

## SSLP320 - Propagation of a crack X-FEM emerging requested in Mode I

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### Summary

The purpose of this test is to validate the calculation of the stress intensity factors ( $K_I$  and  $K_{II}$ ) and the way of propagation of crack with X-FEM in 2D, within the framework of linear elasticity.

This test brings into play a rectangular plate with a crack leading, and subjected to a loading of traction to the edges inferior and superior of the plate.

Three methods to manage the propagation of cracks X-FEM are available. Each one of them is the object of a modeling.

Three modelings are considered:

- modeling a: method grid,
- modeling b: method simplex,
- modeling C: method upwind,
- modeling D: geometrical method,

The relevance of the results is evaluated by comparison of the factors of intensity of the constraints with the analytical values.

One finds a variation enters  $K_I$  and  $K_{II}$  theoretical lower than 1.13 % for the method grid, 1.12 % for the method simplex, 1.13 % for the method upwind, 1.1 % for the geometrical method and 1.1 % for the method upwind & FM.

## 1 Problem of reference

### 1.1 Geometry

The structure 2d is a rectangular plate ( $LX=10\text{ m}$ ,  $LY=30\text{ m}$ ), comprising an emerging crack [Figure 1.1-a]. The length of the initial crack is  $a=5\text{ m}$ .

One calls "lower line", the line in  $y=0$  and "higher line", the line in  $y=Ly$ .

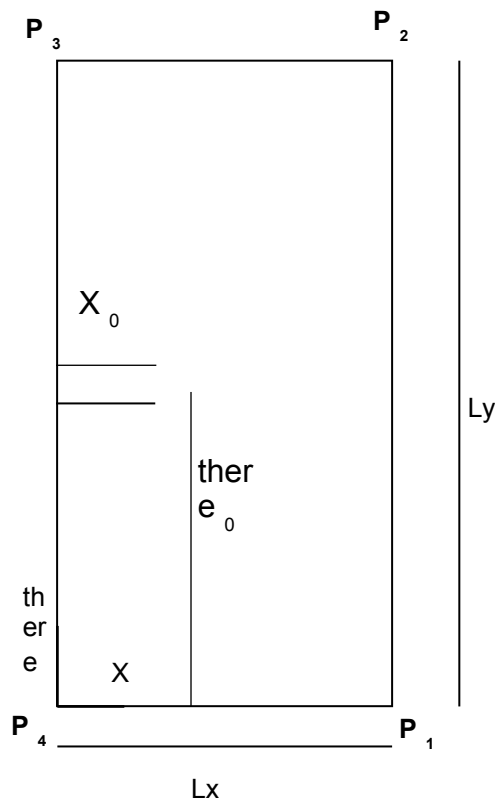


Figure 1.1-a : geometry of the fissured plate

Noted nodes  $P1$  and  $P4$  on Figure 1.1-a are used to impose the boundary conditions, which is clarified in the paragraph [§1.3].

### 1.2 Properties of material

Young modulus:  $E=205\ 10^9\ Pa$   
Poisson's ratio:  $\nu=0$

### 1.3 Boundary conditions and loadings

The loading consists in applying a force distributed to the lines lower and higher  $p=106\ Pa$  and in the direction of the normal external to surface.

In order to block the rigid modes, displacements of the nodes are blocked  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_4$  as follows:

- $DY^{P4}=DY^{P1}=0$  ;

- $DX^{P4} = 0$ .

