SSLP304 - Orthotropic square plate in uniaxial traction out of axes of orthotropism

Summary:

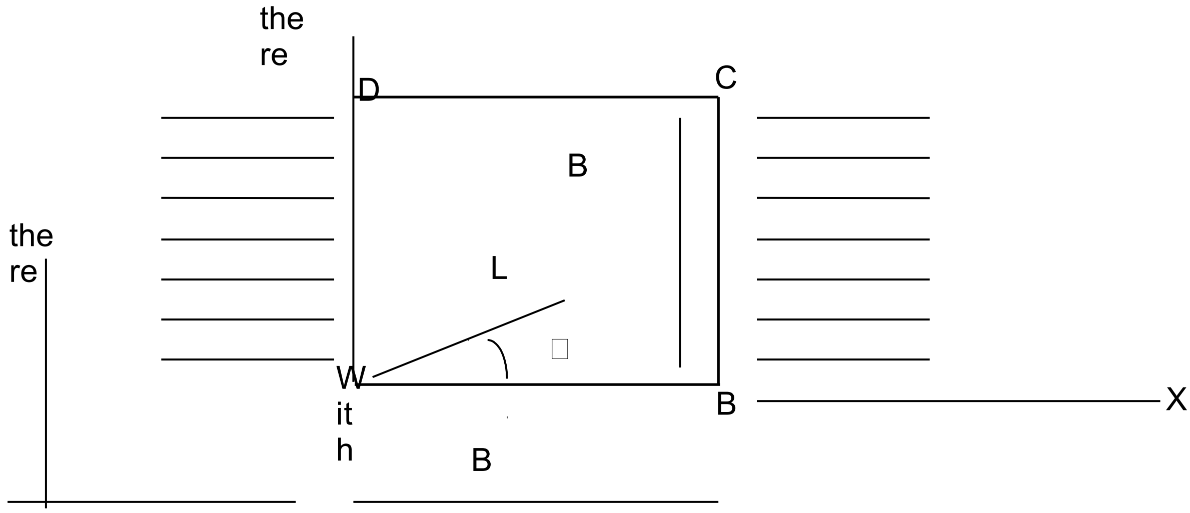
This test represents the static calculation of a square plate, out of orthotropic elastic material, whose axes of orthotropism are tilted of 30 degrees compared to the basic edge, subjected with a uniaxial traction. It makes it possible to validate the good taking into account of orthotropic elastic materials and the change of associated reference mark. 4 modelings are used: C_PLAN with meshs QUAD8 and TRIA6, in a first reference mark, C_PLAN in a second reference mark, COQUE_3D with meshs QUAD9 and TRIA7, in small displacements and COQUE_3D in great displacements. Displacements and the constraints obtained are compared with an analytical reference solution.

The first two modelings of this test result from the validation independent of version 3 of *Code_Aster* (linear static batch).

1 Problem of reference

1.1 Geometry

A square plate, made up of a tilted orthotropic material of 30 degrees compared to the edge AB .



With $b=1\,m$, unspecified thickness (plane constraints), angle of orthotropism: θ = 30 degrees.

1.2 Properties of materials

The properties of materials constituting the plate are:

orthotropic rubber band:

$$E_L=4.E10\,Pa$$
$$E_T=1.E10\,Pa$$
$$G_{LT}=0.45E10\,Pa$$
$$G_{TN}=0.35E10\,Pa$$
$$NU_{LT}=0.3$$

The axis L is tilted of 30 degrees compared to AB .

1.3 Boundary conditions and loadings

- At the point A : $DX=0$, $DY=0$
- At the point B : $DX=0$,

- Linear loading distributed: $Fx=10^4\,Pa$ on BC
- Linear loading distributed: $Fx=-10^4\,Pa$ on DA

1.4 Initial conditions

Without object.

