SSLL400 - Non-prismatic beam, subjected with efforts specific or distributed

Summary:
This test is resulting from the validation independent of version 4 of the models of beams.

This test allows the checking of calculations of right beams in the linear static field. (a modeling with elements of beams `POU_D_E`, right beam of EULER).

One calculates simultaneously 3 beams of the different sections: section rings, right-angled, and general. These beams are subjected to efforts specific or distributed.

The values tested are displacements and rotations, the efforts generalized, and the constraints.

1 Problem of reference

1.1 Geometry

1.1.1 Right beam of variable circular section

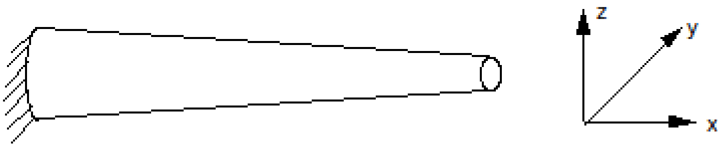

Figure 1.1.1-a : Beam with variable circular section.

Length	: $1\,m$
Ray with embedding	: $0{,}1\,m$
Ray at the loose lead	: $0{,}05\,m$

1.1.2 Right beam of variable rectangular section

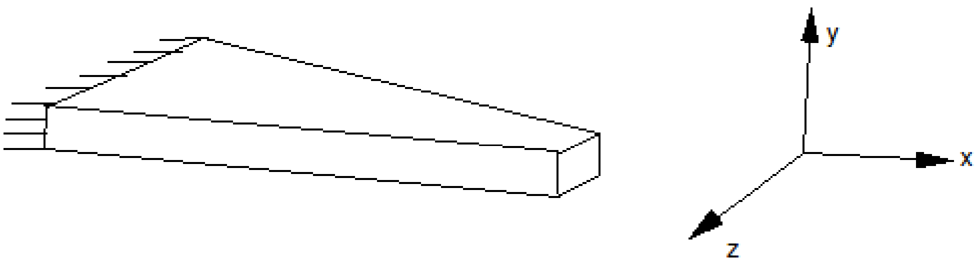

Figure 1.1.2-a : Beam with variable rectangular section

Length	: $1\,m$	
with embedding	: $Hy=0{,}05\,m$	$Hz=0{,}10\,m$
at the loose lead	: $Hy=0{,}05\,m$	$Hz=0{,}05\,m$

1.1.3 Right beam of variable general section

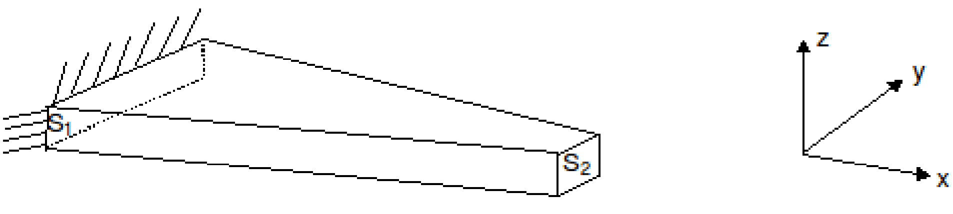

Figure 1.1.3-a : Beam with variable general section

Length	: $1\,m$	
with embedding	: $A=10^{-2}\,m^2$	$Iy=8{,}3333\,10^{-6}\,m^4$
at the loose lead	: $A=2{,}5\,10^{-3}\,m^2$	$Iy=5{,}20833\,10^{-7}\,m^4$

1.2 Properties of materials

Young modulus:	$E=2.10^{11}\,Pa$
Poisson's ratio:	$\nu=0{,}3$
Density:	$\rho=7800\,kg.m^{-3}$

1.3 Boundary conditions and loading

Boundary condition:
Embedded end: $DX = DY = DZ = DRX = DRY = DRZ = 0$

Loading:
To the right beam of variable circular section and to the right beam of variable rectangular section, one applies successively:

Loading case	Nature
1	a following specific effort X at the loose lead, $Fx = 100\,N$
2	a following specific effort Y at the loose lead, $Fy = 100\,N$
3	one specific moment around the axis X at the loose lead, $Mx = 100\,m.N$
4	one specific moment around the axis Z at the loose lead, $Mz = 100\,m.N$
5	a distributed load on the whole of the beam, $fx = 100\,N.m^{-1}$
6	a distributed load on the whole of the beam, $fy = 100\,N.m^{-1}$

