SSLP321 - Propagation of a crack X-FEM in a flexbeam 3 points

Summary

The purpose of this test is to validate the way of propagation of crack with X-FEM in 2D, within the framework of linear elasticity.

This test brings into play a rectangular plate with a crack emerging, and subjected to an inflection 3 points as in the article of Mariani and Perego [1].

Five methods to manage the propagation of cracks X-FEM are available. Each one of them is the object of a modeling.

- modeling A : method grid
- modeling B : method simplex
- modeling C : method upwind
- modeling D : geometrical method

The comparisons are done with the values given by the method grid.

1 Problem of reference

1.1 Geometry

The geometry, dimensions and the materials are taken identical to those of Mariani et al. **[1]**.
The structure $2D$ is a rectangular plate ($230\,mm \times 75\,mm$), comprising an emerging crack [Figure 1.1-a]. The length of the initial crack is $a=19\,mm$.

Noted nodes $P1$, $P2$ and $P3$ are used to impose the boundary conditions, which is clarified in the paragraph [§1.3].

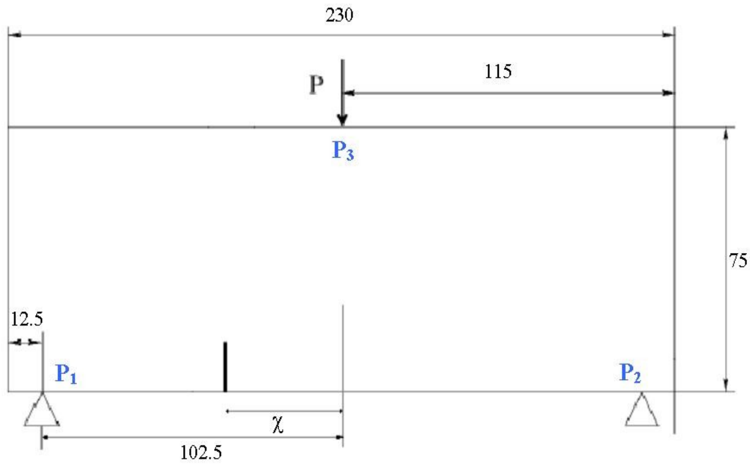


Figure 1.1-a: geometry of the fissured plate

1.2 Properties of material

Young modulus: $E=31{,}37\,10^{9}\,Pa$
Poisson's ratio: $\nu=0.2$

1.3 Boundary conditions and loadings

The loading consists in applying a unit nodal force in $P3$.

In order to block the rigid modes, displacements of the nodes are blocked $P1$ and $P2$ as follows:

- $DY^{P1}=DY^{P2}=0$;
- $DX^{P1}=0$.

2 Reference solution

2.1 Method of calculating

The study of this case is based entirely on the article of Mariani and Perego. Three initial configurations of crack are selected: $\chi=0, 25$ and 50. In this case test, we chose only $\chi=50$. One thus compares the way of propagation compared to the experimental way of the article [bib1].

Expressions of reference of the stress intensity factors K_I and K_{II} are those of the method grid. One will thus compare the values of the methods simplex, upwind and geometrical with the values given by the method grid.

For the propagation of the crack, we use the law of Paris:
$\frac{da}{dN}=C \Delta K^m$ where has is the length of crack, C and m are constants of material, ΔK is the difference between two $FICs$ consecutive and N is the number of cycles.

The criterion of junction used is it *maximum hoop stress criterion*:

$$\beta=2\arctan[\frac{1}{4}(\frac{K_I}{K_{II}}-sign(K_{II})\sqrt{(\frac{K_I}{K_{II}})^2+8})]$$

With the digital values of the test:

Pas de propagation: $0{,}3\,m$
$x_0 : 65\,mm$
$y_0 : 19\,mm$
Many steps of propagation: 13
$RI : 3\,mm$
$RS : 12\,mm$
$RP : 12\,mm$

