SSLL107 - Validation of MACR_CARA_POUTRE

Summary:
The whole of modelings of this test make it possible to validate the macro_commande of calculation of the characteristics of section of beam, MACR_CARA_POUTRE, for all the options suggested. The studied sections are different according to modelings.

- Modeling A validates the calculation of the characteristics of section of a corner type.
- Modeling B validates the calculation of the characteristics of a circular section.
- Modeling C validates the calculation of the characteristics of a rectangular section.
- Modeling D validates the calculation of the characteristics of an alveolate rectangular section.
- Modeling E validates the calculation of the characteristics of an octagonal section.
- Modeling F validates the calculation of the characteristics of a circular section with a sequence on a calculation of beam.
- Modeling G validates the calculation of a network of 2 rectangular beams of section.
- Modeling H validates the calculation of the characteristics of a mean section out of U.
- Modeling I validates the calculation of the constant of torsion for a perforated section.
- Modeling J validates the calculation of the coefficients of shearing for a perforated circular section.
- Modeling K validates the calculation of the coefficients of shearing for a perforated rectangular section.

1 Problem of reference

1.1 Geometry

The geometry of the various sections is provided via a grid plan. It are different for each modeling, and will thus be described in the corresponding paragraphs.

Modeling F also implements the chained calculation of the characteristics of a circular section, and the use of these characteristics in a calculation of a right beam, length $L=1\mathrm{m}$ in pure traction.

1.2 Material properties

Without object, except for modeling F, where the treated beam has a Young modulus of $2.\mathrm{E}11\mathrm{Pa}$ and a Poisson's ratio of 0.3 .

1.3 Boundary conditions and loadings

Without object, except for modeling F: the right beam is embedded at an end, and is subjected at the other end with a tractive effort $F=1000\mathrm{N}$.

2 Reference solution

2.1 Method of calculating used for the reference solution

Since the solutions are specific to each modeling, they are described in the corresponding paragraphs. They are drawn mainly from [bib1] and [bib2];

2.2 Results of reference

One describes here the characteristics calculated by MACR_CARA_POUTRE [R3.08.03]:

- Characteristics geometrical of the sections
  - In the reference mark OYZ of description of the grid $2\mathrm{D}$ for the grid provided by the user
    - surface: A_M
    - position of the centre of gravity: CDG_Y_M, CDG_Z_M
    - moments and product of inertia of surface, in the centre of gravity G in the reference mark GYZ : IY_G_M, IZ_G_M, IYZ_G_M
  - In the same total reference mark, for the grid obtained by symmetrization if SYME_Y or SYME_Z :
    - surface: With
    - position of the centre of gravity: CDG_Y, CDG_Z
    - moments and product of inertia of surface, in the centre of gravity G in the reference mark GYZ : IY_G, IZ_G, IYZ_G
  - In the principal reference mark of inertia Gyz. cross-section, whose denomination corresponds to that used with the description of the elements of neutral fibre beam GX [U4.24.01].
    - principal moments of inertia of surface in the reference mark Gyz, usable for the calculation of the rigidity of inflection of the beam: IY and IZ
    - angle of flow of the reference mark GYZ with the principal reference mark of inertia Gyz : ALPHA
    - characteristic distances, compared to the centre of gravity G section for calculations of maximum constraints: Y_MAX, Y_MIN, Z_MAX, Z_MIN and R_MAX.
  - In the total reference mark, in a point P provided by the user:
    - Y_P, Z_P : not calculation of the moments of inertia
    - IY_P, IZ_P, IYZ_P : moments of inertia in the reference mark PYZ
    - IY_P, IZ_P : moments of inertia in the reference mark Pyz.

- Mechanical characteristics:

Identification	Significance
JX	Constant of torsion
EY	Position of the center of torsion/shearing
EZ	Position of the center of torsion/shearing
$PCTY$	Offsetting of the center of torsion in the reference mark GYZ along the axis Y
$PCTZ$	Offsetting of the center of torsion in the reference mark GYZ along the axis Z
AY	Coefficient of shearing
AZ	Coefficient of shearing
JG	Constant of warping