To the right beam of variable general section, one applies:

Loading case	Nature
7	an effort of gravity according to z with $g = 9{,}81\,m.s^{-2}$

2 Reference solutions

2.1 Method of calculating used for the reference solutions

2.1.1 Circular section

2.1.1.1 Beam subjected to a specific tractive effort Fx

The ‘equation D’ balance is:

$$\frac{\partial}{\partial x}\left(EA(x)\frac{\partial u}{dx}\right)=0 \text{ with } A(x)=A_1\left(1+c\frac{x}{L}\right) \text{ and } c=\sqrt{\frac{A_2}{A_1}}-1 \text{ , } N(L)=F_x$$

While integrating twice [R3.08.01], we obtain displacements according to the force applied, that is to say:

$$u(x)=\frac{LF_x}{EA_1}\left(\frac{x}{L+cx}\right)$$

and thus at the end L of the beam:

$$u(L)=\frac{L}{E\sqrt{A_1A_2}}F_x$$

The internal efforts are given by:

$$N(x)=EA(x)\frac{\partial u}{\partial x}(x)=F_x$$

and constraints by:

$$\sigma_{xx}=\frac{N(x)}{A(x)}$$

2.1.1.2 Beam subjected to a specific bending stress Fy

The ‘equation D’ balances, under the assumption of Euler, is given by the equation:

$$\frac{\partial^2}{\partial x^2}\left[EI_z(x)\frac{\partial^2 v}{\partial x^2}\right]=0 \text{ with } I_z(x)=I_{z_1}\left(1+c\frac{x}{L}\right)^4 \text{ and } c=\left(\frac{I_{z_2}}{I_{z_1}}\right)^{\frac{1}{4}}-1 \text{ , } V_y(L)=F_y$$

We solve the equation by integration by taking account of the law of behavior modified $MF_z=EI_z\frac{\partial^2 v}{\partial x^2}$ and the equilibrium equation $\frac{\partial MF_z}{\partial x}+V_y=0$

Four successive integrations, by taking account for the calculation of the constants of integration that:

$$\frac{\partial}{\partial x}\left[EI_z(x)\frac{\partial^2 v}{\partial x^2}\right](L)=-V_y(L)=-F_y$$

$$\left(EI_z(L)\frac{\partial^2 v}{\partial x^2}\right)(L)=0$$

$$\frac{\partial v}{\partial x}(0)=0$$

$$v(0)=0$$

lead to the expression of:

$$v(x)=+\frac{F_y L^2}{6\ E\ I_{z_1}}\ \frac{x^2(3\ L\ -\ x\ +\ 2\ cx)}{(L+cx)^2}$$

and with the expression of $\theta_z(x)$

$$\theta_z(x)=+\frac{F_y L^2}{6\ E\ I_{z_1}}\ \frac{x\left(6\ L^2-3\ Lx+6\ L\ cx-cx^2+2\ c^2\ x^2\right)}{(L+cx)^3}$$

The internal efforts are given by:

$$V_y(x)=F_y$$
$$MF_z(x)=F_y(L-x)$$

and constraints by:

$$\sigma_{xx}(x)=\left|MF_z(x)\right|\ \frac{R(x)}{I_z(x)}$$

$\sigma_{xy}=\frac{V_y(x)}{A(x)}$ pas de coefficient of correction of shearing in assumption of Euler.

2.1.1.3 Beam subjected to one specific torque Mx

The movement is given by the equation:

$$\frac{\partial}{\partial x}\left[G\ I_p(x)\frac{\partial\theta_x}{\partial x}\right]=0 \text{ with } I_p(x)=I_{p_1}\left(1+c\frac{x}{L}\right)^4 \text{ and } c=\left(\frac{I_{p_2}}{I_{p_1}}\right)^{\frac{1}{4}}-1\ ,\ M_x(L)=M_x$$

After integration, and by taking account owing to the fact that:

$$G\ I_p(L)=\frac{\partial\theta_x}{\partial x}(L)=M_x \text{, and } \theta_x(0)=0$$

we obtain the expression of $\theta_x(x)$:

$$\theta_x(x)=\frac{L\ M_x}{3\ G\ I_{p_1}}\ \frac{x\ (3\ L^2+3\ L\ cx+c^2\ x^2)}{\left(L+cx\right)^3}$$