2.2 Sizes and results of reference

Reference (method grid)			
$x(mm)$	$y(mm)$	$K_I(MPa.m^{0,5})$	$K_{II}(MPa.m^{0,5})$
65	19	2.43961 10^{-1}	4.27722 10^{-2}
66.129	22.313	2.90147 10^{-1}	1.21013 10^{-4}
67.261	25.625	3.30840 10^{-1}	7.10255 10^{-3}
68.533	28.885	3.75984 10^{-1}	1.94683 10^{-3}
69.839	32.132	4.33606 10^{-1}	1.20266 10^{-3}
71.164	35.372	4.96975 10^{-1}	8.82542 10^{-4}
72.5	38.607	5.73785 10^{-1}	-1.23199 10^{-3}
73.821	41.848	6.70222 10^{-1}	-3.54655 10^{-3}
75.109	45.103	7.89716 10^{-1}	-4.54122 10^{-3}
76.359	48.372	9.39463 10^{-1}	-8.18030 10^{-3}
77.552	51.662	1.15201	-1.55772 10^{-2}
78.655	54.984	1.45163	-2.31849 10^{-2}
79.652	58.339	1.91885	-3.52229 10^{-2}

Table 2.2-1 : values of reference for K_I and K_{II}

2.3 Bibliographical references

[1] Mariani S, Perego U – Extended finite element method for quasi-brittle fracture, *International Newspaper for numerical methods in engineering* , 58:103-126 (2003)

3 Modeling A

3.1 Characteristics of modeling

In this modeling, the method grid is tested for the propagation of crack. The level-sets are determined by orthogonal projection on the segments composing the crack.

3.2 Characteristics of the grid

The structure is modelled by a regular grid composed of 90×30 QUAD4, respectively along the axes x , y . The crack is not with a grid.

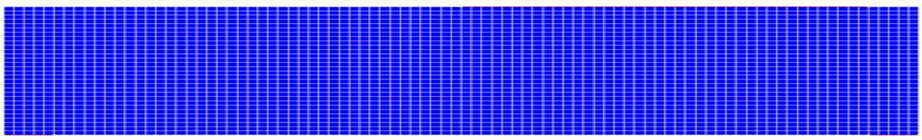


Figure 3.2-a : grid of the fissured plate

3.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

3.3.1 Results on K_I :

One carries out a relative test of nonregression on K_I with a precision of 2.10^{-3} .

Identification	*Code_Aster*	Reference	difference
CALC_G			
KI_1	2.43961 10^{-1}	2.43961 10^{-1}	-
KI_2	2.90147 10^{-1}	2.90147 10^{-1}	-
KI_3	3.30840 10^{-1}	3.30840 10^{-1}	-
KI_4	3.75984 10^{-1}	3.75984 10^{-1}	-
KI_5	4.33606 10^{-1}	4.33606 10^{-1}	-
KI_6	4.96975 10^{-1}	4.96975 10^{-1}	-
KI_7	5.73785 10^{-1}	5.73785 10^{-1}	-
KI_8	6.70222 10^{-1}	6.70222 10^{-1}	-
KI_9	7.89716 10^{-1}	7.89716 10^{-1}	-
KI_10	9.39463 10^{-1}	9.39463 10^{-1}	-
KI_11	1.15201	1.15201	-
KI_12	1.45163	1.45163	-
KI_13	1.91885	1.91885	-

3.3.2 Results on K_{II} :

For this test, it be wished that K_{II} maybe such as $K_{II}=K_{IIref}\pm 10^{-2}$.

Identification	*Code_Aster*	Reference	Difference
CALC_G			
KII_1	4.27722 10^{-2}	4.27722 10^{-2}	-
KII_2	1.21013 10^{-4}	1.21013 10^{-4}	-
KII_3	7.10255 10^{-3}	7.10255 10^{-3}	-
KII_4	1.94683 10^{-3}	1.94683 10^{-3}	-
KII_5	1.20266 10^{-3}	1.20266 10^{-3}	-
KII_6	8.82542 10^{-4}	8.82542 10^{-4}	-
KII_7	-1.23199 10^{-3}	-1.23199 10^{-3}	-
KII_8	-3.54655 10^{-3}	-3.54655 10^{-3}	-
KII_9	-4.54122 10^{-3}	-4.54122 10^{-3}	-
KII_10	-8.18030 10^{-3}	-8.18030 10^{-3}	-
KII_11	-1.55772 10^{-2}	-1.55772 10^{-2}	-
KII_12	-2.31849 10^{-2}	-2.31849 10^{-2}	-
KII_13	-3.52229 10^{-2}	-3.52229 10^{-2}	-

