# SSLS120 - Cylindrical thin hull under hydrostatic pressure

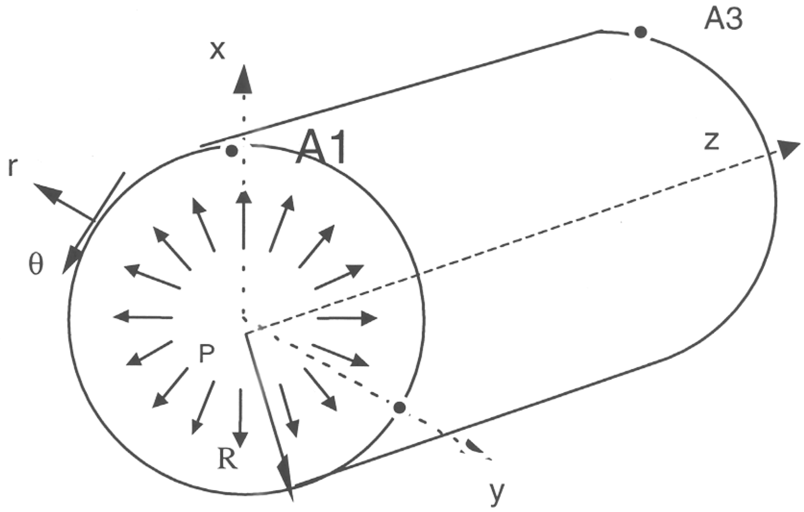
**Summary:**

This test represents a static calculation of thin cylindrical tank filled with water. It makes it possible to validate the good taking into account of the pressures function of the geometry, as well as orthotropic elastic materials. 5 modelings finite elements are used: `AXIS`, `COQUE_3D` with meshs `QUAD9`, `COQUE_3D` with meshs `TRIA7`, and `DKT` with meshs `QUAD4` and `3D` with meshs `HEXA20`. Displacements and the constraints obtained are compared with an analytical reference solution.

# 1 Problem of reference

## 1.1 Geometry

Cylindrical tank of average radius $R=5.7\,m$, thickness $e=0.04\,m$ and height $L=16\,m$, simply supported at its base (rotation is free), and subjected to an internal hydrostatic pressure.




Average radius: $R=5.7\,m$
Thickness: $e=0.04\,m$
Height: $L=16\,m$

## 1.2 Material properties

The properties of materials constituting the plate are:

Material 1 : isotropic rubber band: Young modulus $E=2.1\,10^{11}\,Pa$

Poisson's ratio $\nu=0.3$

Material 2 : orthotropic rubber band: In order to free itself from the dependence of the notations to the reference mark of orthotropism, one gives characteristic materials in the cylindrical reference mark

$E_r=2.1\,E\,11\,Pa$
$E_\theta=2.1\,E\,11\,Pa$
$E_z=4\,E\,11\,Pa$
$\nu_{r\theta}=0.075$
$\nu_{rz}=0.075$
$\nu_{\theta z}=0.075$
$G_{r\theta}=0.35\,E\,10\,Pa$
$G_{rz}=0.45\,E\,10\,Pa$
$G_{\theta z}=0.45\,E\,10\,Pa$

## 1.3 Boundary conditions and loadings

Base $z=0$ simply supported,

Internal pressure varying linearly according to $z$ : $p(z)=P0.(L-z)/L$
with $P0=15000\,Pa$ .

## 1.4 Initial conditions

Without object.

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

- Isotropic material : Analytical solution [bib1], obtained with the mean assumption of hull:

$$\sigma_{zz}=0$$

$$\sigma_{\theta\theta}=P0\,R\frac{(L-z)}{L\,e}$$

$$u_r=\frac{P0\,R^2}{E\,e}\left[1-\frac{z}{L}\right]$$

$$u_z=\frac{-P0\,R\,\nu\,z}{E\,e}\left[1-\frac{z}{2\mathrm{L}}\right]$$

Radial displacement at the base of the cylinder: $u_r(z=0)=\frac{P0\,R^2}{E\,e}$

Vertical displacement at the top of cylinder: $u_z(z=L)=\frac{-P0\,R\,L\,\nu}{2\,E\,e}$

Circumferential constraint in bottom of the cylinder $\sigma_{\theta\theta}(z=0)=\frac{P0\,R}{e}$

- Orthotropic material : The solution can be deduced from the preceding one: the constraints being statically determined, it is enough to amend the law of behavior, and to integrate the deformations. So that the solution is independent of the various notations (the value $E_T$ the same significance does not cover according to the reference mark of orthotropism), one is placed in cylindrical reference mark $(r,\theta,z)$ .