## 2 Reference solution

### 2.1 Method of calculating

Analytical expressions of the stress intensity factors  $K_I$  and  $K_{II}$  are functions of the force distributed  $p$ , length of the crack has, the width of the plate  $Lx$  :

$$K_I = p \sqrt{\pi a} f\left(\frac{a}{Lx}\right)$$

$$K_{II} = 0$$

where the function  $f$  can be given several different manners. We choose that obtained by [1], and which is true for  $\frac{a}{Lx} < 0,6$  :

$$f\left(\frac{a}{Lx}\right) = 1,12 - 0,231\left(\frac{a}{Lx}\right) + 10,55\left(\frac{a}{Lx}\right)^2 - 21,72\left(\frac{a}{Lx}\right)^3 + 30,39\left(\frac{a}{Lx}\right)^4$$

One advances the crack thanks to the law of Paris:

$\frac{da}{dN} = C \Delta K^m$  where  $a$  is the length of crack,  $C$  and  $m$  are constants of material,  $\Delta K$  between two FICs is the difference consecutive and  $N$  is the number of cycles.

With the digital values of the test:

$$\text{Pas de propagation: } 0,25 \text{ m}$$

$$Lx : 10 \text{ m}$$

### 2.2 Sizes and results of reference

Reference		
$a(m)$	$K_I (Pa.m^{0,5})$	$K_{II} (Pa.m^{0,5})$
2.5	4.205998 10 <sup>6</sup>	0
2.75	4.63286 10 <sup>6</sup>	0
3	5.09492 10 <sup>6</sup>	0
3.25	5.59908 10 <sup>6</sup>	0
3.5	6.15349 10 <sup>6</sup>	0
3.75	6.76776 10 <sup>6</sup>	0
4	7.4531 10 <sup>6</sup>	0
4.25	8.2224 10 <sup>6</sup>	0
4.5	9.0905 10 <sup>6</sup>	0
4.75	1.0074 10 <sup>7</sup>	0
5	1.1192 10 <sup>7</sup>	0
5.25	1.2465 10 <sup>7</sup>	0
5.5	1.3916 10 <sup>7</sup>	0
5.75	1.55716 10 <sup>7</sup>	0
6	1.74586 10 <sup>7</sup>	0

Table 2.2-1 : values of reference for  $K_I$  and  $K_{II}$

## 2.3 Uncertainties on the solution

No, analytical solution.

## 2.4 Bibliographical references

- [1] TADA H., PARIS P., IRWIN G.: The stress analysis of aces, Handbook. Del Research Corporation, Hellertown, Pennsylvania, 1973.

## 3 Modeling A

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### 3.1 Characteristics of modeling

In this modeling, the method grid is tested for the propagation of crack. The level-sets are determined by orthogonal projection on the segments composing the crack.

### 3.2 Characteristics of the grid

The structure is modelled by a "healthy" grid regular composed of  $40 \times 101$  QUAD4, respectively along the axes  $x, y$ . The crack is represented by a succession of SEG2, independently of the grid of the structure.

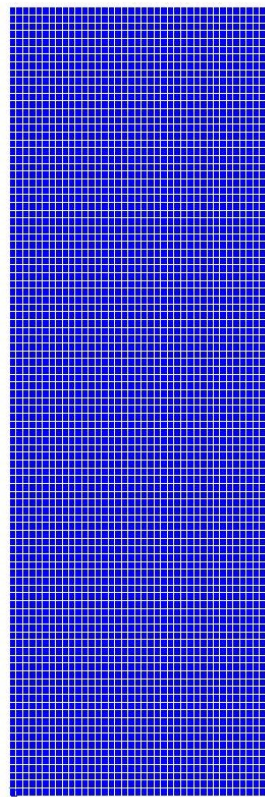


Figure 3.2-a : grid of the fissured plate

### 3.3 Sizes tested and results

For each step of propagation ( $2,5 m$ ), one tests the value of the stress intensity factors  $K_I$  and  $K_{II}$  data by CALC\_G.

One also tests the ordinate of the bottom of crack given by PROPA\_FISS.