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution, obtained with the assumption of uniaxiality of the constraints:

$$\sigma_{xx}(x,y) = F_x \qquad \sigma_{xy}(x,y) = \sigma_{yy}(x,y) = \sigma_{zz}(x,y) = 0$$

maybe in the reference mark (A, L, T) :

$$\sigma_{LL}(x,y) = c^2 F_x, \sigma_{TT}(x,y) = s^2 F_x \qquad \sigma_{LT}(x,y) = - csF_x$$

By the law of elastic behavior orthotropic, by using conventions of *Code_Aster* with regard to NU_{LT}, (cf document of use of the order DEFI_MATERIAU [§3.5.2]), one obtains directly (see for example [bib1]):

$$\varepsilon_{xx}(x,y) = \frac{F_x}{E_x} \text{ , } \varepsilon_{yy}(x,y) = - \frac{\nu_{xy}}{E_x} F_x \text{, } 2\varepsilon_{xy}(x,y) = \frac{\eta_x}{E_x} F_x$$

with:

$$\frac{1}{E_x(\theta)} = \frac{c^4}{E_L} + \frac{s^4}{E_T} + c^2 s^2 \left(\frac{1}{G_{LT}} - 2\frac{\nu_{LT}}{E_T} \right) \qquad \frac{\nu_{xy}}{E_x(\theta)} = (c^4 + s^4)\frac{\nu_{LT}}{E_T} - c^2 s^2 \left(\frac{1}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}} \right)$$

$$\frac{\eta_y}{E_x(\theta)} = - 2cs \left[\left(\frac{c^2}{E_L} - \frac{s^2}{E_T} \right) + (c^2 - s^2) \left(\frac{\nu_{LT}}{E_T} - \frac{1}{2G_{LT}} \right) \right]$$

avec $c = \cos\theta$

$s = \sin\theta$

As the deformations are uniform in the plate one obtains, by integration, displacements in the reference mark (A, x, y) :

$$u_x(x,y) = \varepsilon_{xx} . x$$

$$u_y(x,y) = \varepsilon_{yy} . y + 2\varepsilon_{xy} . x$$

2.2 Results of reference

Displacements in the reference mark (A, x, y) (in m):

Not	B	C	D
u_x	0.	5,917 10^{-7}	5,917 10^{-7}
u_x	–2,292 10^{-7}	–5,028 10^{-7}	–7,319 10^{-7}

Constraints in the reference mark related to the orthotropism:

$$\sigma_{LL}(x,y)=7500\,Pa\,,\qquad \sigma_{TT}(x,y)=2500\,Pa\,,\qquad \sigma_{LT}(x,y)=4330.127\,Pa$$

2.3 Uncertainty on the solution

Analytical solution.

