SSLL104 - Predeformations in a beam right-hand side

Summary:
This test validates the taking into account of predeformations in the elastic design of a right beam. The characteristics of calculation are:

- static analysis,
- linear behavior,
- linear model

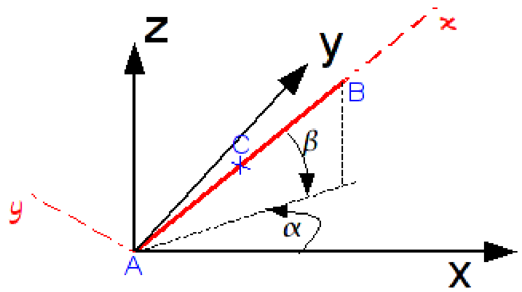
Modeling A validates the taking into account of predeformations indicated by value (AFFE_CHAR_MECA/PRE_EPSI) for elements POU_D_E, POU_D_T, POU_D_TG, POU_D_EM and POU_D_TGM. Lsolution has is analytical.

Modeling B validates the taking into account of predeformations indicated by function (AFFE_CHAR_MECA_F/PRE_EPSI) for elements POU_D_E, POU_D_T, POU_D_TG, POU_D_EM and POU_D_TGM. Lsolution has of reference is a calculation with predeformations indicated by value. It is specified that this loading is authorized for the elements of beam only if the value of the function to the two nodes of the same element is the same one.

Valid modeling C, for the element BAR, the taking into account of predeformations indicated by value, by comparison with the element POU_D_E and the taking into account of predeformations indicated by function by comparison with a calculation with predeformations indicated by value.

1 Problem of reference

1.1 Geometry



A beam AB length $l=100\,mm$ is located on trisecting trihedron (X,Y,Z) : the coordinates of the point B are: $B=\left(\frac{100}{\sqrt{3}},\frac{100}{\sqrt{3}},\frac{100}{\sqrt{3}}\right)$

One defines also a point C medium of A, B.

The local reference mark (A,x,y,z) results from the total reference mark (A,X,Y,Z) by the nautical angles $\begin{cases}\alpha=45°\\ \beta=-35.26°\,solution\,de\cos\beta=\sqrt{\frac{2}{3}}\end{cases}$

1.2 Material properties

The material is elastic linear.

Young modulus $E=1.0\,MPa$ (without influence on the result).

Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

Embedding in A : $DX=DY=DZ=DRX=DRY=DRZ=0$.

For modeling a:

Loading: predeformation in the local reference mark (A,x,y,z)

- elongation according to x : $\epsilon_X^0=0.001$
- curve around y : $\chi_y^0=0.002$
- curve around z : $\chi_z^0=0.003$

1.4 Characteristics of the section of beam

All the characteristics (surface, inertias,…) are taken equal to 1. They are without influence on the result.

2 Reference solution

2.1 Method of calculating used for the reference solution

Modeling a:

The solution is analytical. It is calculated in the local reference mark.
that is to say:

$U=(u,v,w,\theta_x,\theta_y,\theta_z)$ the displacement of the beam and $E=(\epsilon_x,\chi_y,\chi_z,\gamma_{xy},\gamma_{xz})$ generalized deformation.

That is to say the solution:

$$u=\alpha x \qquad v=\gamma\frac{x^2}{2} \qquad w=-\beta\frac{x^2}{2} \qquad \theta_x=0 \qquad \theta_y=\beta x \qquad \theta_z=\gamma x$$

then:

$$\epsilon_x=u_{,x}=\alpha \qquad \chi_y=\theta_{y,x}=\beta \qquad \chi_z=\theta_{z,x}=\gamma \qquad \gamma_{xy}=v_{,x}-\theta_z=0 \qquad \gamma_{xz}=w_{,x}+\theta_y=0$$

If one chooses $\alpha=\epsilon_X(=0.001)$, $\beta=\chi_y^0(=0.002)$, $\gamma=\chi_z^0(=0.003)$ then $E-E_{init}=0$ and Lbe efforts are worthless: balance is thus checked. In addition, the solution checks the boundary conditions of embedding in A. It is thus the solution of the posed problem.

Modeling b:

The functions used to impose the predeformations are of standard function “staircase” depend on the coordinates of the nodes of the meshs. To obtain the reference solution, one affects the values of the predeformations meshs by meshs.

Modeling C:

In modeling C, the meshs of beam are duplicated in meshs of bar.

