

## SSLL403 - Buckling of a beam under the effect of its actual weight

---

### Summary:

This test makes it possible to validate in linear elasticity the loading due to the forces of gravity for a modeling of type right beam of Euler (POU\_D\_E). It also allows the implementation and the validation of the calculation of the geometrical matrix of rigidity.

The reference solution is analytical and the results considered to be satisfactory.

## 1 Problem of reference

### 1.1 Geometry

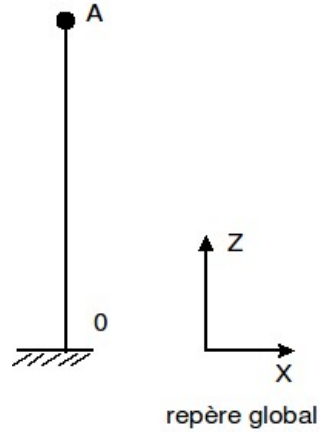


Figure 1.1-a : Vertical beam.

Rectangular section:  $H_y=0.01\text{ m}$  ,  $H_z=0.01\text{ m}$   
Length:  $L=1\text{ m}$

### 1.2 Properties of materials

Young modulus:  $E=2.10^{11}\text{ Pa}$   
Poisson's ratio:  $\nu=0,3$   
Density:  $\rho=7800\text{ kg/m}^3$

### 1.3 Boundary conditions and loading

Boundary condition:  
Embedded end (0) :  $DX = DY = DZ = DRX = DRY = DRZ = 0$  .

**Loading:**

Force of gravity:  $p$  weight per unit of length with  $g=(0, 0, -9.81)$  (given in total reference mark).

## 2 Reference solutions

### 2.1 Method of calculating used for the reference solutions

In local reference mark,  $x$  along the axis  $OA$  beam, bending moment, with the X-coordinate  $x$ , has as an expression:

$$M_{F_y}(x) = p \int_x^L [v(\xi) - v(x)] d\xi.$$

The arrow  $v(x)$  satisfied thus the equation:

$$E I_z \frac{d^2 v}{dx^2} = p \int_x^L [v(\xi) - v(x)] d\xi = -p \left[ \int_x^L v(\xi) d\xi + (L-x)v(x) \right]$$

By deriving the two members, one obtains the differential equation:

$$\frac{d^3 v}{dx^3} + \frac{p}{E I_z} (L-x) \frac{dv}{dx} = 0$$

The function  $v'(x) = \frac{dv}{dx}$  satisfied the linear and homogeneous differential equation with the second order:

$$\frac{d^2 v'}{dx^2} + \frac{p}{E I_z} (L-x) v' = 0,$$

who can be solved using the functions of Bessel. One finds the value of the linear weight then criticizes equalizes with:

$$p_c = 7,837 \frac{E I_z}{L^3}.$$

The analytical solution gives numerically:

$$p_c = 7,837 \cdot 2 \cdot 10^{11} \cdot \frac{10^{-8}}{12} = 1,3061667 \cdot 10^3.$$

### 2.2 Results of reference

The value criticizes multiplier  $\lambda$  :  $\lambda_c = \frac{P_c}{\rho S g}$

### 2.3 Uncertainty on the solution

Analytical solution.

### 2.4 Bibliographical references

- [1] Report n° 2314/A of the Institute Aerotechnics "Proposal and realization for new cases tests missing with the validation of the beams Aster"

## 3 Modeling A

---

### 3.1 Characteristics of modeling

The model is composed of 10 elements right beam of Euler.

### 3.2 Characteristics of the grid

It consists of 10 elements `POU_D_E`.

### 3.3 Sizes tested and results

Eigenvalue of the system  $(K + \lambda K_G) X = 0$  :

	Reference	Aster	Variation %
$\lambda$	- 170,701	- 170.0005	- 0,408

### 3.4 Notice

Since  $p_c = \lambda \rho S g$ , ( $\rho S g$  represent linear prestressing), we have like critical loading:  
 $p_c = 1300,84 \text{ N.m}^{-1}$ .

## 4 Summary of the results

---

The results are very close to the analytical solution (variation: 0,4% for 10 elements). This variation is function of the smoothness of discretization being given the assumptions used for geometrical rigidity (cf [R3.08.01]). This thus validates this kind of loading for the buckling of Euler.