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

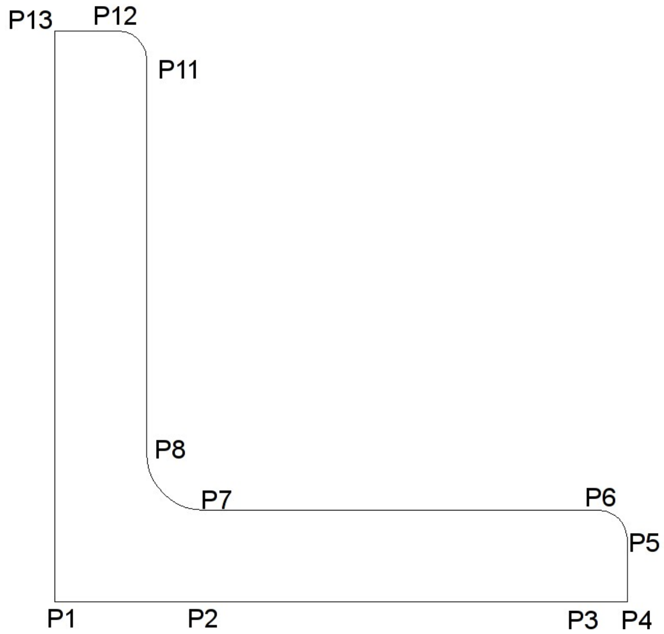
1. PILKEY W.D.: “ Formulated for stress, Structural Strain and Matrices ”. Wiley & Idiots, New York, 1994.

2. D. BLEVINS: Formulated for natural frequency and shape mode.

3 Modeling A

3.1 Characteristics of modeling

Corner section:

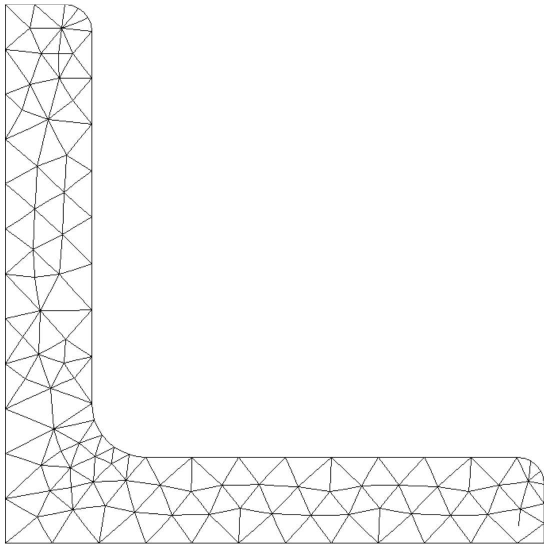


The coordinates of the points are:

P1	0.0	0.0
P2	1.3E-02	0.0
P3	4.75E-02	0.0
P4	5.0E-02	0.0E+00
P5	5.0E-02	5.5E-03
P6	4.750E-02	8.0E-03
P7	1.30E-02	8.0E-03
P10	8.0E-03	1.300E-02
P11	8.0E-03	4.75E-02
P12	5.5E-03	5.0E-02
P13	0.	5.0E-02

3.2 Characteristics of the grid

182 meshs TRIA6.



3.3 Reference solution

Pas de exact analytical solution. The values are values of nonregression.

3.4 Sizes tested and results

Identification	Reference	Tolerance
A	7.39E-04	1,00E-03
$ALPHA$	1.35E+02	1,00E-03
CDG_Y	1.53E-02	1,50E-03
CDG_Z	1.53E-02	1,50E-03
JX	1.43E-08	1,20E-01
EY	0.00E+00	1,00E-03
EZ	– 1.60E-02	1,00E-03
IY_G	1.64E-07	4,00E-03
$IYR2_P$	1.41E-08	1,40E-02
IYZ_G	– 9.50E-08	1,00E-03
IZ_G	1.64E-07	4,00E-03
IY	6.95E-08	4,00E-03
$IZR2_P$	1.41E-08	1,40E-02
IZ	2.60E-07	4,00E-03
$PCTY$	– 4.00E-03	1,00E-03
$PCTZ$	– 4.00E-03	1,00E-02
R_{MAX}	3.79E-02	1,00E-03
Y_{MAX}	3.54E-02	1,00E-03
Y_{MIN}	– 3.54E-02	1,00E-03
Z_{MAX}	2,17E-002	1,00E-03
Z_{MIN}	– 1.83E-02	1,00E-03