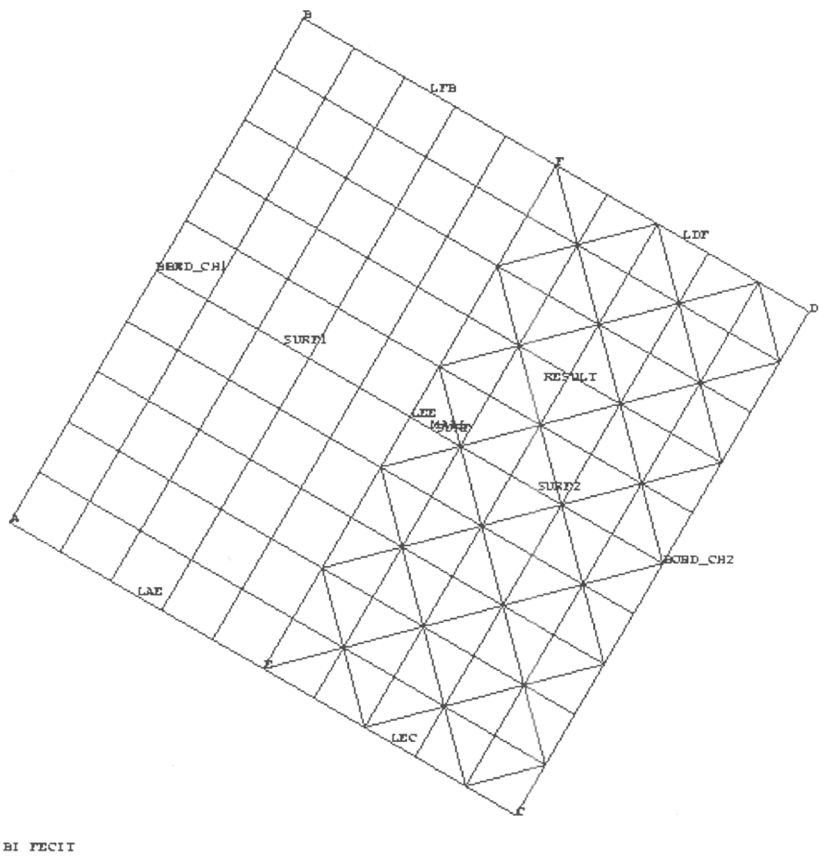
2.4 Bibliographical references

1) GAY D: “Composite materials”; 3ème edition, Hermes

3 Modeling A

3.1 Characteristics of modeling

Modeling C_PLAN. The plate is turned from -30 degrees around Z, i.e. the axis X total is colinéaire with the axis of orthotropism L. Boundary conditions and loadings, to apply in the reference mark (A, x, y) bound to the plate, are thus projected on the total reference mark (A, X, Y) (use of LIAISON_DDL in B).