We must also have for the internal efforts and the constraints:

$$M_x(x)=M_x$$
$$\sigma_{xy}(x)=\frac{M_x(x)}{I_p(x)}R_T(x)$$
$$\sigma_{xz}(x)=\frac{M_x(x)}{I_p(x)}R_T(x)$$

2.1.1.4 Beam subjected to one specific bending moment My

The reasoning to find the solution analytical is the same one as previously. We use the law of behavior $M_y(x)=-EI_y(x)\frac{\partial^2 w}{\partial x^2}$ and the equilibrium equation $\frac{\partial MF_y}{\partial x}-V_z=0$ The calculation of the constants of integration differs: one has $V_z(L)=0$ and $M_{F_y}(L)=M_y$.

One obtains the expression of $w(x)$:

$$w(x)=\frac{L\ M_y}{6\ E\ I_{y_1}}\ \frac{x^2(3\ L+2\,cx)}{(L+cx)^2},$$

and the expression of $\theta_y(x)$:

$$\sigma_y(x)=\frac{L\,M_y}{3\,E\,I_{y1}}\ \frac{x\left(3\,L^2+3\,L\,cx+c^2\,x^2\right)}{(L+cx)^3}$$

One must also have for the internal efforts and the constraints:

$$V_z(x)=0$$
$$MF_y(x)=My$$
$$\sigma_{xx}(x)=\left|MF_y(x)\right|\ \frac{R(x)}{I_y(x)}$$

2.1.1.5 Beam subjected to a tractive effort distributed regularly fx

Balance is described by the equation

$$\frac{\partial}{\partial x}\left(EA(x)\ \frac{\partial u}{\partial x}\right)=-f_x \text{ with } A(x)=A_1\left(1+c\,\frac{x}{L}\right)^2 \text{ and } c=\left(\frac{A_2}{A_1}\right)^{\frac{1}{2}}-1$$

By integrating for the first time this equation, we obtain:

$$E\,A(x)\frac{\partial u}{\partial x}=-f_x\,x+c_1$$

The limiting condition $N(L)=0$ imply $c_1=f_x\,L$. We thus have:

$$\frac{\partial u}{\partial x}=-f_x\frac{(L-x)}{E\ A(x)}$$

that is to say:

$$u(x)=f_x\int\frac{(L-x)}{E\ A(x)}\,dx+c_2$$

c_2 is given so that $u(0)=0$

Taking everything into account, we have:

$$u(x)=\frac{L^2\,f_x}{E\ A_1\,c^2}\ \frac{c\ x+c^2\,x+(L+c\ x)\log\frac{L}{L+c\ x}}{L+c\ x}.$$

The internal efforts are deduced from the law of behavior $N(x)=E\ A(x)\frac{\partial u}{\partial x}$:

$$N(x)=f_x(L-x)$$

and the constraints are given by:

$$\sigma_{xx}(x)=\frac{N(x)}{A(x)}=\frac{f_x(L-x)}{\left[\sqrt{A_1}+\left(\sqrt{A_2}-\sqrt{A_1}\right)\frac{x}{L}\right]^2}$$

2.1.1.6 Beam subjected to a bending stress distributed regularly fy

On the basis of the equilibrium equation:

$$\frac{\partial^2}{\partial x^2}\left[E\ I_z(x)\frac{\partial^2 v}{\partial x^2}\right]=-f_y \text{ with } I_z(x)=I_{z_1}\left(1+c\frac{x}{L}\right)^4 \text{ and } c=\left(\frac{I_{z_2}}{I_{z_1}}\right)^{\frac{1}{4}}-1$$

we carry out four successive integrations. The determination of the constants of integration is made starting from the following limiting conditions:

$$V_y(L)=0$$
$$M_z(L)=0$$
$$\frac{\partial v}{\partial x}(0)=0$$
$$v(0)=0$$

The analytical expression for $v(x)$ and $\theta(z)$ in the presence of a loading distributed is, taking everything into account:

$$v(x)=\frac{-f_y L^3}{12\,EI_{z_1}c^4(L+cx)^2}\left[\begin{array}{c}-6L^2cx+x^2\left(-9Lc^2-3Lc^4\right)+x^3\left(-2c^3+2c^4-2c^5\right)\\+\log\left(1+c\frac{x}{L}\right)(6L^3+12L^2cx+6Lc^2x^2)\end{array}\right]$$

$$\theta_z(x)=\frac{+L^3 f_y x}{6EI_{z_1}(L+cx)^3}\left[3L^2-3Lx+3Lcx+x^2\left(1-c+c^2\right)\right]$$