Radial displacement at the base of the cylinder: $u_r(z=0)=\frac{P0\,R^2}{E_r e}$

Vertical displacement at the top of cylinder: $u_z(z=L)=\frac{-P0\,R\,L\,\nu_{\theta z}}{2\,E_\theta e}$

Circumferential constraint in bottom of the cylinder $\sigma_{\theta\theta}(z=0)=\frac{P0\,R}{e}$

## 2.2 Results of reference

Isotropic material :

| | |
|---|---|
| Radial displacement at the base of the cylinder: | $Ur(A1)=5.8017857\mathrm{E}-05\,m$ |
| Vertical displacement at the top of cylinder: | $Uz(A3)=-2.442857\mathrm{E}-05\,m$ |
| Circumferential constraint in bottom of the cylinder: | $Stt(A1)=2.1375\mathrm{E}+06\,Pa$ |

Orthotropic material :

| | |
|---|---|
| Radial displacement at the base of the cylinder: | $Ur(A1)=5.8017857\mathrm{E}-05\,m$ |
| Vertical displacement at the top of cylinder: | $Uz(A3)=-6.107143\mathrm{E}-06\,m$ |
| Circumferential constraint in bottom of the cylinder: | $Stt(A1)=2.1375\mathrm{E}+06\,Pa$ |

## 2.3 Uncertainty on the solution

Analytical solution.

## 2.4 Bibliographical references

1) PILKEY W.D.: “Formulated for stress, Strain and Structural Matrices”. Wiley & Idiots, New York, 1994.

# 3 Modeling A

## 3.1 Characteristics of modeling

Modeling AXIS. One nets only one generator of the cylinder. 2 meshs QUAD8 in the thickness and 400 on the height.

## 3.2 Characteristics of the grid

Many nodes: 3206

Many meshs and types: 800 QUAD8

## 3.3 Values tested

Isotropic material

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DX(PM)$ | 5.8018E – 05 |
| $Uz(z=L)$ | $DY(A3)$ | – 2.4429E – 05 |
| $Uz(z=L)$ | $DY(A4)$ | – 2.4429E – 05 |
| $SigmaTT(z=0)$ | $SIZZ(PM)$ | 2.1375E+06 |

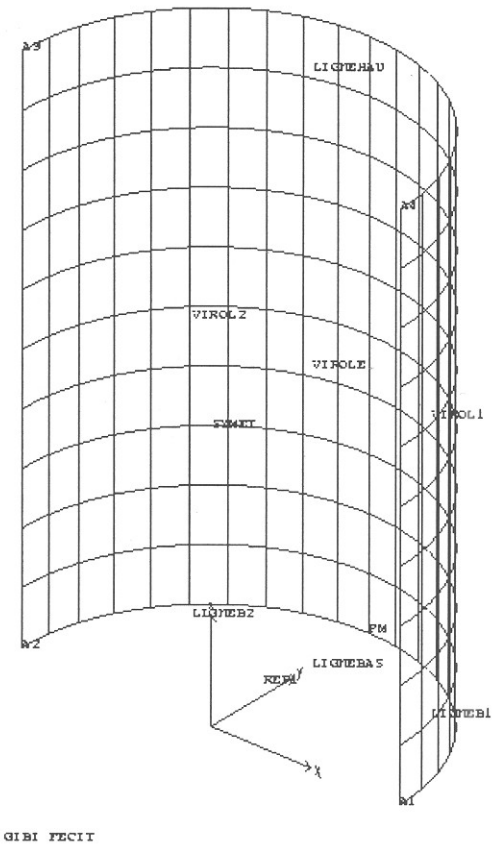
Orthotropic material

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E – 05 |
| $Uz(z=L)$ | $DY(A3)$ | – 6.10714E – 06 |
| $Uz(z=L)$ | $DY(A4)$ | – 2.4429E – 05 |
| $SigmaTT(z=0)$ | $SIZZ(PM)$ | 2.1375E+06 |

# 4 Modeling B

## 4.1 Characteristics of modeling

Modeling `COQUE_3D`. One nets only half of the cylinder (symmetry compared to the plan $y=0$ ) 10 meshs QUAD9 in the height and 20 on the semicircumference.




