SSLP101 - Rate of refund of energy in plane constraints

Summary:

It is about a test of breaking process in statics for a two-dimensional problem. One considers a plate fissured in plane constraints, the features tested are:

- the rate of refund of energy G,
- the rate of refund of energy calculated starting from the calculation of the coefficients of constraints K_1 and K_2.

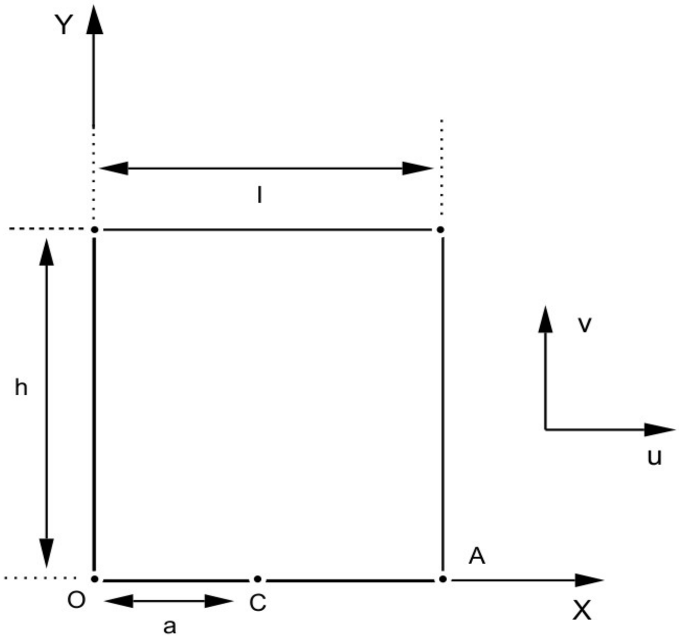
The interest of the test is to compare the value of G classic and the value of G (IRWIN) obtained from K_1 and K_2. It also makes it possible to test the invariance of calculation compared to the crowns of integration.

This test contains 3 different modelings: the modeling A which treated calculation of the integral of Rice is supported more since version 3.

1 Problem of reference

1.1 Geometry

Rectangular plate with crack OC emerging.



For reasons of symmetry, the model is tiny room to the half-structure $y \geq 0$.

Height plates: $h = 250\ mm$

Width plates: $I = 100\ mm$

Depth fissures: $a = 37.5\ mm$ (OC)

1.2 Material properties

$E = 200000\ MPa \quad \nu = 0.3$
Assumption of the plane constraints.

1.3 Boundary conditions and loadings

- Constraint imposed in $Y = h$: $\sigma = 1\ MPa$
- Displacement for the edge CA defined by: $a \leq X \leq l$ and $y = 0$ $v = 0.$
- Not fixes A : $u = v = 0.$

For modeling C one replaces the constraint imposed by a pressure on the lips of the crack.

2 Reference solution

2.1 Method of calculating used for the reference solution

Reference solution of BROWN & STRAWLEY [bib1]:

$J=F^2 \pi a \sigma^2 / E$ with $F=1.98$

a in mm

σ and E in N/mm^2

2.2 Results of reference for G

Results of reference $G=1.98^2 \times \pi \times 37.5 \times 0.5\,10^{-5}=2.3093\,10^{-3}\,Mpa.mm$

The formula G (IRWIN) = $\frac{1}{E}\left(K_1^2+K_2^2\right)$ conduit, like $K_2=0$, with $K_1=21.491\,MPa.mm^{1/2}$

2.3 Results of reference for the derivative of G

While varying the Young modulus and the loading Fy, it is noted that:

$G=\alpha F_Y^2$ with $\alpha=2.3\,10^{-3}$ that is to say $\frac{\partial G}{\partial F_Y}=2\alpha F_Y$

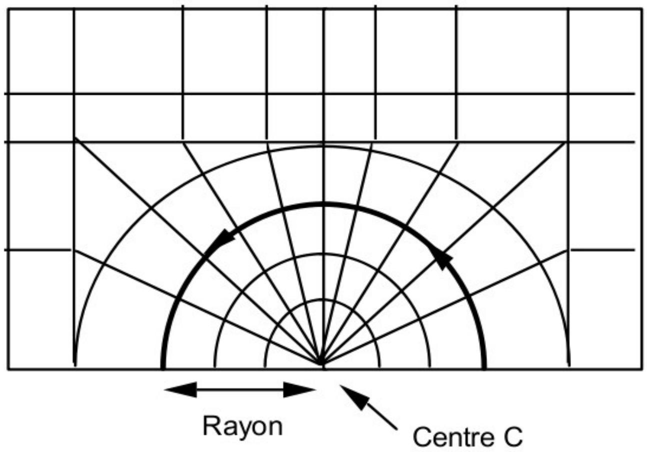
$G=\frac{\beta}{E}$ with $\beta=460.$ that is to say $\frac{\partial G}{\partial E}=-\frac{G}{E}$

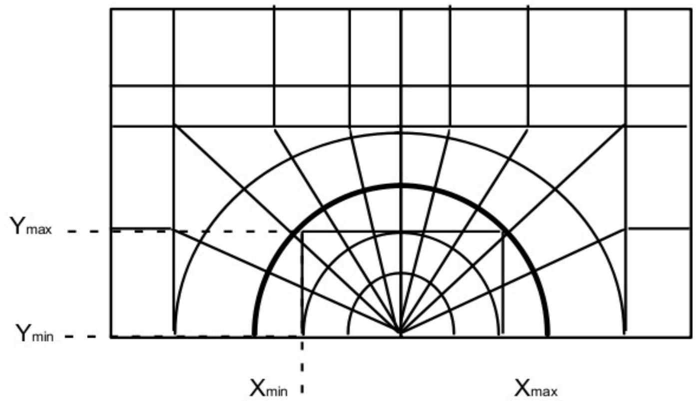
2.4 Bibliographical reference

1) Special BROWN-STAWLEY ASTM Technical Publication n° 410 (1966)

3 Modeling B

3.1 Characteristics of modeling





The field is calculated θ. then the rate of refund of energy G, coefficients of constraints K_1 and K_2, the rate of refund of energy obtained by the formula of IRWIN, direction of propagation of the crack.

