 3 found values?

Aster find the good critical loads, but in the middle of much of others... for example for case 3, the 3 sought critical loads correspond to `NUME_MODE` : 1.10 and 19.

This is true for two modelings.