3.4 Complementary results

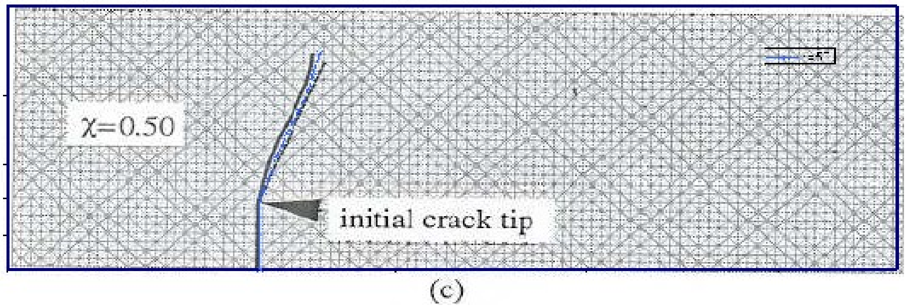


Figure 3.4-a: Comparison between the way obtained and the method grid with the ways of the study of Mariani and Perego

On Figure 3.4-a, one can see: in black digital results of Mariani and Perego, in dotted lines experimental results and blue results of the method grid of Code_Aster. The method grid gives results close to the experimental data.

4 Modeling B

4.1 Characteristics of modeling

In this modeling, the method simplex is tested for the propagation of crack.
The level-sets are determined by resolution of the equations of reactualization.

4.2 Characteristics of the grid

One uses here the same grid as in modeling A .

4.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

4.3.1 Results on KI:

One carries out a relative test of nonregression on K_I compared to $K_{I\,maillage}$ with a precision of 5% .

Identification	*Code_Aster*	Reference	Difference ($\%$)
CALC_G			
KI_1	2.43961 10^{-1}	2.43961 10^{-1}	2.03 10^{-4} %
KI_2	0.29056	2.90147 10^{-1}	0.1%
KI_3	0.3308	3.30840 10^{-1}	3.9 10^{-4} %
KI_4	0.3759	3.75984 10^{-1}	0.02%
KI_5	0.43355	4.33606 10^{-1}	7.9 10^{-4} %
KI_6	0,497	4.96975 10^{-1}	0,025%
KI_7	0,573	5.73785 10^{-1}	0.12%
KI_8	0.6705	6.70222 10^{-1}	0.03%
KI_9	0.7899	7.89716 10^{-1}	0.03%
KI_10	0.93757	9.39463 10^{-1}	0.2%
KI_11	1.1508	1.15201	0.11%
KI_12	1.4472	1.45163	0.30%
KI_13	1.8923	1.91885	1.38%

4.3.2 Results on K_{II} :

For this test, it be wished that K_{II} maybe such as $K_{II}=K_{IIref}\pm 5.10^{-2}$. (absolute test)

Identification	*Code_Aster*	Reference	Difference
CALC_G			
KII_1	0.04277	$4.27722\ 10^{-2}$	$3.92\ 10^{-8}$
KII_2	-0.00019	$1.21013\ 10^{-4}$	$3.1\ 10^{-4}$
KII_3	0.00864	$7.10255\ 10^{-3}$	$1.54\ 10^{-3}$
KII_4	0.00068	$1.94683\ 10^{-3}$	0.0013
KII_5	0.001988	$1.20266\ 10^{-3}$	$7.85\ 10^{-4}$
KII_6	-0.0003965	$8.82542\ 10^{-4}$	$1.27\ 10^{-3}$
KII_7	-0.001049	$-1.23199\ 10^{-3}$	$1.82\ 10^{-4}$
KII_8	-0.01141	$-3.54655\ 10^{-3}$	$7.86\ 10^{-3}$
KII_9	0.006572	$-4.54122\ 10^{-3}$	0.0111
KII_10	0.001981	$-8.18030\ 10^{-3}$	0.01
KII_11	-0.038036	$-1.55772\ 10^{-2}$	0.0224
KII_12	-0.012783	$-2.31849\ 10^{-2}$	0.0104
KII_13	-0.02314	$-3.52229\ 10^{-2}$	0,012

5 Modeling C

5.1 Characteristics of the grid

In this modeling, the method upwind is tested for the propagation of crack. The level-sets are determined by resolution of the equations of reactualization per diagram to the finished differences.

5.2 Characteristics of the grid

One uses here the same grid as in modeling A (§2.1).

5.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

5.3.1 Results on K_I :

One carries out a relative test of nonregression on K_I compared to $K_{I\,maillage}$ with a precision of 3% .