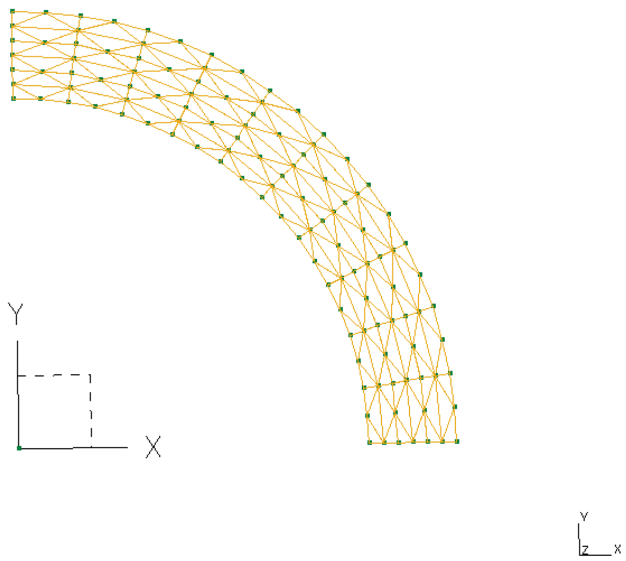
4 Modeling B

4.1 Characteristics of modeling

It is the section of an external circle of radius $R=0.025\text{m}$ and thickness 0.005m. Only one quarter of the section is represented. This modeling makes it possible to test the keyword CARA_GEOM of POST_ELEM, employee also by MACR_CARA_POUTRE to calculate the geometrical characteristics of a plane surface.

4.2 Characteristics of the grid

30 meshs QUAD8.



4.3 Reference solution

For the complete section:

$$A=\pi\left[R^2-(R-ep)^2\right]=1.76714\text{E-}04\,m^2$$

$$I_y=I_z=\frac{\pi}{4}\left[R^4-(R-ep)^4\right]=1.81132\text{E-}7\,m^4$$

$$I_p=2\,I_y=2\,I_z$$

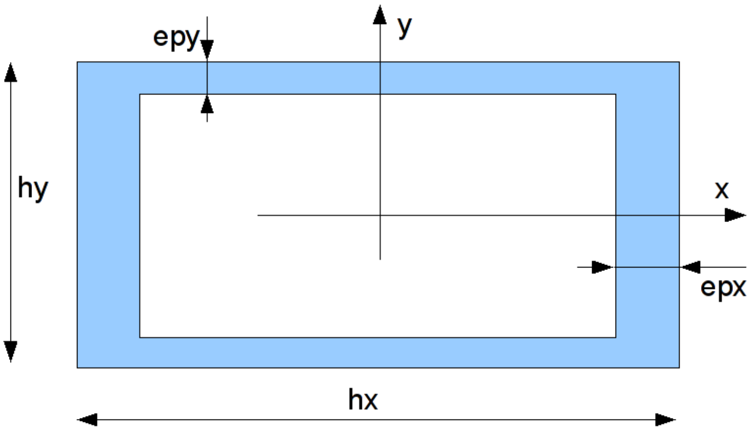
4.4 Sizes tested and results

Identification	Reference	% difference
A_M	1.76714E-04	– 7.76E-05
$CDG_{\text{Y-M}}$	1.438288E-02	– 1.25E-04
$CDG_{\text{Z-M}}$	1.438288E-02	– 1.25E-04
$IY_{\text{G-M}}$	8.7265757E-09	– 2.78E-04
$IZ_{\text{G-M}}$	8.7265757E-09	– 2.78E-04
$IYZ_{\text{G-M}}$	– 7.72837E-09	– 3.83E-04
A	7.0685745E-04	– 7.76E-05
CDG_Y	0.00000E+00	0
CDG_Z	0.00000E+00	0
IY_G	1.81132E-07	– 4.19E-06
IZ_G	1.81132E-07	– 4.19E-06
IYZ_G	0.00000E+00	
IY	1.81132E-07	– 4.19E-06
IZ	1.81132E-07	– 4.19E-06
Y_{MIN}	– 2.50000E-02	0.00E+00
Y_{MAX}	2.50000E-02	0.00E+00
Z_{MIN}	– 2.50000E-02	0.00E+00
Z_{MAX}	2.50000E-02	0.00E+00

5 Modeling C

5.1 Characteristics of modeling

Rectangular section digs which one represents a quarter. This modeling makes it possible to test the keyword `CARA_GEOM` of `POST_ELEM`, employee also by `MACR_CARA_POUTRE` to calculate the geometrical characteristics of a plane surface.