3.2 Characteristics of the grid

Many nodes: 391

Many meshs and types: 50 QUAD8, 100 TRIA6

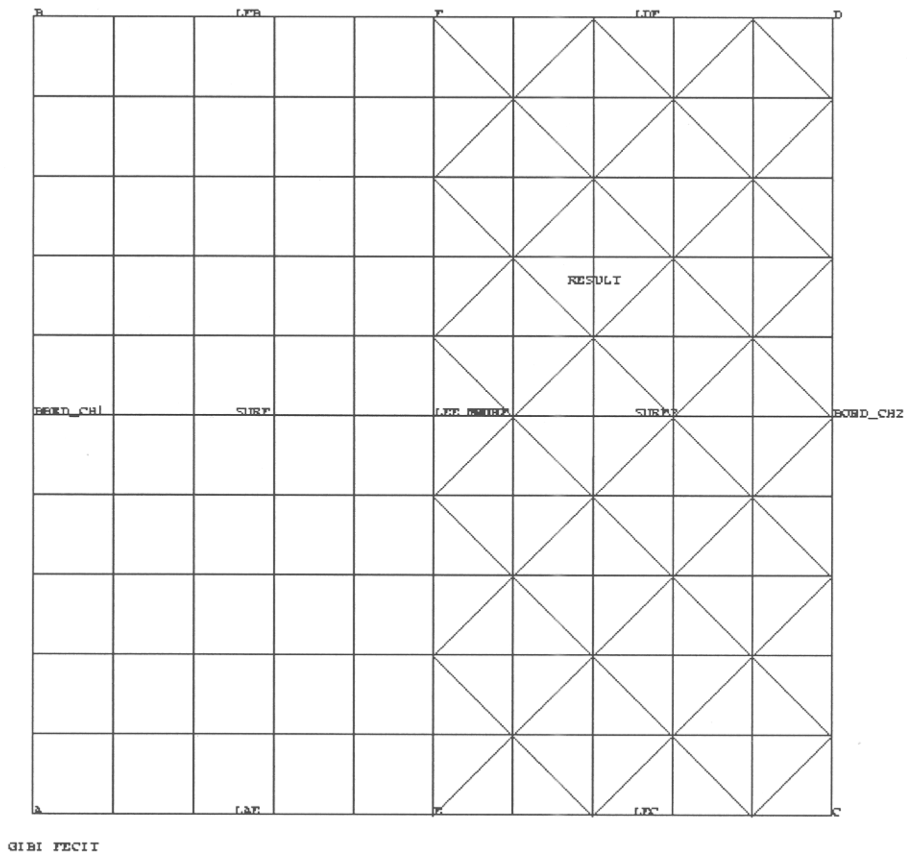
3.3 Values tested

Value	Identification	Reference
$Ux(c)=Ux(D)$	$DX(C)$	$5{,}917\ 10^{-7}$
$Uy(B)$	$DY(B)$	$-2{,}292\ 10^{-7}$
$Uy(C)$	$DY(C)$	$-5{,}028\ 10^{-7}$
$Uy(D)$	$DY(D)$	$-7{,}319\ 10^{-7}$
$Sigma$ LL	$SIXX$ (any point)	7500
$Sigma$ TT	$SIYY$ (any point)	2500
$Sigma$ LL	$SIXY$ (any point)	4300.127

4 Modeling B

4.1 Characteristics of modeling

Modeling `C_PLAN`. The plate is parallel to the total axes, i.e. the axis X total is colinéaire with the axis x. It is thus the axis of orthotropism L who is to be directed (using the keyword `SOLID MASS` of `AFFE_CARA_ELEM`).



4.2 Characteristics of the grid

Many nodes: 391

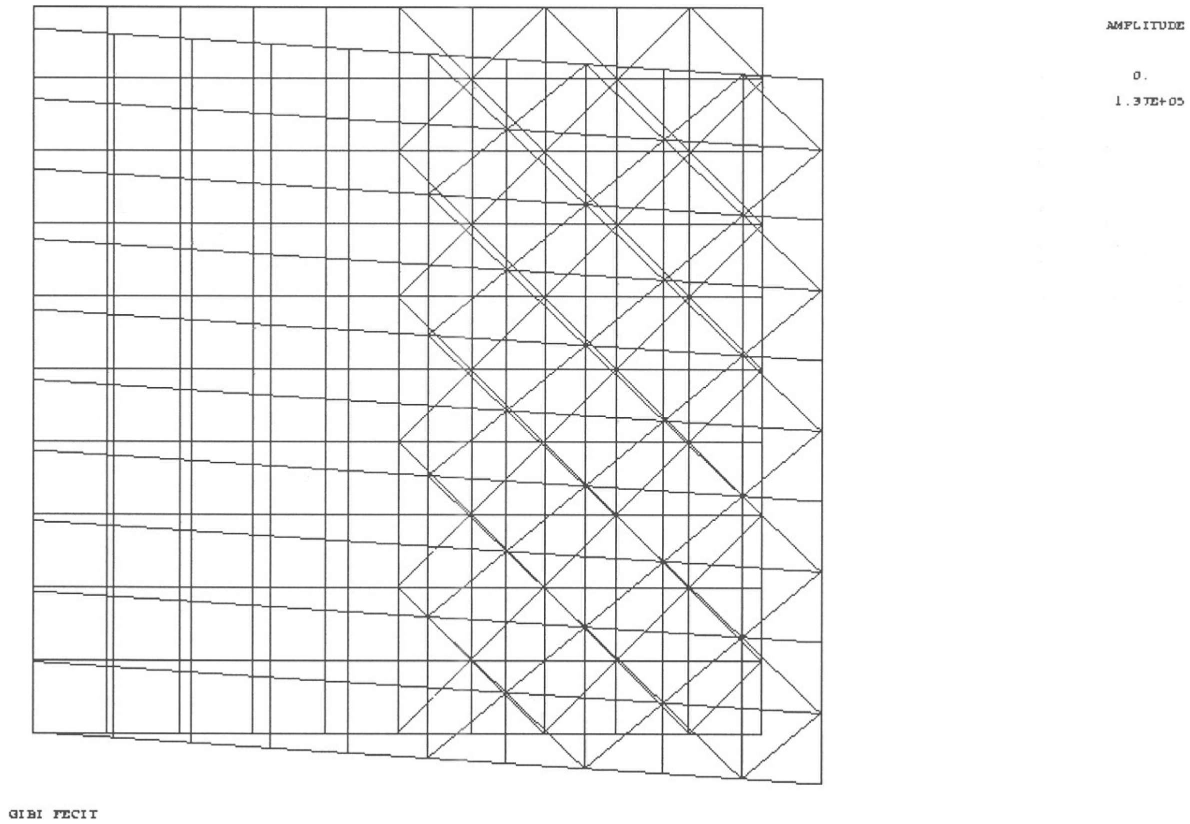
Many meshs and types: 50 QUAD8, 100 TRIA6