Identification	*Code_Aster*	Reference	Difference ($\%$)
CALC_G			
KI_1	0.243960	$2.43961\ 10^{-1}$	2.03E-04%
KI_2	0.290552	$2.90147\ 10^{-1}$	0.14%
KI_3	0.330871	$3.30840\ 10^{-1}$	9.645E-03%
KI_4	0.375936	$3.75984\ 10^{-1}$	0.0126%
KI_5	0.433524	$4.33606\ 10^{-1}$	0.02%
KI_6	0.496884	$4.96975\ 10^{-1}$	0.02%
KI_7	0.572963	$5.73785\ 10^{-1}$	0.14%
KI_8	0.673340	$6.70222\ 10^{-1}$	0.46%
KI_9	0.788694	$7.89716\ 10^{-1}$	0.13%
KI_10	0.94306	$9.39463\ 10^{-1}$	0.39%
KI_11	1.151406	1.15201	0.05%
KI_12	1.44998	1.45163	0.11%
KI_13	1.92206	1.91885	0.17%

5.3.2 Results on K_{II} :

For this test, it be wished that K_{II} maybe such as $K_{II}=K_{IIref}\pm 3.10^{-2}$. (absolute test)

Identification	*Code_Aster*	Reference	Difference
CALC_G			
KII_1	0.0427721	$4.27722\ 10^{-2}$	3.92E-08
KII_2	-0.0001827	$1.21013\ 10^{-4}$	3.04 E-04
KII_3	0.0086350	$7.10255\ 10^{-3}$	1.53E-03
KII_4	0.0006797	$1.94683\ 10^{-3}$	1.27E-03
KII_5	0.0019676	$1.20266\ 10^{-3}$	7.64E-04
KII_6	-0.0001195	$8.82542\ 10^{-4}$	1.00E-03
KII_7	-0.0009619	$-1.23199\ 10^{-3}$	2.70E-04
KII_8	-0.0002964	$-3.54655\ 10^{-3}$	3.84E-03
KII_9	-0.0027228	$-4.54122\ 10^{-3}$	1.82E-03
KII_10	-0.0258299	$-8.18030\ 10^{-3}$	0,017
KII_11	-0.0041077	$-1.55772\ 10^{-2}$	0.01146
KII_12	-0.0283523	$-2.31849\ 10^{-2}$	5.17E-03
KII_13	-0.0304681	$-3.52229\ 10^{-2}$	4.75E-03

6 Modeling D

6.1 Characteristics of modeling

In this modeling, the geometrical method is tested for the propagation of crack. The level-sets are recomputed with each step of propagation.

6.2 Characteristics of the grid

One uses here the same grid as in modeling A .

6.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} given by CALC_G.

6.3.1 Results on K_I :

One carries out a relative test of nonregression on K_I compared to $K_{I\,maillage}$ with a precision of 3% .

Identification	*Code_Aster*	Reference	Difference ($\%$)
CALC_G			
KI_1	2.4396 10^{-1}	2.43961 10^{-1}	-2.03 10^{-4} %
KI_2	2.9026 10^{-1}	2.90147 10^{-1}	0.04%
KI_3	3.3057 10^{-1}	3.30840 10^{-1}	0.08%
KI_4	3.7665 10^{-1}	3.75984 10^{-1}	0.18%
KI_5	4.3352 10^{-1}	4.33606 10^{-1}	0.01%
KI_6	4.966710^{-1}	4.96975 10^{-1}	0.06%
KI_7	5.7348 10^{-1}	5.73785 10^{-1}	0.05%
KI_8	6.7175 10^{-1}	6.70222 10^{-1}	0.23%
KI_9	7.8989 10^{-1}	7.89716 10^{-1}	0.02%
KI_10	9.3925 10^{-1}	9.39463 10^{-1}	0.02%
KI_11	1.15158	1.15201	0.04%
KI_12	1.45290	1.45163	0.09%
KI_13	1.92063	1.91885	0.09%