The normal on the hull is directed towards the interior of the cylinder.

## 4.2 Characteristics of the grid

Many nodes: 664

Number of meshs and type: 200 QUAD9

## 4.3 Values tested

1. Isotropic material

The reference mark USER on the hull is defined by the nautical angles (α=-90°, β=20°).
The component All tensor of the constraints is obtained by carrying out a change of reference mark (reference mark USER towards the CYLINDRICAL reference mark).

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DX(PM)$ | 5.8018E – 05 |
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E – 05 |
| $Ur(z=0)$ | $DX(A2)$ | – 5.8018E – 05 |

| | | |
|---|---|---|
| $Uz(z=L)$ | $DZ(A3)$ | – 2.4429E-05 |
| $Uz(z=L)$ | $DZ(A4)$ | – 2.4429E-05 |
| $SigmaTT(z=0)$ | $SIZZ(PM)$ | 2.1375E+06 |

2. Orthotropic material

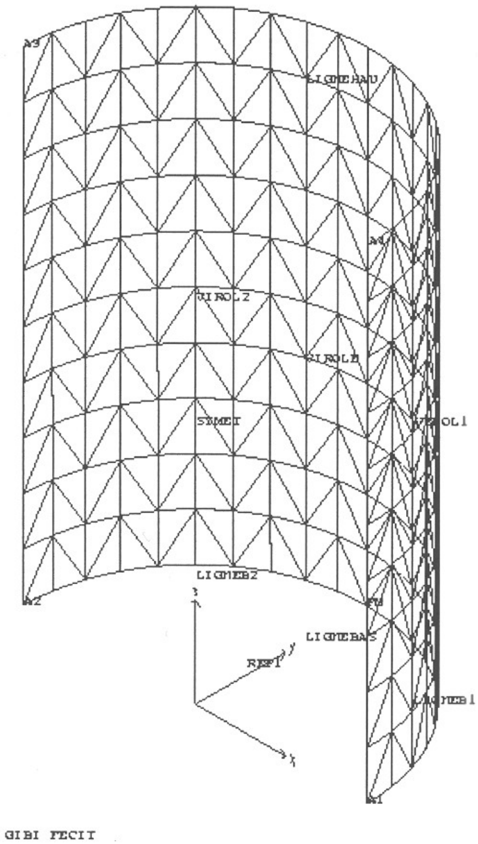
The reference mark USER on the hull is defined by the nautical angles (α=0°, β=-90°). The second vector of the reference mark USER is roughly equal to the tangential vector of the cylindrical reference mark associated with the hull. The component All tensor of the constraints is approached by SIYY.

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DX(PM)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A2)$ | – 5.8018E-05 |
| $Uz(z=L)$ | $DZ(A3)$ | – 6.10714E-06 |
| $Uz(z=L)$ | $DZ(A4)$ | – 6.10714E-06 |
| $SigmaTT(z=0)$ | $SIYY(PM)$ | 2.1375E+06 |

# 5 Modeling C

## 5.1 Characteristics of modeling

Modeling `COQUE_3D`. One nets only half of the cylinder (symmetry compared to the plan $y=0$ ) 10 meshs TRIA7 in the height and 20 on the semicircumference.




## 5.2 Characteristics of the grid

Many nodes: 864

Many meshs and types: 400 TRIA7

## 5.3 Values tested

Isotropic material

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DY(PM)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A2)$ | – 5.8018E-05 |
| $Uz(z=L)$ | $DZ(A3)$ | – 2.4429E-05 |
| $Uz(z=L)$ | $DZ(A4)$ | – 2.4429E-05 |
| $SigmaTT(z=0)$ | $SIZZ(PM)$ | 2.1375E+06 |