The internal efforts are given by:

$$V_y(x)=f_y(L-x) \text{ and } Mf_z(x)=\frac{1}{2}f_y(L-x)^2$$

constraints by:

$$\sigma_{xy}(x)=\frac{V_y(x)}{A(x)}$$
$$\sigma_{xx}(x)=\left|Mf_z(x)\right|\frac{R(x)}{I_z(x)}$$

2.1.2 Rectangular section

2.1.2.1 Beam subjected to a specific tractive effort Fx

The equilibrium equation is:

$$\frac{\partial}{\partial x}\left(EA(x)\frac{\partial u}{\partial x}\right)=0 \text{ with } A(x)=A_1+(A_2-A_1)\frac{x}{L},\ N(L)=F_x$$

While integrating twice, and by taking account owing to the fact that:

$$E\ A(L)\frac{\partial u}{\partial x}(L)=F_x,\ u(0)=0$$

for the determination of the constants of integration, we obtain the analytical expression of $u(x)$, that is to say:

$$u(x)=\frac{F_x L}{A_1 Ec}\log\left(1+c\frac{x}{L}\right)$$

For the internal and forced efforts, we have:

$$N(x)=F_x$$

$$\sigma_{xx}=\frac{N(x)}{A(x)}$$

2.1.2.2 Beam subjected to a specific bending stress Fy

The movement is given by the equation:

$$\frac{\partial^2}{\partial x^2}\left[E\ I_z(x)\frac{\partial^2 v}{\partial x^2}\right]=0 \text{ with } I_z(x)=I_{z_1}\left(1+c\frac{x}{L}\right)^3 \text{ and } c=\left(\frac{I_{z_2}}{I_{z_1}}\right)^{\frac{1}{3}}-1\ ,\ V_y(L)=F_y$$

The same reasoning that for the circular section leads to the following result:

$$v(x)=-\frac{F_y L^2}{2EI_{z_1}c^3}\ \frac{\left[2Lcx+c^2x^2-c^3x^2+2L(L+cx)\log\left(\frac{L}{L+cx}\right)\right]}{(L+cx)}$$

$$\theta_z(x)=\frac{F_y L^2}{2EI_{z_1}}\frac{x(2L-x+cx)}{(L+cx)^2}$$

We must have for the internal efforts and the constraints:

$$V_y(x)=F_y$$

$$M_{F_z}(x)=F_y(L-x)$$

$$\sigma_{xx}(x)=\frac{H_y(x)M_{F_x}(x)}{2\mathrm{I}_z(x)}$$

$$\sigma_{xy}(x)=\frac{V_y(x)}{A(x)}$$

2.1.2.3 Beam subjected to one specific torque Mx

The movement is given by the equation:

$$\frac{\partial}{\partial x}\left[G\ I_p(x)\frac{\partial\theta_x}{\partial x}\right]=0 \text{ with } I_p(x)=I_{p_1}\left(1+c\frac{x}{L}\right)^3 \text{ and } c=\left(\frac{I_{p_2}}{I_{p_1}}\right)^{\frac{1}{3}}-1\ ,\ M_x(L)=M_x$$

By the same reasoning as the beam with circular section, we obtain the analytical expression of $\theta_x(x)$:

$$\theta_x(x)=\frac{L\ M_x x(2L+cx)}{2\ I_{p_1}G(L+cx)^2}$$

I_{p_1} and I_{p_2} are calculated according to the formulas given in the reference material [R3.08.01].
The internal efforts and the constraints are given by:

$$M_x(x)=M_x$$

$$\sigma_{xy}(x)=\frac{M_x(x)}{I_p(x)}R_T(x)=\sigma_{xz}$$

2.1.2.4 Beam subjected to one specific bending moment My

The same reasoning is taken again that previously, one obtains the following analytical expressions for $w(x)$ and $\theta_y(x)$:

$$w(x)=-\frac{L\ M_y x^2}{2\,EI_{y_1}(L+cx)}$$

$$\theta_y(x)=\frac{L\ M_y\, x(2\,L+cx)}{2\ E\ I_{y_1}(L+cx)^2}$$

for the efforts:

$$V_z(x)=0$$
$$MF_y(x)=M_y$$

and for the constraints:

$$\sigma_{xx}(x)=\frac{H_z(x)\ \ MF_y(x)}{2\ I_y\ (x)}$$

2.1.2.5 Beam subjected to a tractive effort distributed regularly Fx

The equilibrium equation is:

$$\frac{\partial}{\partial x}\left[EA(x)\frac{\partial u}{\partial x}\right]=-f_x \text{ with } A(x)=A_1\left(1+c\frac{x}{L}\right) \text{ and } c=\left(\frac{A_2}{A_1}\right)-1\ .$$

After two integrations and by taking account owing to the fact that $N(L)=0$ to determine the first constant of integration, and $u(0)=0$ to determine the second, we obtain the analytical expression of $u(x)$:

$$u(x)=\frac{-L\,f_x}{E\ A_1\ c^2}\left[c\ x+(L+L_c)\log\left(\frac{L}{L+c\ x}\right)\right]$$

The internal efforts are known by the following expression:

$$N(x)=f_x(L-x)$$

and constraints by:

$$\sigma_{xx}(x)=\frac{f_x(L-x)}{A(x)}$$

2.1.2.6 Beam subjected to a bending stress distributed regularly Fy

The equilibrium equation is:

$$\frac{\partial^2}{\partial x^2}\left[EI_z(x)\frac{\partial^2 v}{\partial x^2}\right]=-f_y \text{ with } I_z(x)=I_{z_1}\left(1+c\frac{x}{L}\right)^3 \text{ and } c=\left(\frac{I_{z_2}}{I_{z_1}}\right)^{\frac{1}{3}}-1$$

We integrate successively four times this equation. The constants of integration are calculated by taking account owing to the fact that:

$$V_y(L)=0$$
$$MF_z(L)=0$$
$$\frac{\partial v}{\partial x}(0)=0$$
$$v(0)=0$$

The analytical result for the arrow and rotation in L is the following:

$$v(x)=\frac{L^3 f_y}{4\ E\ I_{z_1}\ c^4(L+c\ x)}\left[x\left(6\ Lc+4\ Lc^2\right)+x^2\left(5c^2+2c^3-c^4\right)\right]$$
$$+\left(6L^2+4L^2c+8Lcx+4Lc^2x+2c^2x^2\right)\log\left(\frac{L}{L+cx}\right)$$
$$\theta_z(x)=\frac{L^3 f_y}{4\ EI_{z_1}\ c^3(L+cx)^2}\left[x\left(2\ Lc+2\ Lc^3\right)+x^2\left(3\ c^2+2\ c^3-c^4\right)\right]$$
$$+\left(2\ L^2+Lcx+2\ c^2\ x^2\right)\log\left(\frac{L}{L+cx}\right)$$

The internal efforts are given by the following expressions:

$$V_y(x)=f_y(L-x)\ ,\ Mf_z(x)=\frac{1}{2}f_y(L-x)^2$$

constraints by:

$$\sigma_{xy}(x)=\frac{V_y(x)}{A(x)}$$
$$\sigma_{xx}(x)=\left|\frac{Mf_z(x)h_y}{I_z(x)2}\right|$$

2.1.3 General section

2.1.3.1 Beam subjected to the forces of gravity

The efforts of gravity are applied along the axis z. The movement of the beam induced by these efforts is thus a movement of inflection in the plan $(x\,o\,z)$.
The equilibrium equation is given by the expression:

$$\frac{\partial^2}{\partial x^2}\left(E\ I_y(x)\frac{\partial^2 w}{\partial x^2}\right)=\underbrace{\rho\,A(x)\ g}_{poids\ linéique}$$

with $A(x)=A_1\left(1+c\frac{x}{L}\right)^2$, $c=\left(\frac{A_2}{A_1}\right)^{\frac{1}{2}}-1$ and $I_y(x)=I_{y1}\left(1+d\frac{x}{L}\right)^4$ $d=\left(\frac{I_{y2}}{I_{y1}}\right)^{\frac{1}{4}}-1$

While integrating for the first time, we obtain the shearing action intern:

$$V_z(x)=-\int\rho\ A(x)\ g\ dx+C_1$$

C_1 is given so that $V_z(L)=0$.

