

## SSLS10 – Torus under pressure interns uniform

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### Summary:

The objective of this test is to validate the calculation of displacements, and of the constraints in subjected torus a pressure has interns uniform.

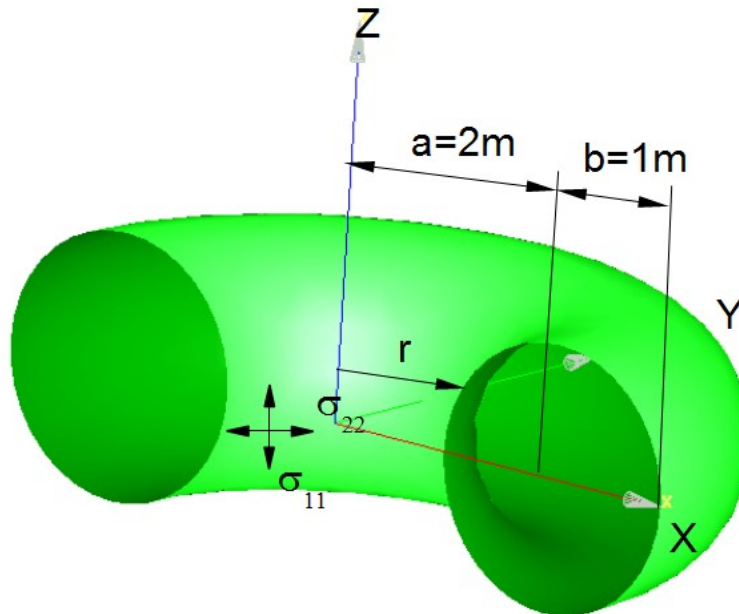
### Modelings :

- Modeling *A* : DKT with meshes QUAD4/TRIA3
- Modeling *B* : DST with meshes QUAD4/TRIA3
- Modeling *C* : Q4G with meshes QUAD4/TRIA3
- Modeling *D* : COQUE\_3D with meshes QUAD9/TRIA7

## 1 Problem of reference

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### 1.1 Geometry



Thickness:  $h=0,02\text{ m}$

### 1.2 Properties of material

The material is elastic isotropic:

- $E=2,1 \times 10^{11}\text{ Pa}$
- $\nu=0.3$

### 1.3 Boundary conditions and loadings

Free conditions

Loading:

- Internal pressure:  $p=10^4\text{ Pa}$

### 1.4 Initial conditions

Nothing

## 2 Reference solution

### 2.1 Method of calculating

The reference solution is a digital solution [1].

If  $a - b \leq r < a + b$

- Displacement:  $\delta_r = \frac{pb}{2Eh}(r - \nu(r + a))$
- Constraints:  $\sigma_{11} = \frac{pb}{2h} \times \frac{r + a}{r}$        $\sigma_{22} = \frac{pb}{2h}$

These formulas are applicable only to the thin tori, such as  $\frac{b}{h} > 10$  and of radius of curvature such as

$r \times \pi < 100 \sqrt{\frac{I_x}{A}}$  with  $I_x$  = moment of inertia and  $A$  = surface of the geometrical section of the torus

### 2.2 Sizes and results of reference

- Displacements

Not	$DX (m)$
$r = a - b$	$\delta_r = 1.19 \times 10^{-7}$
$r = a + b$	$\delta_r = 1.79 \times 10^{-6}$

- Constraints

Not	Constraints (Pa)
$\forall r$	$\sigma_{22} = 2,5 \times 10^5$
$r = a - b$	$\sigma_{11} = 7.5 \times 10^5$
$r = a + b$	$\sigma_{11} = 4.17 \times 10^5$