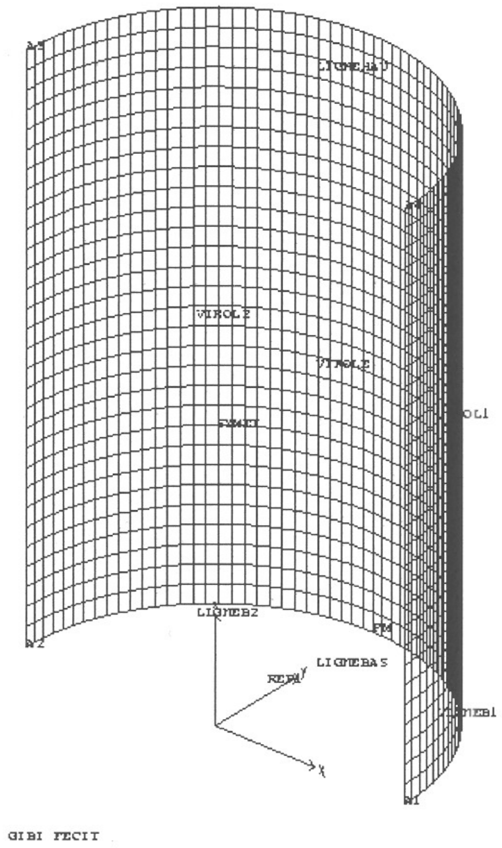
Orthotropic material

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DX(PM)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A2)$ | – 5.8018E-05 |
| $Uz(z=L)$ | $DZ(A3)$ | – 6.10714E-06 |
| $Uz(z=L)$ | $DZ(A4)$ | – 6.10714E-06 |
| $SigmaTT(z=0)$ | $SIZZ(PM)$ | 2.1375E+06 |

# 6 Modeling D

## 6.1 Characteristics of modeling

Modeling DKT. One nets only half of the cylinder (symmetry compared to the plan $y=0$ ) 30 meshs QUAD4 in the height and 60 on the semicircumference.




## 6.2 Characteristics of the grid

Many nodes: 1894

Many meshs and types: 1800 QUAD4

Note:
To obtain a precise solution of this problem, it is necessary to use refined an enough grid (here 1800 QUAD4).

One observes the following errors according to the discretization:

| Many elements | Maximum error on displacement |
|---|---|
| 450 QUAD4 | 0.5 % |
| 1800 QUAD4 | 0.2 % |
| 900 TRIA3 | 17 % |
| 3600 TRIA3 | 1.4 % |

It is seen that for this problem, the grid in quadrangles is preferable.

## 6.3 Values tested

Isotropic material

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DY(PM)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A2)$ | – 5.8018E-05 |
| $Uz(z=L)$ | $DZ(A3)$ | – 2.4429E-05 |
| $Uz(z=L)$ | $DZ(A4)$ | – 2.4429E-05 |
| $SigmaTT(z=0)$ | $SIZZ(PM)$ | 2.1375E+06 |

Orthotropic material

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DY(PM)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A2)$ | – 5.8018E-05 |
| $Uz(z=L)$ | $DZ(A3)$ | – 6.10714E-06 |
| $Uz(z=L)$ | $DZ(A4)$ | – 6.10714E-06 |
| $SigmaTT(z=0)$ | $SIZZ(PM)$ | 2.1375E+06 |

# 7 Modeling E

## 7.1 Characteristics of modeling

Modeling 3D. One nets only half of the cylinder (symmetry compared to the plan $y=0$ ) 10 meshs HEXA20 in the height, 40 on the semicircumference and 2 in the thickness.

## 7.2 Characteristics of the grid

Many nodes: 4725

Many meshs and types: 800 HEXA20

## 7.3 Values tested

Orthotropic material by MECA_STATIQUE

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DY(PM)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E-05 |
| $Uz(z=L)$ | $DZ(A3)$ | – 6.10714E-06 |
| $Uz(z=L)$ | $DZ(A4)$ | – 6.10714E-06 |

Orthotropic material by STAT_NON_LINE

| Value | Identification | Reference |
|---|---|---|
| $Ur(z=0)$ | $DY(PM)$ | 5.8018E-05 |
| $Ur(z=0)$ | $DX(A1)$ | 5.8018E-05 |
| $Uz(z=L)$ | $DZ(A3)$ | – 6.10714E-06 |
| $Uz(z=L)$ | $DZ(A4)$ | – 6.10714E-06 |

# 8 Summary of the results

The results of five modelings are very close to the analytical solution: to the maximum 0.4% variation for modelings COQUE_3D and DKT, and less than 2% of variation for axisymmetric modeling and 3D, which is explained by the fact why the analytical solution is a mean solution hull.

This test thus validates on the one hand the efforts of pressure varying linearly with the geometry, for thin hulls, and on the other hand the taking into account of orthotropic elasticity.