We obtain:

$$V_z(x)=\frac{L A_1 \rho g}{3c}\left[-\left(1+c\frac{x}{L}\right)^3+(1+c)^3\right].$$

While integrating for the second time, we obtain the internal bending moment:

$$M_y(x)=\int V_z(x)\,dx+c_2$$

C_2 is calculated so that $M_y(L)=0$

We obtain:

$$M_y(x)=\frac{A_1 \rho g}{12 L^2}(L-x)^2\left(6L^2+8L^2c+3L^2c^2+4Lcx+2Lcx+\frac{2Lc^2x}{+c^2x^2}\right)$$

We calculate then rotation starting from the law of behavior $M_y(x)=EI_y(x)\frac{\partial \theta_y}{\partial x}$

We thus have $\theta_y(x)=\int \frac{M_y(x)}{EI_y(x)}dx+C_3$ with $\theta_y(0)=0$

The arrow $w(x)$ is given starting from the relation of Euler: $\theta_y=-\frac{\partial w}{\partial x}$

We calculate $w(x)$ by integration of $\theta_y(x)$: $w(x)=-\int \theta_y(x)+C_4$

with C_4 such as $w(0)=0$.

Analytical expressions of $\theta_y(x)$ and $w(x)$ are not retranscribed here because they are too much heavy. They were calculated, like the preceding ones, by the formal computation software MATHEMATICA.

2.2 Results of reference

- Displacements and rotations at the loose lead
- Interior efforts at the two ends
- Constraints at the two ends

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

[1] Report n° 2314/A of the Institute Aerotechnics “Proposal and realization for new cases tests missing with the validation of the beams ASTER”

3 Modeling A

3.1 Characteristics of modeling

The model is composed of 10 elements right beam of Euler.

S1 section: variable circular section
with embedding, $R1=0.1\,m$ (full section)
at the loose lead, $R2=0.05\,m$ (full section)

S2 section: variable rectangular section
with embedding, $H_{y1}=0.05\,m$ $H_{z1}=0.10\,m$
at the loose lead, $H_{y2}=0.05\,m$ $H_{z2}=0.05\,m$

S3 section: variable general section
with embedding, $A_1=10^2\,m^4$ $I_{y1}=8.3333\,10^6\,m^4$
at the loose lead, $A_2=2.5\,10^3\,m^2$ $I_{y2}=5.20833\,10^7\,m^4$

3.2 Characteristics of the grid

3 sections $\times$ 10 elements `POU_D_E`

3.3 Sizes tested and results

Loading case	Section	Identification	Reference	Tolerance %
1	$S1$	$u(l)$	3.1831E-08	1.00E-05
		$n(0)$	1.0000E+02	1.00E-05
		$n(l)$	1.0000E+02	1.00E-05
		$\sigma_{xx}(0)$	3.1831E+03	1.00E-05
		$\sigma_{xx}(l)$	1.2732E+04	1.00E-05
2	$S1$	$v(l)$	4.2441E-06	1.00E-05
		$\theta_z(l)$	8.4882E-06	1.00E-05
		$v_y(0)$	1.0000E+02	1.00E-05
		$v_y(l)$	1.0000E+02	1.00E-05
		$mf_z(0)$	1.0000E+02	1.00E-05
		$mf_z(l)$	0.0000E+00	1.00E-05
		$\sigma_{xx}(0)$	1.2732E+05	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-05
3	$S1$	$\theta_x(l)$	3.8621E-05	1.00E-05
		$m_x(0)$	1.0000E+02	1.00E-05
		$m_x(l)$	1.0000E+02	1.00E-05
4	$S1$	$w(l)$	– 8.4882E-06	1.00E-05
		$\theta_y(l)$	2.9708E-05	1.00E-05
		$v_z(0)$	0.0000E+00	1.00E-05
		$v_z(l)$	0.0000E+00	1.00E-05
		$mf_y(0)$	1.0000E+02	1.00E-05
		$mf_y(l)$	1.0000E+02	1.00E-05
		$\sigma_{xx}(0)$	1.2732E+05	1.00E-05
		$\sigma_{xx}(l)$	1.0185E+06	1.00E-05