### 2.3 Uncertainties on the solution

Analytical solution

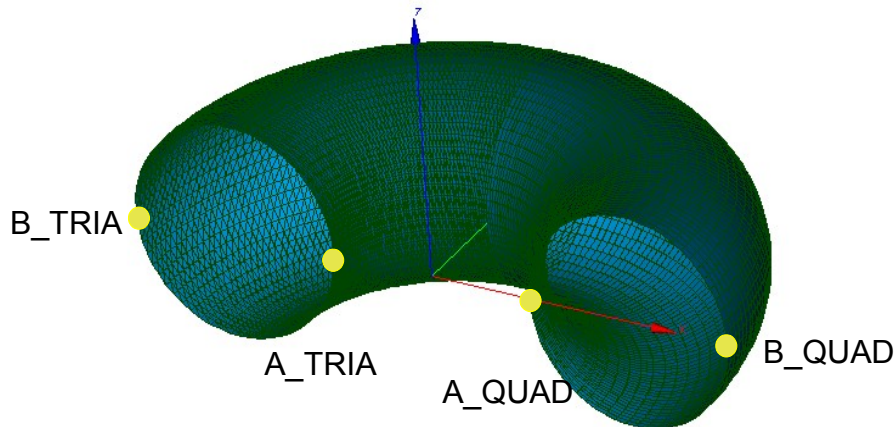
### 2.4 Bibliographical references

- [1] Guide VPCS - Edition 1990.

## 3 Modeling A

### 3.1 Characteristics of modeling

A modeling is used DKT .



### 3.2 Characteristics of the grid

The grid contains 7260 nodes and 10800 meshes of which:

- 7200 meshes of the type TRIA3,
- 3600 meshes of the type QUAD4.

### 3.3 Sizes tested and results

- Displacements

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>DX</i>	'ANALYTICAL'	$\delta_r = 1.19 \times 10^{-7} m$	3.0
<i>A_TRIA</i>		'ANALYTICAL'	$\delta_r = 1.19 \times 10^{-7} m$	2.0
<i>B_QUAD</i>		'ANALYTICAL'	$\delta_r = 1.79 \times 10^{-6} m$	1.5
<i>B_TRIA</i>		'ANALYTICAL'	$\delta_r = 1.79 \times 10^{-6} m$	1.5

- Constraints

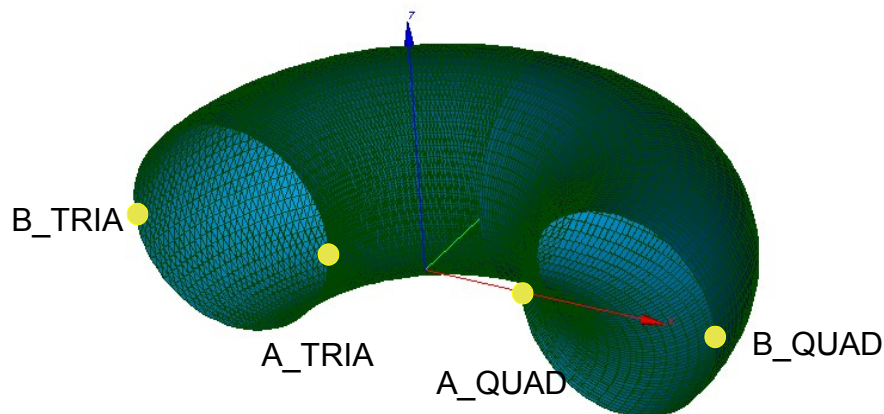
Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>SIXX</i>	'ANALYTICAL'	$7.5 \times 10^5 Pa$	5.0
<i>A_TRIA</i>		'ANALYTICAL'	$7.5 \times 10^5 Pa$	5.0
<i>B_QUAD</i>		'ANALYTICAL'	$4.17 \times 10^5 Pa$	3.0
<i>B_TRIA</i>		'ANALYTICAL'	$4.17 \times 10^5 Pa$	3.0

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>SIYY</i>	'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	12.0
<i>A_TRIA</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	12.0
<i>B_QUAD</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	4.0
<i>B_TRIA</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	4.0

## 4 Modeling B

### 4.1 Characteristics of modeling

A modeling is used `DST`.