### 3.3.1 Results on $K_I$ :

Identification	Code_Aster	Reference	difference
CALC_G			
KI_1	4.17448 10 <sup>6</sup>	4.205998 10 <sup>6</sup>	-0,749%
KI_2	4.60197 10 <sup>6</sup>	4.63286 10 <sup>6</sup>	-0,667%
KI_3	5.0668 10 <sup>6</sup>	5.09492 10 <sup>6</sup>	-0,552%
KI_4	5.575 10 <sup>6</sup>	5.59908 10 <sup>6</sup>	-0,43%
KI_5	6.1334 10 <sup>6</sup>	6.15349 10 <sup>6</sup>	-0,326%
KI_6	6.7499 10 <sup>6</sup>	6.76776 10 <sup>6</sup>	-0,264%
KI_7	7.4338 10 <sup>6</sup>	7.4531 10 <sup>6</sup>	-0,259%
KI_8	8.19599 10 <sup>6</sup>	8.2224 10 <sup>6</sup>	-0,322%
KI_9	9.0497 10 <sup>6</sup>	9.0905 10 <sup>6</sup>	-0,449%
KI_10	1.0011 10 <sup>7</sup>	1.0074 10 <sup>7</sup>	-0,627%
KI_11	1.1099 10 <sup>7</sup>	1.1192 10 <sup>7</sup>	-0,828%
KI_12	1.2339 10 <sup>7</sup>	1.2465 10 <sup>7</sup>	-1,011%
KI_13	1.37603 10 <sup>7</sup>	1.3916 10 <sup>7</sup>	-1,121%
KI_14	1.54018 10 <sup>7</sup>	1.55716 10 <sup>7</sup>	-1,09%
KI_15	1.7313 10 <sup>7</sup>	1.74586 10 <sup>7</sup>	-0,834%

### 3.3.2 Results on $K_{II}$ :

For this test, it be wished that  $K_{II}$  that is to say lower than  $10^{-4} K_I$ . Thus, one makes sure that  $K_{II}$  is rather close to zero, the value of reference.

Identification	Code_Aster	Reference
CALC_G		
KII_1	-2.7313 10 <sup>2</sup>	0
KII_2	-8.5062 10 <sup>1</sup>	0
KII_3	-2.6061 10 <sup>2</sup>	0
KII_4	1.5995 10 <sup>2</sup>	0
KII_5	-2.7309 10 <sup>2</sup>	0
KII_6	-2.3176 10 <sup>2</sup>	0
KII_7	-3.1276 10 <sup>2</sup>	0
KII_8	3.1327 10 <sup>2</sup>	0
KII_9	-3.8393 10 <sup>2</sup>	0
KII_10	-4.1916 10 <sup>2</sup>	0
KII_11	-4.986 10 <sup>2</sup>	0
KII_12	-5.6998 10 <sup>2</sup>	0
KII_13	-6.7642 10 <sup>2</sup>	0
KII_14	-7.9542 10 <sup>2</sup>	0
KII_15	-9.5344 10 <sup>2</sup>	0

### 3.3.3 Results on the ordinate of the bottom of crack:

It is checked that the coordinates in ordinate of the successive funds of crack are close to the initial value. This checking gives the same indications as the test on  $K_{II}$ .

Identification	Code_Aster	Reference	Difference
CALC_G			
y_1	15	15	0%
y_2	15	15	2.18 10 <sup>-4</sup> %
y_3	15	15	2.8 10 <sup>-4</sup> %
y_4	15	15	4.51 10 <sup>-4</sup> %
y_5	15	15	5.47 10 <sup>-4</sup> %
y_6	15	15	6.95 10 <sup>-4</sup> %
y_7	15	15	8.1 10 <sup>-4</sup> %
y_8	15	15	9.5 10 <sup>-4</sup> %
y_9	15.0002	15	0,001%
y_10	15.0002	15	0,001%
y_11	15.0002	15	0,001%
y_12	15.0002	15	0,002%
y_13	15.0002	15	0,002%
y_14	15.0003	15	0,002%
y_15	15.0003	15	0,002%