5	*S1*	$u(l)$	1.2296E-08	1.00E-02
		$n(0)$	1.0000E+02	1.00E-05
		$n(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	3.1831E+03	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-03 (absolute)
6	*S1*	$v(l)$	1.3486E-06	1.00E-02
		$\theta_z(l)$	2.1220E-06	1.00E-02 (absolute)
		$v_y(0)$	1.0000E+02	1.00E-05
		$v_y(l)$	0.0000E+00	1.00E-03 (absolute)
		$mf_z(0)$	5.0000E+01	1.00E-02
		$mf_z(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	6.3662E+04	1.00E-05
		$\sigma_{xy}(0)$	3.1831E+03	1.00E-05

Loading case	Section	Identification	Reference	Variation %
1	*S2*	$u(l)$	1.3862E-07	1.00E-05
		$n(0)$	1.0000E+02	1.00E-05
		$n(l)$	1.0000E+02	1.00E-05
		$\sigma_{xx}(0)$	2.0000E+04	1.00E-05
		$\sigma_{xx}(l)$	4.0000E+04	1.00E-05
2	*S2*	$v(l)$	1.8969E-04	2.30E-02
		$\theta_z(l)$	3.0238E-04	2.70E-02
		$v_y(0)$	1.0000E+02	1.00E-05
		$v_y(l)$	1.0000E+02	1.00E-05
		$mf_z(0)$	1.0000E+02	1.00E-05
		$mf_z(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	2.4000E+06	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xy}(0)$	2.0000E+04	1.00E-05
		$\sigma_{xy}(l)$	4.0000E+04	1.00E-05
3	*S2*	$\theta_x(l)$	8.3506E-04	5.70E-02
		$m_x(0)$	1.0000E+02	1.00E-05
		$m_x(l)$	1.0000E+02	1.00E-05
		$\sigma_{xy}(0)$	1.5600E+06	1.00E-05
		$\sigma_{xy}(l)$	4.0371E+06	5.00E-02
		$\sigma_{xz}(0)$	1.5600E+06	1.00E-05
		$\sigma_{xz}(l)$	4.0371E+06	5.00E-02
4	*S2*	$w(l)$	– 1.2000E-04	3.00E-03
		$\theta_y(l)$	3.600E-04	4.00E-03
		$v_z(0)$	0.0000E+00	1.00E-03 (absolute)
		$v_z(l)$	0.0000E+00	1.00E-03 (absolute)
		$mf_y(0)$	1.0000E+02	1.00E-05
		$mf_y(l)$	1.0000E+02	1.00E-05
		$\sigma_{xx}(0)$	1.2000E+06	1.00E-05

		$\sigma_{xx}(l)$	4.8000E+06	1.00E-05
5	$S2$	$u(l)$	6.1370E-08	1.00E-05
		$n(0)$	1.0000E+02	1.00E-05
		$n(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	2.0000E+04	1.00E-05
		$\sigma_{xx}(l/2)$	1.3333E+04	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-03 (absolute)
6	$S2$	$v(l)$	6.8626E-05	2.00E-02
		$\theta_z(l)$	9.4847E-05	2.40E-02
		$v_y(0)$	1.0000E+02	1.00E-05
		$v_y(l)$	0.0000E+00	1.00E-03 (absolute)
		$mf_z(0)$	5.0000E+01	1.00E-02
		$mf_z(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	1.2000E+06	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-03 (absolute)
7	$S3$	$w(l)$	– 3.8259E-05	1.00E-02
		$\theta_y(l)$	5.7388E-05	1.00E-02
		$v_z(0)$	– 4.4633E+02	1.00E-03
		$mf_y(0)$	1.7535E+02	1.00E-02

3.4 Remarks

Modeling being made in beams of Euler, the coefficients of shearing are $ky=kz=1$.

4 Summary of the results

The got results confirm that the elements POU_D_E with variable section present a good degree of reliability.
For the circular section, the results all are exact with the nodes (one finds the properties of the element with constant section) except for the efforts distributed where the effect of the smoothness of discretization is felt.
For a rectangular section and a general section, it is necessary to discretize finely to have a correct solution.