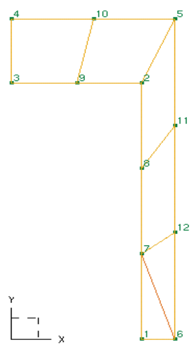


$Hx=0.02\text{m}$, $Hy=0.05\text{m}$, $epx=0.002\text{m}$, $epy=0.005\text{m}$.

5.2 Characteristics of the grid

The coordinates of the nodes are:

N1	8.00E-03	0.00E+00
N2	8.00E-03	2.00E-02
N3	0.00E+00	2.00E-02
N4	0.00E+00	2.50E-02
N5	1.00E-02	2.50E-02
N6	1.00E-02	0.00E+00
N7	8.00E-03	6.6667E-03
N8	8.00E-03	1.3333E-02
N9	4.00E-03	2.00E-02
N10	5.00E-03	2.50E-02
N11	1.00E-02	1.66667E-02
N12	1.00E-02	8.3333E-03



5.3 Reference solution

$$A=h_y h_z-(h_y-2\,ep_y)(h_z-2\,ep_z)=3.6\text{E-}04\,m^2$$

$$I_x=\frac{1}{12}\left[h_y h_z^3-(h_y-2\,ep_y)(h_z-2\,ep_z)^3\right]=1.23\text{E-}7\,m^4$$

$$I_y=\frac{1}{12}\left[h_z h_y^3-(h_z-2\,ep_z)(h_y-2\,ep_y)^3\right]=1.968\text{E-}8\,m^4$$

5.4 Sizes tested and results

Identification	Reference	% difference
A	3.60000E-04	– 2.26E-13
$ALPHA$	9.00000E+01	0.00E+00
CDG_Y	0.00000E+00	0
CDG_Z	0.00000E+00	0.00E+00
IY_G	1.23000E-07	– 6.46E-14
$IYZ_{\text{G-M}}$	– 1.11111E-09	1.00E-04
IZ_G	1.96800E-08	– 2.52E-13
IY	1.96800E-08	– 2.52E-13
IZ	1.23000E-07	– 6.46E-14
R_{MAX}	2.69258E-02	– 6.54E-04
Y_{MAX}	2.50000E-02	0.00E+00
Y_{MIN}	– 2.50000E-02	0.00E+00
Z_{MAX}	1.00000E-02	0.00E+00
Z_{MIN}	– 1.00000E-02	0.00E+00

6 Modeling D

6.1 Characteristics of modeling

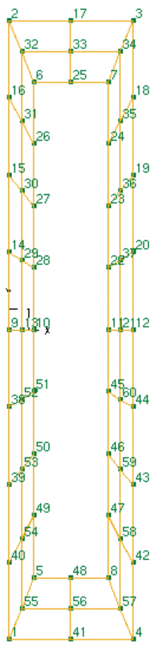
Hollow rectangular section. This modeling makes it possible to test `MACR_CARA_POUTRE` to calculate the geometrical and mechanical characteristics of a plane surface.
Two calculations are carried out:

- first is carried out with the keyword `SYME_Z = 'YES'`, i.e. that the section considered is obtained by symmetry around the axis Z (alveolate section). Moreover inertias are calculated compared to the point of coordinates $(0,-0.025)$ (keyword `ORIG_INER`),
- second is carried out without symmetry, on the section with a grid, with a calculation of inertias in the center of the grid, C coordinates $(0.005,0)$, and 2 different groups of meshs, which correspond each one to the vertical half of the grid (on both sides of the axis Cz).

6.2 Characteristics of the grid

40 meshs `QUAD4`.
The coordinates of the nodes tops of the rectangle are:

$N1$	0.00E+00	– 2.50E-02
$N2$	0.00E+00	2.50E-02
$N3$	1.00E-02	2.50E-02
$N4$	1.00E-02	– 2.50E-02
$N5$	2.00E-03	– 2.00E-02
$N6$	2.00E-03	2.00E-02
$N7$	8.01E-03	2.00E-02
$N8$	8.01E-03	– 2.00E-02
$N9$	0.00E+00	0.00E+00



6.3 Sizes tested and results

For the section symmetrized according to OY, the geometrical characteristics are:

Identification	Reference	% difference
A_M	2.600E-04	– 1.25E-13
A	5.200E-04	– 1.25E-13
$ALPHA$	9.000E+01	0.00E+00
$CDG_{\text{Y-M}}$	5.000E-03	– 5.20E-14
CDG_Y	0.000E+00	0.00E+00
$CDG_{\text{Z-M}}$	0.000E+00	1.40E-18
CDG_Z	0.000E+00	1.40E-18
$IY_{\text{G-M}}$	7.21667E-08	– 4.62E-05
IY_G	1.44333E-07	2.31E-04
IY_P	4.69333E-07	7.10E-05
$IYZ_{\text{G-M}}$	0.000E+00	– 4.33E-26
IYZ_G	0.000E+00	– 4.33E-26
$IZ_{\text{G-M}}$	3.44667E-09	– 9.67E-05
IZ_G	1.98933E-08	1.68E-04
IY	1.98933E-08	1.68E-04
IY_P	1.98933E-08	1.68E-04
IZ	1.44333E-07	2.31E-04
R_{MAX}	2.69260E-02	– 6.54E-04
Y_{MAX}	2.500E-02	0.00E+00
Y_{MIN}	– 2.500E-02	0.00E+00
Z_{MAX}	1.000E-02	1.73E-14
Z_{MIN}	– 1.000E-02	1.73E-14

For the not symmetrized section, the geometrical characteristics are:

Place	Identification	Reference	% difference
$TOUT$	IY_P	3.60833E-08	9.24E-05
$GR1$	IY_P	3.60833E-08	9.24E-05
$GR2$	IY_P	7.21667E-08	– 4.62E-05
$TOUT$	IZ_P	1.72333E-09	1.93E-04
$GR1$	IZ_P	1.72333E-09	1.93E-04
$GR2$	IZ_P	3.44667E-09	– 9.67E-05

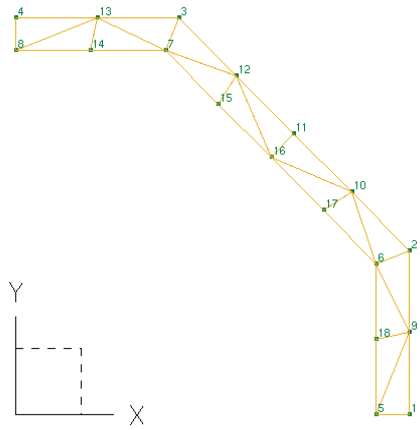
7 Modeling E

7.1 Characteristics of modeling

Hollow octagonal section, which one nets a quarter.

7.2 Characteristics of the grid

```
N1  2.30969E-02  0.00000E+00
N2  2.30969E-02  9.56708E-03
N3  9.56708E-03  2.30969E-02
N4  0.00000E+00  2.30969E-02
N5  2.11835E-02  0.00000E+00
N6  2.11835E-02  8.77452E-03
N7  8.77452E-03  2.11835E-02
N8  0.00000E+00  2.11835E-02
```



7.3 Sizes tested and results

For the section symmetrized according to OZ , the geometrical characteristics are:

Identification	Reference	% difference
$ALPHA$	9.00000E+01	0.00E+00
IYZ_G	0.00000E+00	0.00E+00
IZ_G	7.28824E-08	0,003
IY	7.28824E-08	0,003
IZ	7.28824E-08	0,003
R_{MAX}	2.50000E-02	4.58E-13
Y_{MAX}	2.30967E-02	0,001
Y_{MIN}	– 2.30967E-02	0,001
Z_{MAX}	2.30967E-02	0,001
Z_{MIN}	– 2.30967E-02	0,001

8 Modeling F

8.1 Characteristics of modeling

Full circular section, of ray $0.025\mathrm{m}$. The characteristics calculated are then used directly in a calculation of right beam (length $L=1\mathrm{m}$), in pure traction ($F=1000\mathrm{N}$). The Young modulus is worth $2.\mathrm{E}11\,Pa$. The characteristics of the section are given to AFFE_CARA_ELEM via the keyword TABLE_CARA.

8.2 Characteristics of the grid

Many meshs: 52 TRIA6, 299 QUAD8

8.3 Reference solution

$$A=\pi R^2=1.9635\mathrm{E}\text{-}3\,m^2\;;\; I_y=I_z=\frac{\pi}{4}R^4=3.06796\mathrm{E}\text{-}7\,m^4\;;\; A_y=A_z=\frac{10}{9}\;;\; C=I_x=2I_y=2I_z$$

Pure traction of a beam of full circular section, length $L=1\mathrm{m}$, subjected to a force $F=1000\mathrm{N}$:

$$u(x)=\frac{F\,x}{E\,A}\quad u(L)=\frac{F\,L}{E\,A}=2.54648\mathrm{E}\text{-}6\,m$$