### 3.4 Complementary results

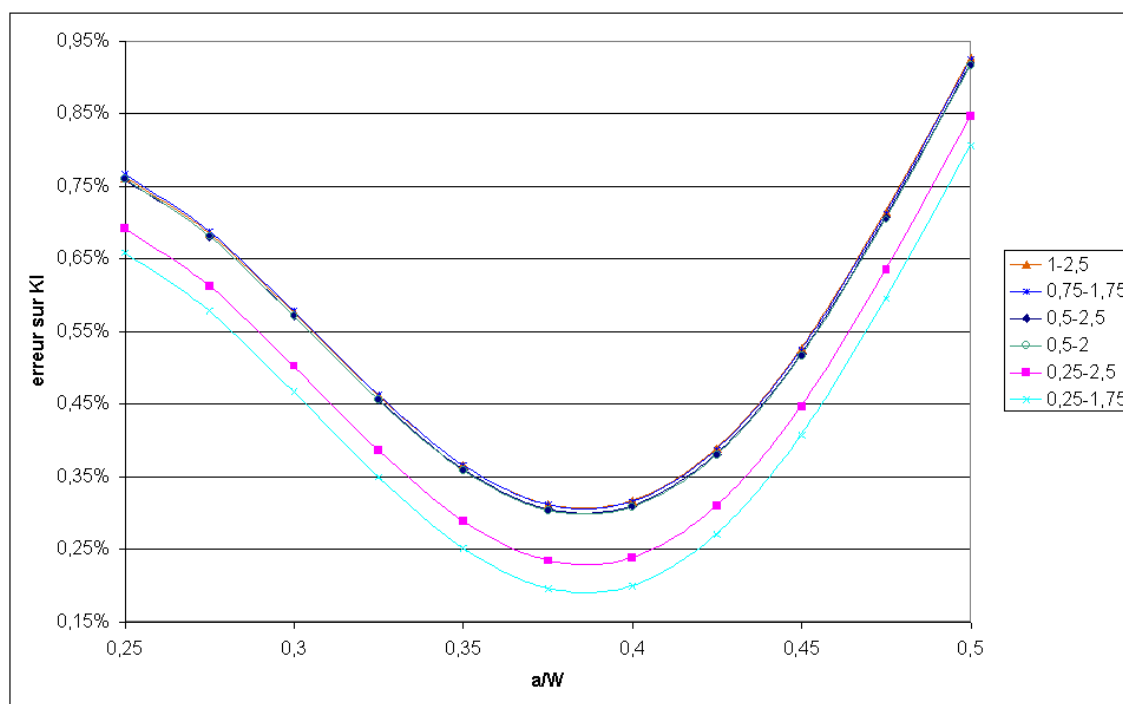


Figure 3.4-a : Influence of the choice of crowns IH and RS on the error on KI



We can see here that the configuration most adapted for the choice of  $RI$  and  $RS$  (crowns lower and higher of the field theta) is:  $RI = 2 * L_0$  and  $RS = 7 * L_0$  where  $L_0$  is the smallest edge of the grid.

## 4 Modeling B

### 4.1 Characteristics of modeling

In this modeling, the method simplex is tested for the propagation of crack.  
The level-sets are determined by resolution of the equations of reactualization.

### 4.2 Characteristics of the grid

One uses here the same grid as in modeling A.

