SSLP103 - Calculation of the coefficients of intensity of constraints K_I and K_{II} for a circular plate fissured in linear elasticity

Summary

It is about a test of breaking process in static linear elasticity for a two-dimensional problem. One considers a fissured circular plate (with a tilted crack of 30 degrees compared to the x-axis) for which one calculates:

- coefficients of intensity of constraints K_I and K_{II} ,
- the rate of refund of energy G starting from the formula of IRWIN.

The interest of the test is to know the analytical solution which gives the coefficients of intensity of constraints and to have a tilted crack.

Two modelings are:
- Modeling a: FEM for elements C_PLAN, D_PLAN
- Modeling b: FEM for elements D_PLAN_INCO_UPG, D_PLAN_INCO_UP

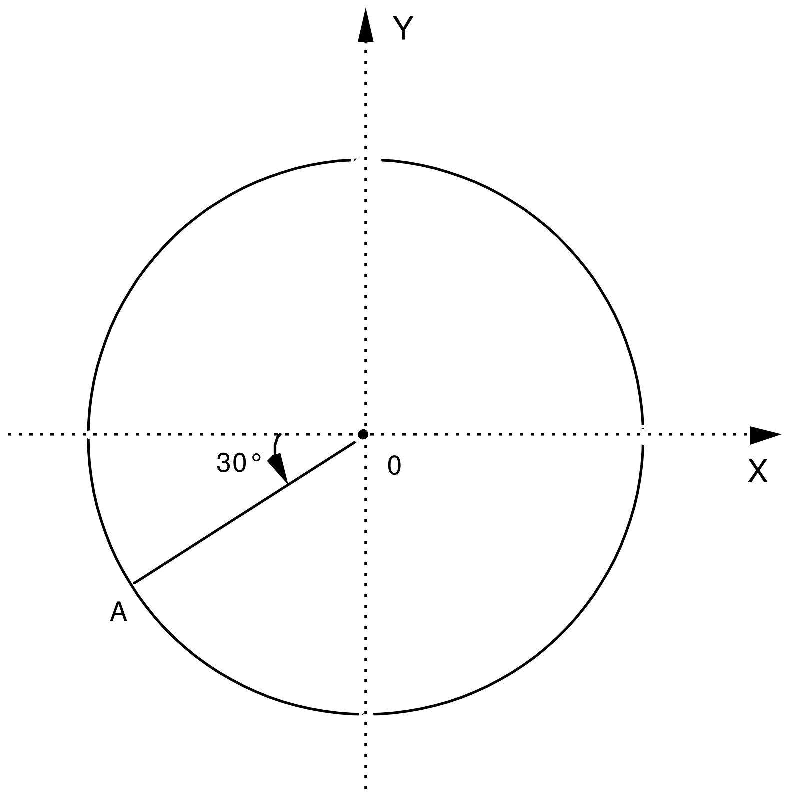
This test understands a modeling which treats successively the plane strains and the plane stresses (elements of continuous mediums).

The digital results do not deviate more 1% with 2% values of reference.

1 Problem of reference

1.1 Geometry

It is about a circular plate of ray $OA=100\,mm$, with a tilted crack of 30 degrees compared to the x-axis.



1.2 Material properties

The characteristics of material are the following ones:

$E=200\,000\,MPa$

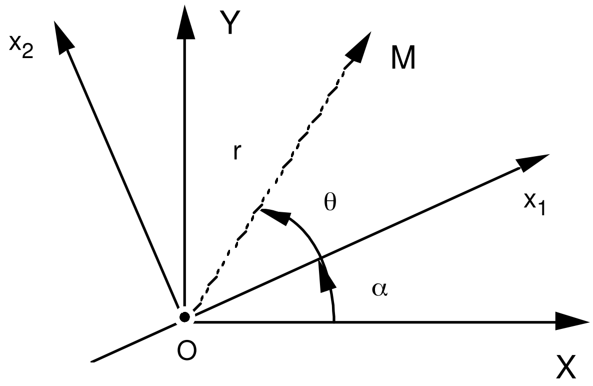
$\nu=0.3$

1.3 Boundary conditions and loadings

Displacements are imposed on the contour of the plate. They result from the singular analytical solution in mixed mode (with $K_I=2.$ and $K_{II}=1.$).

2 Reference solution

2.1 Method of calculating used for the reference solution



In plane strains or plane stresses, the distribution of displacements is given in this reference mark $(0, x_{1,} x_2)$ by:

$$\begin{cases} u_1 = \frac{1+\nu}{E}\sqrt{\frac{r}{2\pi}}\left(K_I \cos\frac{\theta}{2}(k-\cos\theta) + K_{II}\sin\left(\frac{\theta}{2}\right)(k-\cos\theta+2)\right) \\ u_2 = \frac{1+\nu}{E}\sqrt{\frac{r}{2\pi}}\left(K_I \sin\frac{\theta}{2}(k-\cos\theta) - K_{II}\cos\left(\frac{\theta}{2}\right)(k+\cos\theta-2)\right) \end{cases}$$

with $k = 3-4\nu$ in plane deformations

$k = \frac{3-\nu}{1+\nu}$ in plane constraints

or in the reference mark (O, X, Y) by: $\begin{cases} u_x = \cos\alpha\, u_1 - \sin\alpha\, u_2 \\ u_y = \sin\alpha\, u_1 + \cos\alpha\, u_2 \end{cases}$

On the contour of the plate, one a: $r = OA = 100\,mm$.