8.4 Sizes tested and results

For the section symmetrized according to OY, the geometrical characteristics are:

Identification	Reference	Value	Tolerance
A	ANALYTICAL	1,96E-03	0,50%
CDG_Y	ANALYTICAL	0,00E+00	0,10%
CDG_Z	ANALYTICAL	0,00E+00	0,10%
IY_G	ANALYTICAL	3,07E-07	0,90%
IZ_G	ANALYTICAL	3,07E-07	0,90%
IYZ_G	ANALYTICAL	0,00E+00	0,10%
IY	ANALYTICAL	3,07E-07	0,90%
IZ	ANALYTICAL	3,07E-07	0,90%
Y_{MIN}	ANALYTICAL	-2,50E-02	0,10%
Y_{MAX}	ANALYTICAL	2,50E-02	0,10%
Z_{MIN}	ANALYTICAL	-2,50E-02	0,10%
Z_{MAX}	ANALYTICAL	2,50E-02	0,10%
JX	ANALYTICAL	6,14E-07	0,90%
AY	ANALYTICAL	1,17E+00	0,10%
AZ	ANALYTICAL	1,17E+00	0,10%
EY	ANALYTICAL	0,00E+00	0,10%
EZ	ANALYTICAL	0,00E+00	0,10%
JG	ANALYTICAL	0,00E+00	0,10%

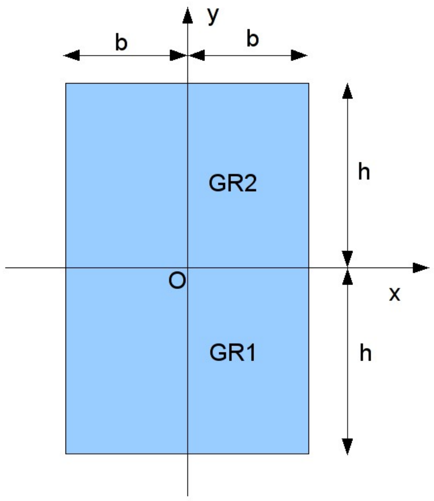
For the calculation of traction of beam, the result is:

Identification	Reference	Value	Tolerance
DEPL	ANALYTICAL	2,55E-06	1,00E-03
FORC_NODWith	ANALYTICAL	1,00E+03	1,00E-03

9 Modeling G

9.1 Characteristics of modeling

Full rectangular section, of width 0.02m and height 0.05m. It is divided into two rectangles respective heights 0.025m, in order to to test the calculation of the characteristics on groups meshs for a network here made up of two parallel beams, ranging between two floors distant of $L=0.0002\text{m}$ (what makes it possible to obtain characteristics (coefficient of shearing) very close to that of the complete section).



9.2 Characteristics of the grid

Many meshs: 32 QUAD8

9.3 Reference solution

Geometrical characteristics for the complete section and each half-section:

PLACE	With	CDG_Y	CDG_Z	IY_G	IZ_G	IYZ_G
All	1.00E-03	0.0	0.0	2.08E-07	3.33E-08	0.0
GR1	5.00E-04	0.0	-1.25E-02	2.60E-08	1.67E-08	0.0
GR2	5.00E-04	0.0	1.25E-02	2.60E-08	1.67E-08	0.0

PLACE	Y_P	Z_P	IY_P	IZ_P	IYZ_P	IY	IZ
All	0.00E+00	0.00E+00	2.08E-07	3.33E-08	0.0	3.33E-08	2.08E-07
GR1	0.00E+00	0.00E+00	1.04E-07	1.67E-08	0.0	1.67E-08	1.04E-07
GR2	0.00E+00	0.00E+00	1.04E-07	1.67E-08	0.0	1.67E-08	1.04E-07

Coefficients of shearing: for each rectangular section: $A_y=A_z=1.2$

9.4 Sizes tested and results

For the complete section, the geometrical and mechanical characteristics are:

Identification	Reference	% difference
A	1.0000000E-03	0.00E+00
$ALPHA$	9.0000000E+01	0.00E+00
AY	1.2000000E+00	– 0,004
AZ	1.2000000E+00	– 0,065
CDG_Y	0.0000000E+00	– 1.03E-19
CDG_Z	0.0000000E+00	– 2.67E-19
JX	9.9805000E-08	– 0,124
EY	0.0000000E+00	1.55E-18
EZ	0.0000000E+00	– 4.79E-18
IY_G	2.0833333E-07	1.60E-06
IYZ_G	0.0000000E+00	– 1.40E-24
IZ_G	3.3333330E-08	1.00E-05
$PCTY$	0.0000000E+00	4.90E-18
$PCTZ$	0.0000000E+00	1.82E-18
Y_{MAX}	2.5000000E-02	0.00E+00
Y_{MIN}	– 2.5000000E-02	0.00E+00
Z_{MAX}	1.0000000E-02	1.73E-14
Z_{MIN}	– 1.0000000E-02	1.73E-14

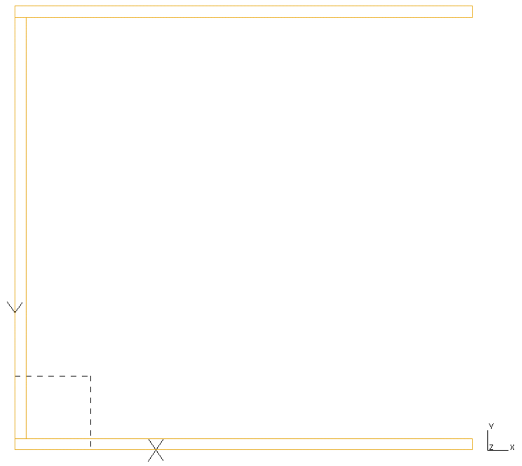
For the two disjoined groups, one obtains:

Place	Identification	Reference	% difference
$GR2$	A	5.00000E-04	2.17E-14
$GR1$	A	5.00000E-04	4.34E-14
$TOUT$	AY	1.20000E+00	– 0,064
$GR1$	AY	1.20000E+00	– 0,065
$GR2$	AY	1.20000E+00	– 0,065
$GR1$	AZ	1.20000E+00	– 0,065
$GR2$	AZ	1.20000E+00	– 0,065
$GR1$	CDG_Y	0.00000E+00	1.59E-19
$GR2$	CDG_Y	0.00000E+00	2.11E-19
$GR1$	CDG_Z	1.25000E-02	– 1.39E-14
$GR2$	CDG_Z	– 1.25000E-02	– 4.16E-14
$GR1$	IY_G	2.60417E-08	– 1.28E-04
$GR2$	IY_G	2.60417E-08	– 1.28E-04
$GR1$	IYZ_G	0.00000E+00	– 1.58E-24
$GR2$	IYZ_G	0.00000E+00	1.98E-24
$GR1$	IZ_G	1.66667E-08	– 2.00E-04
$GR2$	IZ_G	1.66667E-08	– 2.00E-04

10 Modeling H

10.1 Characteristics of modeling

Section in U, of dimension $l=20\text{mm}$, and thickness $e=0.5\text{mm}$



10.2 Characteristics of the grid

Many meshs: 236 QUAD8

10.3 Reference solution

The approximate analytical values result from [bib1].

10.4 Sizes tested and results

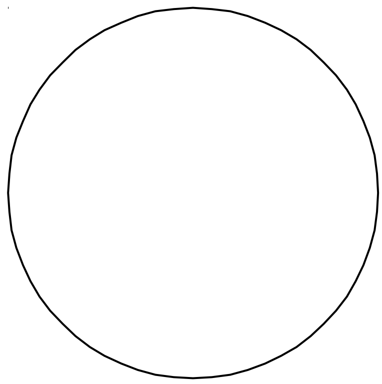
For the complete section, the geometrical and mechanical characteristics are:

Identification	Reference	% difference
A	29.5	0.00E+00
AY	4.25300E+00	5,483
AZ	1.61800E+00	18,472
CDG_Y	6.8602	– 0,015
CDG_Z	10	– 2.31E-13
JX	2.4984	– 1.9
EY	0	– 4.18E-11
EZ	– 15.43	– 0,089
JG	8.69 E+04 [bib4]	0,253

11 Modeling I

11.1 Characteristics of modeling

Hollow circular section, of external ray 10mm , and thickness 1mm .