4.3 Values tested

Value	Identification	Reference
$Ux(c)=Ux(D)$	$DX(C)$	5,917 10^{-7}
$Uy(B)$	$DY(B)$	– 2,292 10^{-7}
$Uy(C)$	$DY(C)$	– 5,028 10^{-7}
$Uy(D)$	$DY(D)$	– 7,319 10^{-7}
$Sigma\,\mathrm{LL}$	$SIXX$ (any point)	7500
$Sigma\,\mathrm{TT}$	$SIYY$ (any point)	2500
$Sigma\,\mathrm{LL}$	$SIXY$ (any point)	4300.127

4.4 Remarks

Pace of the deformation: nonsymmetrical because of the orthotropism.



5 Modeling C

5.1 Characteristics of modeling

Modeling COQUE_3D. The plate is parallel to the total axes, i.e. the axis X total is colinéaire with the axis x . It is thus the axis of orthotropism L who is to be directed (using the keyword SOLID MASS of AFFE_CARA_ELEM). The grid is identical to that of modeling B.

5.2 Characteristics of the grid

Many nodes: 541

Many meshs and types: 50 QUAD9, 100 TRIA7

5.3 Values tested

Value	Identification	Reference
$Ux(c)=Ux(D)$	$DX(C)$	$5{,}917\ 10^{-7}$
$Uy(B)$	$DY(B)$	$-2{,}292\ 10^{-7}$
$Uy(C)$	$DY(C)$	$-5{,}028\ 10^{-7}$
$Uy(D)$	$DY(D)$	$-7{,}319\ 10^{-7}$
$Sigma\,\mathrm{LL}$	$SIXX$ (any point)	7500
$Sigma\,\mathrm{TT}$	$SIYY$ (any point)	2500
$Sigma\,\mathrm{LL}$	$SIXY$ (any point)	4300.127

6 Modeling D

6.1 Characteristics of modeling

Modeling COQUE_3D in great displacements. The plate is parallel to the total axes, it is - with - to say that the axis X total is colinéaire with the axis x. It is thus the axis of orthotropism L who is to be directed (using the keyword SOLID MASS of AFFE_CARA_ELEM). The grid is identical to that of modeling B.

6.2 Characteristics of the grid

Many nodes: 541

Many meshs and types: 50 QUAD9, 100 TRIA7

6.3 Values tested

Value	Identification	Reference
$Ux(c)=Ux(D)$	$DX(C)$	5,917 10^{-7}
$Uy(B)$	$DY(B)$	– 2,292 10^{-7}
$Uy(C)$	$DY(C)$	– 5,028 10^{-7}
$Uy(D)$	$DY(D)$	– 7,319 10^{-7}
$Sigma$ LL	$SIXX$ (any point)	7500
$Sigma$ TT	$SIYY$ (any point)	2500
$Sigma$ LL	$SIXY$ (any point)	4300.127

7 Summary of the results

The results of four modelings are very close to the analytical solution: to the maximum 0,015% of variation.

This test thus validates the taking into account of orthotropic elasticity.