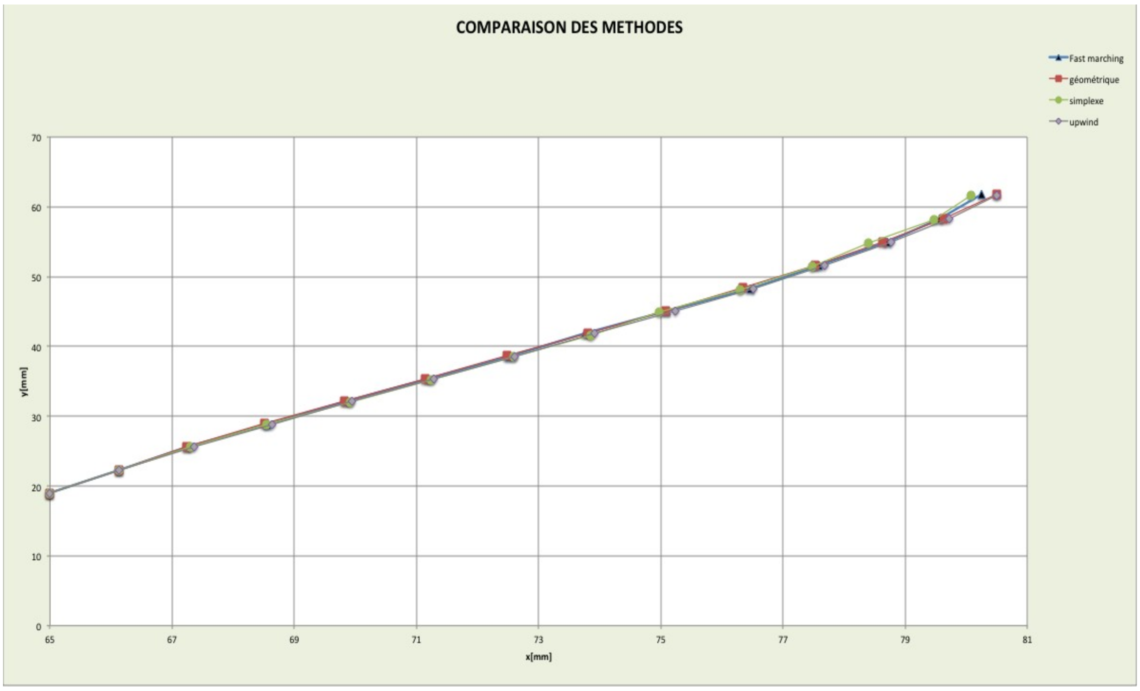
6.3.2 Results on K_{II} :

For this test, it be wished that K_{II} maybe such as $K_{II}=K_{IIref}\pm 3.10^{-2}$ (test in absolute).

Identification	*Code_Aster*	Reference	Difference %
CALC_G			
KII_1	4.27721 10^{-2}	4.27722 10^{-2}	3.92 10^{-8}
KII_2	5.49728 10^{-5}	1.21013 10^{-4}	6.6 10^{-5}
KII_3	8.31292 10^{-3}	7.10255 10^{-3}	1.21 10^{-3}
KII_4	1.41042 10^{-3}	1.94683 10^{-3}	5.36 10^{-4}
KII_5	1.93652 10^{-4}	1.20266 10^{-3}	7.34 10^{-4}
KII_6	7.04563 10^{-4}	8.82542 10^{-4}	1.78 10^{-4}
KII_7	0.0	-1.23199 10^{-3}	1.23 10^{-3}
KII_8	0.0	-3.54655 10^{-3}	3.55 10^{-3}
KII_9	0.0	-4.54122 10^{-3}	4.54 10^{-3}
KII_10	0.0	-8.18030 10^{-3}	8.18 10^{-3}
KII_11	0.0	-1.55772 10^{-2}	1.56
KII_12	0.0	-2.31849 10^{-2}	2.32
KII_13	0.0	-3.52229 10^{-2}	3.52

7 Summaries of the results

One can compare the ways which the four methods give (geometrical simplex, upwind, upwind&FMM and):



Four methods all the same course of propagation gives, which is very close with the experimental data (Figure 3.4-a).

One can compare the computing time for the same number of steps of propagation (13) of the four methods. For the methods simplex and upwind, one checked the performance while using the restriction of the field of calculation. It is noticed that the restriction of the field makes it possible to strongly reduce the computing time of these two methods and to return the performance of the methods grid, comparable simplex and upwind.

Grid	Method	Time (S)
quadrangles	Grid	19.0
	Simplex	13.0
	Upwind	16
	Geometrical	15.3

The results make it possible to validate on a simple case the calculation of the stress intensity factors in mode I for the elements X-FEM for the various methods.