For the validation of the predeformations by value, a first calculation is done in which the predeformations are imposed on the meshs of beam, it is used as reference to the second calculation in which the predeformations are imposed on the meshs of bar.

For the validation of the predeformations by function, the reference solution is obtained by affecting on each mesh the value of the function depending on the geometry (coordinated point of Gauss).

2.2 Results of reference

Modeling a:

The results expressed in the local reference mark are:

In B :

$$Dx=0.10\,mm\,;Dy=15.0\,mm\,;Dz=-10.0\,mm\,;DRx=0.0\,rd\,;DRy=0.2\,rd\,;DRz=0.30\,rd$$

In C :

$$Dx=0.05\,mm\,;Dy=3.75\,mm\,;Dz=-2.50\,mm\,;DRx=0.0\,rd\,;DRy=0.1\,rd\,;DRz=0.15\,rd$$

In the total reference mark, one finds at the points B and C :

$$DX(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}\left(-3\sqrt{6}+2\sqrt{2}\right)[mm] \qquad DX(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}\left(-3\sqrt{6}+2\sqrt{2}\right)[mm]$$

$$DY(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}\left(3\sqrt{6}+2\sqrt{2}\right)[mm] \qquad DY(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}\left(3\sqrt{6}+2\sqrt{2}\right)[mm]$$

$$DZ(B)=\frac{\sqrt{3}}{30}+5\frac{\sqrt{3}}{6}\left(-4\sqrt{2}\right)[mm] \qquad DZ(C)=\frac{\sqrt{3}}{60}+5\frac{\sqrt{3}}{24}\left(-4\sqrt{2}\right)[mm]$$

$$DRX(B)=\frac{1}{20}\left(-\sqrt{6}-2\sqrt{2}\right)[rd] \qquad DRX(C)=\frac{1}{40}\left(-\sqrt{6}-2\sqrt{2}\right)[rd]$$

$$DRY(B)=\frac{1}{20}\left(-\sqrt{6}+2\sqrt{2}\right)[rd] \qquad DRY(C)=\frac{1}{40}\left(-\sqrt{6}+2\sqrt{2}\right)[rd]$$

$$DRZ(B)=\frac{1}{20}\left(2\sqrt{6}\right)[rd] \qquad DRZ(C)=\frac{1}{40}\left(2\sqrt{6}\right)[rd]$$

Modeling B and C:

Results coming from other calculations carried out in this same test.

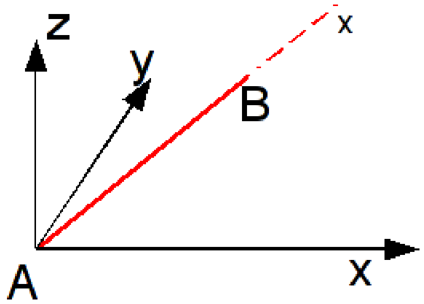
2.3 Uncertainty on the solution

Modeling a:

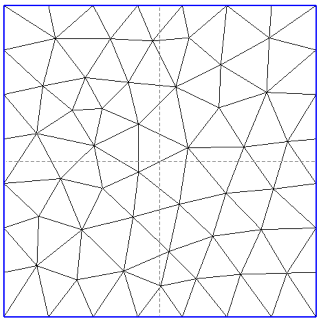
The solution is exact for the theory of the beams of Euler (or of Timoshenko because there is no shearing). Torsion not intervening, the solution is also valid for the elements `POU_D_TG` and `POU_D_TGM`.

3 Modeling A

3.1 Characteristics of modeling



- The segment AB is cut out in 10 of the same elements length (10.). (Only one element would be sufficient).
- 5 identical calculations are successively done on this grid with 4 different modelings:
  - with 10 elements `POU_D_E`
  - with 10 elements `POU_D_T`
  - with 10 elements `POU_D_TG`
  - with 10 elements `POU_D_EM`
  - with 10 elements `POU_D_TGM`