One chooses to take $K_I = 2.$ and $K_{II} = 1.$ and to impose displacements on the contour of the circular plate.

2.2 Results of reference

$K_I = 2.$
$K_{II} = 1.$
$G = 2.275\,10^{-5}$ in plane deformations
$G = 2.5\,10^{-5}$ in plane constraints

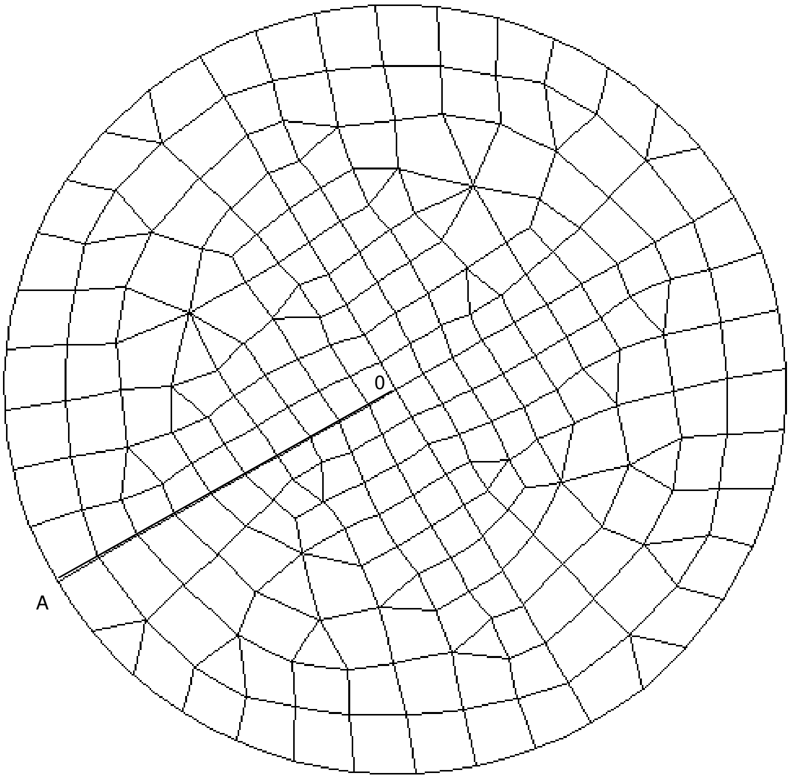
2.3 Bibliographical references

1) H.D. BUI Mécanique of Brittle fracture - ED. Masson 1978

3 Modeling a: FEM for elements D_PLAN and C_PLAN

3.1 Characteristics of modeling

Calculation is carried out in plane constraints (C_PLAN) then in plane deformations (D_PLAN).



3.2 Characteristics of the grid

Many nodes: 737

Many meshs and types: 204 meshs QUAD8, 30 meshs TRIA6

3.3 Sizes tested and results

The values tested are the coefficients of intensity of constraints K_I and K_{II} and the rate of refund of energy G calculated by the formula of IRWIN:

Identification	Reference	*Aster*	% difference
Plane constraints			
K_I	2.0	2.0067	0.33
K_{II}	1.0	0.9877	1.23
G	2.5 10 $^{-5}$	2.5213 10 $^{-5}$	0.85
Plane deformations			

K_I	2.0	2.0030	0.15
K_{II}	1.0	0.9960	0.39
G	2,275 10 $^{-5}$	2.2968 10 $^{-5}$	0.96

3.4 Remarks

The formula of IRWIN gives: $G=\frac{(1-\nu^2)}{E}(K_I^2+K_{II}^2)$ in plane deformations

and $G=\frac{1}{E}(K_I^2+K_{II}^2)$ in plane constraints.

Calculations are carried out with a crown of lower integration of ray 10.0 and of higher ray 20.0 .

4 Modeling b: FEM for elements D_PLAN_INCO_UPG, D_PLAN_INCO_UP

4.1 Characteristics of modeling

Identical to modeling A except the use of the elements D_PLAN_INCO_UPG, D_PLAN_INCO_UP.

4.2 Characteristics of the grid

Identical to modeling A

4.3 Sizes tested and results

The values tested are the coefficients of intensity of constraints K_I and K_{II} and the rate of refund of energy G calculated by the formula of IRWIN:

Identification	Method	Reference	Type of reference	% tolerance
G	CALC_G	2.275E-5	ANALYTICAL	2
$K1$	CALC_K_G	2	ANALYTICAL	2
$K2$	CALC_K_G	1	ANALYTICAL	7
G	CALC_K_G	2.275E-5	ANALYTICAL	2
G_{IRWIN}	CALC_K_G	2.275E-5	ANALYTICAL	3

5 Summaries of the results

The digital values of the coefficients of intensity of constraints and the rate of refund of energy do not deviate more than 1 with 2% values of reference, which is satisfactory.

The grid could be improved, in particular in the vicinity of the bottom of crack.