11.2 Characteristics of the grid

Many meshs: 300 QUAD8

11.3 Reference solution

$$C=I_x=\pi\left[\frac{R^4}{2}-\frac{(R-e)^4}{2}\right]=5401.97\,mm^4$$

11.4 Sizes tested and results

The constant of torsion is worth:

Identification	Reference	% difference
JX	5401.97	– 0,194

12 Modeling J

12.1 Characteristics of modeling

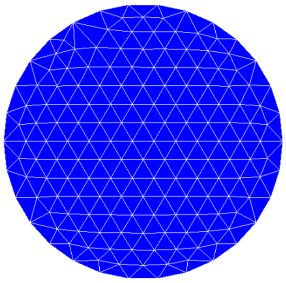
Three sections:

- circular full with ray 2 Mr.
- hollow circular, of external ray 2 m and parameter ALPHA = 0.5.
- hollow circular, of external ray 2 m and parameter ALPHA = 0.9.

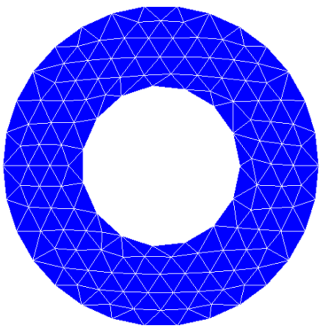
Note: ALPHA = (Rext – thickness)/Rext

12.2 Characteristics of the grids

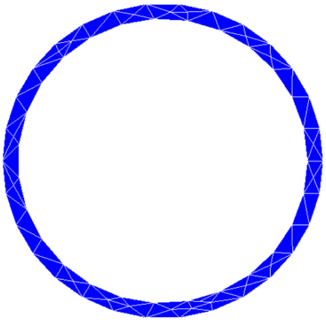
1. Many meshs: 318 `TRIA6`



2. Many meshs: 259 `TRIA6`



3. Many meshs: 95 `TRIA6`



12.3 Sizes tested and results

Tests to guarantee the not-regression of the code are carried out on the calculated coefficients of shearing.

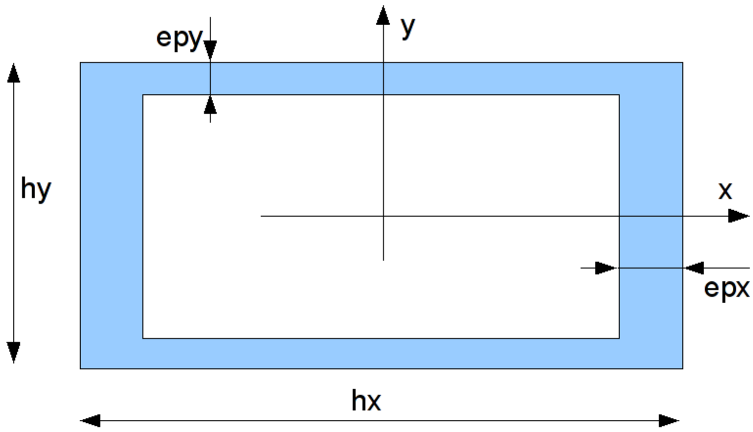
13 Modeling K

13.1 Characteristics of modeling

Three sections:

- square full on side 4 Mr.
- square digs, on side 4 m and parameter ALPHA = 0.5/BETA = 0,525.
- square digs, on side 4 m and parameter ALPHA = 0.9/BETA = 0,945.

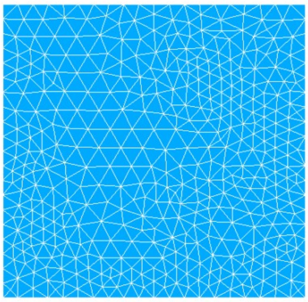
Note:



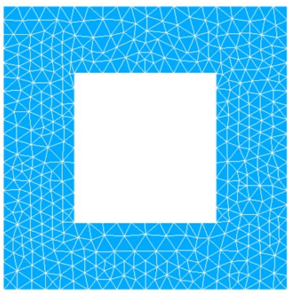
*ALPHA = (HX – 2*EPX) /HX*
*BETA = (HY – 2*EPY)/HY*

13.2 Characteristics of the grids

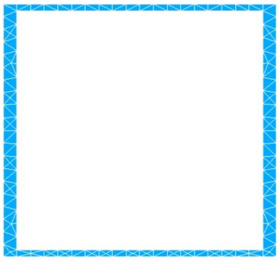
1. Many meshs: 844 `TRIA6`



2. Many meshs: 762 `TRIA6`



3. Many meshs: 208 TRIA6



13.3 Sizes tested and results

Tests to guarantee the not-regression of the code are carried out on the calculated coefficients of shearing.

14 Summary of the results

This test makes it possible to check the good performance of the order simultaneously MACR_CARA_POUTRE for various types of sections.