### 4.2 Characteristics of the grid

The grid contains 7260 nodes and 10800 meshes of which:

- 7200 meshes of the type `TRIA3`,
- 3600 meshes of the type `QUAD4`.

### 4.3 Sizes tested and results

- Displacements

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>DX</i>	'ANALYTICAL'	$\delta_r = 1.19 \times 10^{-7} m$	3.0
<i>A_TRIA</i>		'ANALYTICAL'	$\delta_r = 1.19 \times 10^{-7} m$	2.0
<i>B_QUAD</i>		'ANALYTICAL'	$\delta_r = 1.79 \times 10^{-6} m$	1.5
<i>B_TRIA</i>		'ANALYTICAL'	$\delta_r = 1.79 \times 10^{-6} m$	1.5

- Constraints

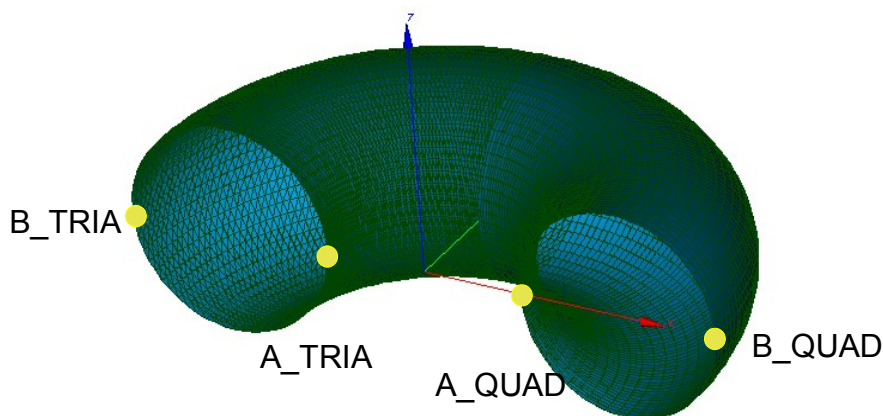
Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>SIXX</i>	'ANALYTICAL'	$7.5 \times 10^5 Pa$	5.0
<i>A_TRIA</i>		'ANALYTICAL'	$7.5 \times 10^5 Pa$	5.0
<i>B_QUAD</i>		'ANALYTICAL'	$4.17 \times 10^5 Pa$	3.0
<i>B_TRIA</i>		'ANALYTICAL'	$4.17 \times 10^5 Pa$	3.0

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>SIYY</i>	'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	12.0
<i>A_TRIA</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	12.0
<i>B_QUAD</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	4.0
<i>B_TRIA</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	4.0

## 5 Modeling C

### 5.1 Characteristics of modeling

A modeling is used Q4G.



### 5.2 Characteristics of the grid

The grid contains 7260 nodes and 10800 meshes of which:

- 7200 meshes of the type TRIA3,
- 3600 meshes of the type QUAD4.

### 5.3 Sizes tested and results

- Displacements

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>DX</i>	'ANALYTICAL'	$\delta_r = 1.19 \times 10^{-7} m$	3.0
<i>A_TRIA</i>		'ANALYTICAL'	$\delta_r = 1.19 \times 10^{-7} m$	2.0
<i>B_QUAD</i>		'ANALYTICAL'	$\delta_r = 1.79 \times 10^{-6} m$	1.5
<i>B_TRIA</i>		'ANALYTICAL'	$\delta_r = 1.79 \times 10^{-6} m$	1.5

- Constraints

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>SIXX</i>	'ANALYTICAL'	$7.5 \times 10^5 Pa$	5.0
<i>A_TRIA</i>		'ANALYTICAL'	$7.5 \times 10^5 Pa$	5.0
<i>B_QUAD</i>		'ANALYTICAL'	$4.17 \times 10^5 Pa$	3.0
<i>B_TRIA</i>		'ANALYTICAL'	$4.17 \times 10^5 Pa$	3.0