### 4.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors  $K_I$  and  $K_{II}$  data by CALC\_G.

#### 4.3.1 Results on $K_I$ :

Identification	Code_Aster	Reference	difference
CALC_G			
KI_1	4.1749 10 <sup>6</sup>	4.205998 10 <sup>6</sup>	0,73%
KI_2	4.6025 10 <sup>6</sup>	4.63286 10 <sup>6</sup>	0,65%
KI_3	5.0675 10 <sup>6</sup>	5.09492 10 <sup>6</sup>	0,54%
KI_4	5.5758 10 <sup>6</sup>	5.59908 10 <sup>6</sup>	0,41%
KI_5	6.1344 10 <sup>6</sup>	6.15349 10 <sup>6</sup>	0,31%
KI_6	6.7511 10 <sup>6</sup>	6.76776 10 <sup>6</sup>	0,24%
KI_7	7.4352 10 <sup>6</sup>	7.4531 10 <sup>6</sup>	0,24%
KI_8	8.1976 10 <sup>6</sup>	8.2224 10 <sup>6</sup>	0,30%
KI_9	9.0516 10 <sup>6</sup>	9.0905 10 <sup>6</sup>	0,42%
KI_10	1.0013 10 <sup>6</sup>	1.0074 10 <sup>7</sup>	0,60%
KI_11	1.1101 10 <sup>6</sup>	1.1192 10 <sup>7</sup>	0,80%
KI_12	1.2341 10 <sup>6</sup>	1.2465 10 <sup>7</sup>	0,98%
KI_13	1.37608 10 <sup>6</sup>	1.3916 10 <sup>7</sup>	1,09%
KI_14	1.5405 10 <sup>6</sup>	1.55716 10 <sup>7</sup>	1,06%
KI_15	1.7317 10 <sup>6</sup>	1.74586 10 <sup>7</sup>	0,80%

## 4.3.2 Results on $K_{II}$ :

Identification	Code_Aster	Reference
CALC_G		
KII_1	-294.543283239	0
KII_2	140.53299141	0
KII_3	-92.1854404834	0
KII_4	31.5966858116	0
KII_5	-22.0812184567	0
KII_6	1.80888843609	0
KII_7	-14.6528361549	0
KII_8	-12.9336699382	0
KII_9	-21.8747247036	0
KII_10	-27.5009059699	0
KII_11	-36.8193114189	0
KII_12	-47.1435134216	0
KII_13	-60.5512354886	0
KII_14	-77.2532857738	0
KII_15	-98.7961435219	0

## 5 Modeling C

### 5.1 Characteristics of modeling

In this modeling, the method upwind fast marching UPWIND is tested for the propagation of crack. The level-sets are determined by resolution of the equations of reactualization per diagram to the finished differences.

### 5.2 Characteristics of modeling

One uses here the same grid as in modeling A (§2.1).

### 5.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors  $K_I$  and  $K_{II}$  data by CALC\_G.

#### 5.3.1 Results on KI:

Identification	Code_Aster	Reference	difference
CALC_G			
KI_1	4.174911 10 <sup>6</sup>	4.205998 10 <sup>6</sup>	0,739%
KI_2	4.602545 10 <sup>6</sup>	4.632857 10 <sup>6</sup>	0,654%
KI_3	5.067525 10 <sup>6</sup>	5.094923 10 <sup>6</sup>	0,538%
KI_4	5.575881 10 <sup>6</sup>	5.599079 10 <sup>6</sup>	0,414%
KI_5	6.134452 10 <sup>6</sup>	6.153487 10 <sup>6</sup>	0,309%
KI_6	6.751139 10 <sup>6</sup>	6.767759 10 <sup>6</sup>	0,24%
KI_7	7.435204 10 <sup>6</sup>	7.453097 10 <sup>6</sup>	0,24%
KI_8	8.197634 10 <sup>6</sup>	8.222429 10 <sup>6</sup>	0,302%
KI_9	9.051616 10 <sup>6</sup>	9.090524 10 <sup>6</sup>	0,428%
KI_10	1.0013134 10 <sup>7</sup>	1.0074102 10 <sup>7</sup>	0,605%
KI_11	1.1101774 10 <sup>7</sup>	1.1191940 10 <sup>7</sup>	0,805%
KI_12	1.2341792 10 <sup>7</sup>	1.2464967 10 <sup>7</sup>	0,99%
KI_13	1.3763566 10 <sup>7</sup>	1.3916354 10 <sup>7</sup>	1,098%
KI_14	1.5405615 10 <sup>7</sup>	1.5571606 10 <sup>7</sup>	1,065%
KI_15	1.7317423 10 <sup>7</sup>	1.7458645 10 <sup>7</sup>	0,809%