Grid of the section:
- 77 nodes
- 124 `TRIA3`

3.2 Characteristics of the grid

Many nodes: 11
Many meshs and types: 10 `SEG2`

3.3 Sizes tested and results

Modeling	Identification		Reference	% difference
POU_D_E	B	DX	– 6.4664E+00	< 1.0E-9
		DY	1.4747E+01	< 1.0E-9
		DZ	– 8.1072E+00	< 1.0E-9
		DRX	– 2.6390E-01	< 1.0E-9
		DRY MARTINI	1.8947E-02	< 1.0E-9
		DRZ	2.4495E-01	< 1.0E-9
	C	DX	– 1.6022E+00	< 1.0E-9
		DY	3.7011E+00	< 1.0E-9
		DZ	– 2.0124E+00	< 1.0E-9
		DRX	– 1.3195E-01	< 1.0E-9
		DRY MARTINI	9.4734E-03	< 1.0E-9
		DRZ	1.2247E-01	< 1.0E-9

Modeling	Identification		Reference	% difference
POU_D_T	B	DX	– 6.4664E+00	< 1.0E-9
		DY	1.4747E+01	< 1.0E-9
		DZ	– 8.1072E+00	< 1.0E-9
		DRX	– 2.6390E-01	< 1.0E-9
		DRY MARTINI	1.8947E-02	< 1.0E-9
		DRZ	2.4495E-01	< 1.0E-9
	C	DX	– 1.6022E+00	< 1.0E-9
		DY	3.7011E+00	< 1.0E-9
		DZ	– 2.0124E+00	< 1.0E-9
		DRX	– 1.3195E-01	< 1.0E-9
		DRY MARTINI	9.4734E-03	< 1.0E-9
		DRZ	1.2247E-01	< 1.0E-9

Modeling	Identification		Reference	% difference
POU_D_TG	B	DX	– 6.4664E+00	< 1.0E-9
		DY	1.4747E+01	< 1.0E-9
		DZ	– 8.1072E+00	< 1.0E-9
		DRX	– 2.6390E-01	< 1.0E-9
		DRY MARTINI	1.8947E-02	< 1.0E-9
		DRZ	2.4495E-01	< 1.0E-9
	C	DX	– 1.6022E+00	< 1.0E-9
		DY	3.7011E+00	< 1.0E-9
		DZ	– 2.0124E+00	< 1.0E-9
		DRX	– 1.3195E-01	< 1.0E-9
		DRY MARTINI	9.4734E-03	< 1.0E-9
		DRZ	1.2247E-01	< 1.0E-9

Modeling	Identification		Reference	% difference
POU_D_EM	B	DX	– 6.4664E+00	< 1.0E-6
		DY	1.4747E+01	< 1.0E-5
		DZ	– 8.1072E+00	< 1.0E-5
		DRX	– 2.6390E-01	< 1.0E-5
		DRY MARTINI	1.8947E-02	< 1.0E-4
		DRZ	2.4495E-01	< 1.0E-5
	C	DX	– 1.6022E+00	< 1.0E-6
		DY	3.7011E+00	< 1.0E-5
		DZ	– 2.0124E+00	< 1.0E-5
		DRX	– 1.3195E-01	< 1.0E-5
		DRY MARTINI	9.4734E-03	< 1.0E-4
		DRZ	1.2247E-01	< 1.0E-5

Modeling	Identification		Reference	% difference
POU_D_TGM	B	DX	– 6.4664E+00	< 1.0E-8
		DY	1.4747E+01	< 1.0E-8
		DZ	– 8.1072E+00	< 1.0E-8
		DRX	– 2.6390E-01	< 1.0E-8
		DRY MARTINI	1.8947E-02	< 1.0E-8
		DRZ	2.4495E-01	< 1.0E-9
	C	DX	– 1.6022E+00	< 1.0E-10
		DY	3.7011E+00	< 1.0E-8
		DZ	– 2.0124E+00	< 1.0E-8
		DRX	– 1.3195E-01	< 1.0E-8
		DRY MARTINI	9.4734E-03	< 1.0E-7
		DRZ	1.2247E-01	< 1.0E-8

4 Modeling B

4.1 Characteristics of modeling

Identical to modeling A

4.2 Characteristics of the grid

Many nodes: 11
Many meshs and types: 10 SEG2

4.3 Loading

For the components of predeformation, the functions are the following ones:

EPX	$1\text{E-3}.\mathit{escalier}(X).\mathit{INST}$
KY	$2\text{E-3}.\mathit{escalier}(Y).\mathit{INST}$
KZ	$3\text{E-3}.\mathit{escalier}(Z).\mathit{INST}$

For each node of the grid the coordinate X is equal to the coordinate Y, itself equalizes with the coordinate Z. The function staircase is made so that its value is:
- 1 on the meshs E1 and E2
- 2 on the meshs E4 and E5
- 3 on the meshs E7 and E8

There is no predeformation imposed on the other meshs: E3, E6, E9 and E10.