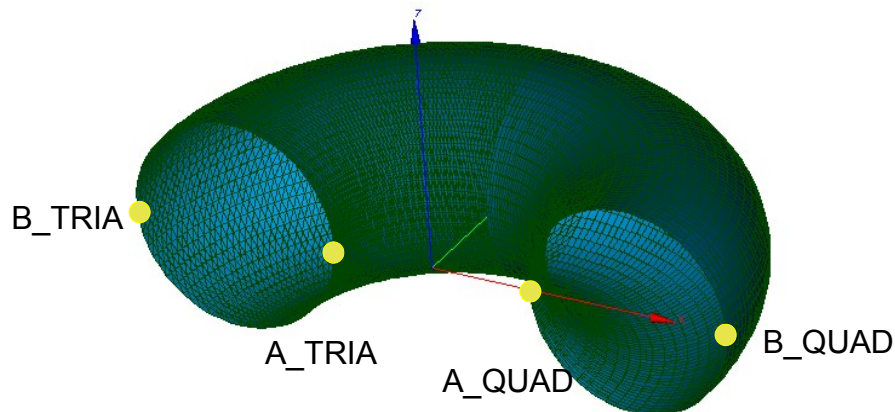


Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>SIYY</i>	'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	12.0
<i>A_TRIA</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	12.0
<i>B_QUAD</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	4.0
<i>B_TRIA</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	4.0

## 6 Modeling D

### 6.1 Characteristics of modeling

A modeling is used COQUE\_3D.



### 6.2 Characteristics of the grid

The grid contains 7260 nodes and 36120 meshes of which:

- 7200 meshes of the type TRIA7,
- 3600 meshes of the type QUAD9.

### 6.3 Sizes tested and results

- Displacements

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>DX</i>	'ANALYTICAL'	$\delta_r = 1.19 \times 10^{-7} m$	3.0
<i>A_TRIA</i>		'ANALYTICAL'	$\delta_r = 1.19 \times 10^{-7} m$	2.0
<i>B_QUAD</i>		'ANALYTICAL'	$\delta_r = 1.79 \times 10^{-6} m$	1.5
<i>B_TRIA</i>		'ANALYTICAL'	$\delta_r = 1.79 \times 10^{-6} m$	1.5

- Constraints

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>SIXX</i>	'ANALYTICAL'	$7.5 \times 10^5 Pa$	0.1
<i>A_TRIA</i>		'ANALYTICAL'	$7.5 \times 10^5 Pa$	16.0
<i>B_QUAD</i>		'ANALYTICAL'	$4.17 \times 10^5 Pa$	0.1
<i>B_TRIA</i>		'ANALYTICAL'	$4.17 \times 10^5 Pa$	3.5

Identification		Type of reference	Value of reference	Tolerance (%)
Not	Size			
<i>A_QUAD</i>	<i>SIYY</i>	'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	0.5
<i>A_TRIA</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	46.0
<i>B_QUAD</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	0.5
<i>B_TRIA</i>		'ANALYTICAL'	$\sigma_{11} = 2.5 \times 10^5 Pa$	5.0

## 7 Summary of the results

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**Displacements** : some is the type of mesh used (TRIA3, QUAD4) , results got for 4 modelings (DKT, DST, Q4G and COQUE\_3D) are satisfactory. Compared to the analytical solution, one observes a maximum change of 5%.

**Constraints** :

- Modelings DKT, DST and Q4G : some is the type of mesh used (TRIA3, QUAD4), the got results are correct. Compared to the analytical solution, one observes a maximum change of 12%. These modelings use elements with facet, by refining the grid one should get better results.
- Modeling COQUE\_3D : the mesh of the type QUAD9 give very good performances, the variation observed is of 0.1%. On the other hand the variation is important (46%) for the mesh of the type TRIA7.