## 5.3.2 Results on KII:

Identification	Code_Aster	Reference
CALC_G		
KII_1	-294.54	0
KII_2	-310.00	0
KII_3	-330.90	0
KII_4	-357.05	0
KII_5	-389.44	0
KII_6	-428.98	0
KII_7	-476.90	0
KII_8	-534.75	0
KII_9	-604.47	0
KII_10	-688.57	0
KII_11	-790.30	0
KII_12	-913.84	0
KII_13	-1064.72	0
KII_14	-1250.30	0
KII_15	-1480.53	0

## 6 Modeling D

### 6.1 Characteristics of modeling

In this modeling, the geometrical method is tested for the propagation of crack.

### 6.2 Characteristics of modeling

One uses here the same grid as in modeling A.

### 6.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors  $K_I$  and  $K_{II}$  data by CALC\_G.

#### 6.3.1 Results on KI:

Identification	Code_Aster	Reference	difference
CALC_G			
KI_1	4.205998 10 <sup>6</sup>	4.205998 10 <sup>6</sup>	0,739%
KI_2	4.602538 10 <sup>6</sup>	4.632857 10 <sup>6</sup>	0,654%
KI_3	5.067526 10 <sup>6</sup>	5.094923 10 <sup>6</sup>	0,538%
KI_4	5.575879 10 <sup>6</sup>	5.599079 10 <sup>6</sup>	0,414%
KI_5	6.134453 10 <sup>6</sup>	6.153487 10 <sup>6</sup>	0,309%
KI_6	6.751140 10 <sup>6</sup>	6.767759 10 <sup>6</sup>	0,246%
KI_7	7.435205 10 <sup>6</sup>	7.453097 10 <sup>6</sup>	0.240%
KI_8	8.197636 10 <sup>6</sup>	8.222429 10 <sup>6</sup>	0,302%
KI_9	9.051618 10 <sup>6</sup>	9.090524 10 <sup>6</sup>	0,428%
KI_10	1.0013136 10 <sup>7</sup>	1.0074102 10 <sup>7</sup>	0.61%
KI_11	1.1101777 10 <sup>7</sup>	1.1191940 10 <sup>7</sup>	0.805%
KI_12	1.2341796 10 <sup>7</sup>	1.2464967 10 <sup>7</sup>	0,99%
KI_13	1.3763571 10 <sup>7</sup>	1.3916354 10 <sup>7</sup>	1.098%
KI_14	1.5405621 10 <sup>7</sup>	1.5571606 10 <sup>7</sup>	1,065%
KI_15	1.7317438 10 <sup>7</sup>	1.7458645 10 <sup>7</sup>	0,808%

## 6.3.2 Results on KII:

Identification	Code_Aster	Reference
CALC_G		
KII_1	-294.543283255	0
KII_2	140.53345257	0
KII_3	-92.1815420989	0
KII_4	31.588424802	0
KII_5	-22.0719386916	0
KII_6	1.80251453051	0
KII_7	-14.6478888153	0
KII_8	-12.9364628022	0
KII_9	-21.8732802716	0
KII_10	-27.5019633622	0
KII_11	-36.819305336	0
KII_12	-47.1442361154	0
KII_13	-60.5518993996	0
KII_14	-77.2544634291	0
KII_15	-98.7975849383	0

## 7 Summaries of the results

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One can compare the computing time for the same number of steps of propagation (15) of the three methods.

Grid	Method	Time ( s )
40×101	Grid	21.7
	Simplex	18.8
	Upwind	23.7
	geometrical	21.2

The results make it possible to validate on a simple case the calculation of the stress intensity factors in mode  $I$  for elements X-FEM for the four methods grid, geometrical simplex, upwind and.