3.2 Characteristics of the grid

Many nodes: 673

Many meshs and types: 112 meshs `QUAD8` and 142 meshs `TRIA6`

3.3 Sizes tested and results

The values tested are the rate of refund of energy calculated by the method theta and the rate of refund of energy calculated by the formula of IRWIN starting from the coefficients of intensity of constraints K_1 and K_2.

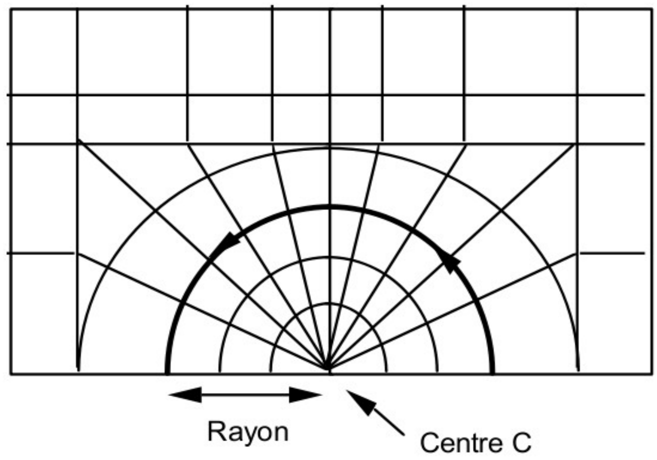
Identification	Reference	Tolerance
Crowns 1 to 6 G	$2.3093\,10^{-3}$	<1%
Crowns 1 to 6 G (IRWIN)	$2.3093\,10^{-3}$	<1%
Crowns 1 to 6 K_1	24.491	<1%
Crowns 1 to 6 K_2	$0.$	absolute

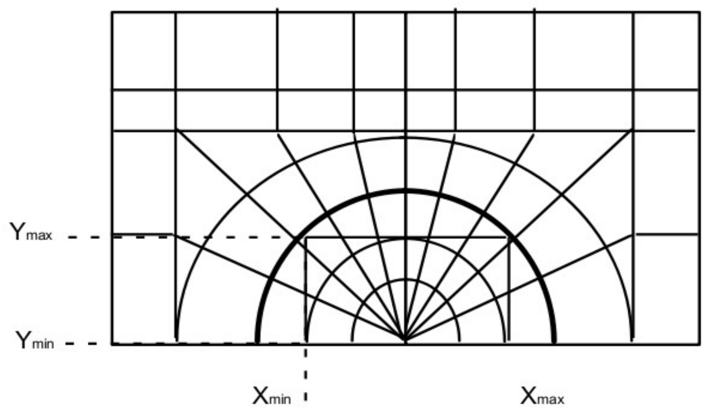
3.4 Notice

The calculation of G, K_1, K_2, G (IRWIN) $=\frac{1}{E}(K_1^2+K_2^2)$ was carried out starting from 6 fields θ different, corresponding each one to a circular ring centered in C.

4 Modeling C

4.1 Characteristics of modeling





The loading differs:

- one relieves the stress imposed in $Y=h$,
- a pressure is imposed $p=-1$ on the lips of the crack.

4.2 Characteristics of the grid

Many nodes: 673

Many meshs and types: 112 meshs `QUAD8` and 142 meshs `TRIA6`

4.3 Sizes tested and results

Values of G

Identification	Reference	Tolerance
Crowns 1 to 6 G	$2.3093\,10^{-3}$	<1%
Crowns 1 to 6 G (IRWIN)	$2.3093\,10^{-3}$	<1%
Crowns 1 to 6 K_1	22.529	<1%
Crowns 1 to 6 K_2	0.	absolute

4.4 Notice

The calculation of G, K_1, K_2 and G (IRWIN) $=\frac{1}{E}(K_1^2+K_2^2)$ was carried out starting from the same fields θ that for preceding modeling. The results are identical.

5 Modeling E

5.1 Characteristics of modeling

The loading considered here is a variable loading along the lips of the crack. One imposes a variable pressure on the lips of the crack:

$$p=\frac{x-100}{37,5}.$$

One also imposes in a second resolution an equivalent force of contour on the lips. Theoretically, the results are the same ones.

5.2 Characteristics of the grid

Grid of modeling C.

5.3 Sizes tested and results

Values of G exits of CALC_G, option CALC_G.
Values of G, G_{IRWIN}, K_1 and K_2 exits of CALC_G, option CALC_K_G.

One tests these values for the 2 loadings quoted with the §5.1.

Identification	Reference	Tolerance
Crowns 1 to 6 G	$6,0\,10^{-4}$	$<0,5\%$
Crowns 1 to 6 G (IRWIN)	$6,0\,10^{-4}$	$<0,55\%$
Crowns 1 to 6 K_1	$10,95$	$<0,5\%$
Crowns 1 to 6 K_2	$0.$	absolute

6 Summary of the results

The calculation of G is not sensitive to the choice of the field of integration.