In the calculation of reference, one thus affects the following values of predeformation:

	E1, E2	E4, E5	E7, E8
EPX	1E-3	2nd-3	3rd-3
KY	2nd-3	4th-3	6th-3
KZ	3rd-3	6th-3	9th-3

The taking into account of the dependence at time is made by the addition of a multiplying function to the loading.

4.4 Sizes tested and results

Modeling POU_D_E :
Calculation of reference:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'NON_REGRESSION'	-	-
Moment 2, Node B , DY	'NON_REGRESSION'	-	-

The second calculation:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'AUTRE_ASTER'	-7.75966054341	1E-6
Moment 2, Node B , DY	'AUTRE_ASTER'	35.3923671581	1E-6

Modeling POU_D_T :
Calculation of reference:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'NON_REGRESSION'	-	-
Moment 2, Node B , DY	'NON_REGRESSION'	-	-

The second calculation:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'AUTRE_ASTER'	-7.75966054341	1E-6
Moment 2, Node B , DY	'AUTRE_ASTER'	35.3923671581	1E-6

Modeling POU_D_TG :
Calculation of reference:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'NON_REGRESSION'	-	-
Moment 2, Node B , DY	'NON_REGRESSION'	-	-

The second calculation:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'AUTRE_ASTER'	-7.75966054343	1E-6
Moment 2, Node B , DY	'AUTRE_ASTER'	35.3923671581	1E-6

Modeling POU_D_EM :
Calculation of reference:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'NON_REGRESSION'	-	-
Moment 2, Node B , DY	'NON_REGRESSION'	-	-

The second calculation:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'AUTRE_ASTER'	-7.75966051002	1E-6
Moment 2, Node B , DY	'AUTRE_ASTER'	35.392366221	1E-6

Modeling POU_D_TGM :
Calculation of reference:

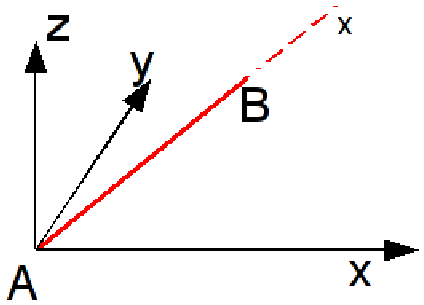
Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'NON_REGRESSION'	-	-
Moment 2, Node B , DY	'NON_REGRESSION'	-	-

The second calculation:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'AUTRE_ASTER'	-7.75966051062	1E-6
Moment 2, Node B , DY	'AUTRE_ASTER'	35.3923662219	1E-6

5 Modeling C

5.1 Characteristics of modeling



The segment AB is cut out in 10 of the same elements length (10.) who form the group BEAM. The meshs are duplicated to form the group BAR.

5.2 Characteristics of the grid

Many nodes: 11
Many meshs and types: 2 times 10 SEG2

5.3 Loading

5.3.1 Calculations with assignment by value

In the calculation of reference, the predeformation is affected on the elements of beam.

EPX	1E-3

In the second calculation, the predeformation is affected on the elements of bar.

EPXX	1E-3

5.3.2 Calculations with assignment by function

The function of predeformation is the following one:

EPXX	$1E\text{-}5.(X+Y+Z).INST$

For the calculation of reference (assignment by value), one calculates and one affects the value of $1E\text{-}5.(X+Y+Z)$ for each mesh of bar by taking the coordinates of its point of Gauss (medium of the mesh). The dependence at time is made by the addition of a multiplying function to the loading.

5.4 Sizes tested and results

5.4.1 Calculations with assignment by value

Calculation of reference:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'NON_REGRESSION'	-	-

The second calculation:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'AUTRE_ASTER'	0.0288675134603	1E-6

5.4.2 Calculations with assignment by function

Calculation of reference:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'NON_REGRESSION'	-	-
Moment 2, Node B , DY	'NON_REGRESSION'	-	-

The second calculation:

Identification	Type of reference	Values of Référence	Tolerance
Moment 1, Node B , DX	'AUTRE_ASTER'	0.0250000000004	1E-6
Moment 2, Node B , DY	'AUTRE_ASTER'	0.0499999999999	1E-6

6 Summary of the results

As one could expect it, the results are very precise. They validate the good taking into account of the predeformations in the elements of beam and of bar as well by assignment by value as by function.