SSLS124 - Beam in inflection with various twinges

Summary:

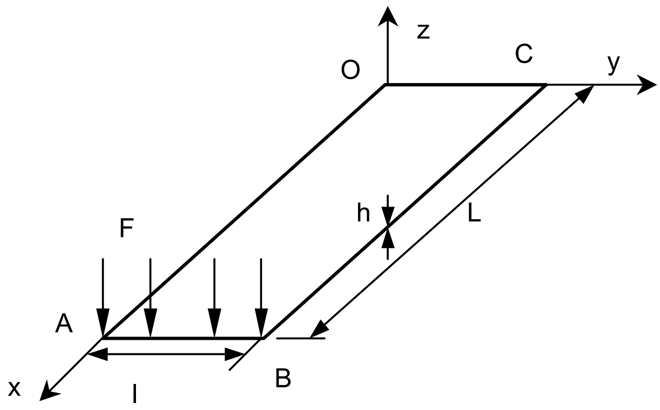
This test represents a calculation quasi-static of a beam in inflection, embedded at an end, and subjected to a vertical force at the other end. This test makes it possible to validate, for a linear elastic design, up to five values of twinge (variable thicknesses), in each following modeling:

- Finite elements SHB8 for a regular grid (modeling A)
- Finite elements SHB8 for a nonregular grid (modeling B)
- Finite elements SHB6 for a regular grid (modeling C)
- Finite elements SHB20 for a regular grid (modeling D)
- Finite elements SHB20 for a nonregular grid (modeling E)
- Finite elements SHB15 for a regular grid (modeling F)
- Finite elements SHB15 for a nonregular grid (modeling G)

Displacements obtained are compared with the elastic analytical solution of a beam in inflection. This test makes it possible to show the limits of the elements in term of twinge, on the one hand, and to show their good convergence for a very irregular grid, on the other hand.

1 Problem of reference

1.1 Geometry



Length $L=100\,m$, width $l=10\,m$.
Thickness: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$, **case 4** $h=0.05\,m$, **case 5** $h=0.02\,m$

1.2 Properties of materials

An elastic material is considered:
$E=2.10^{11}\,Pa$
$\nu=0.3$

1.3 Boundary conditions and loadings

Embedding on the side OC : $u=v=w=0$, $\theta_x=\theta_y=\theta_z=0$
At the end AB , a load uniformly distributed of resultant:
$F_z=1\,N$

2 Reference solution

2.1 Method of calculating used for the reference solution

The results of reference are got by the theory of the elastic beams.
Vertical displacement at the end AB is given by:

$$U_y = \frac{F L^3}{3 E I_z}$$

With:

$$I_z = \frac{l h^3}{12}$$

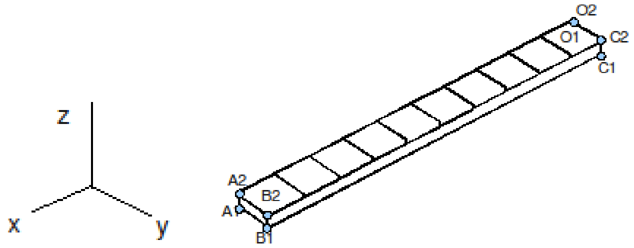
2.2 Results of reference

Displacement of the points A and B according to Z.

3 Modeling A

3.1 Characteristics of modeling

Element SHB8



Cutting: a regular grid is considered in this modeling.

Regular grid:
10 meshs SHB8 : 1 according to the width, 10 according to the length, 1 according to the thickness

Five values thickness are considered in this modeling: **case 1** $h=10m$, **case 2** $h=1m$, **case 3** $h=0.1m$, **case 4** $h=0.05m$, **case 5** $h=0.02m$.

Boundary conditions:
- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: following blocked displacement Z

Loading:
- in $A2$: nodal force according to X : $FX=0,5$
- in $B2$: nodal force according to Y : $FY=0,5$

Name of the nodes:

Not $O1$	$N40$	Not $O2$	$N44$
Not $A1$	$N03$	Not $A2$	$N01$
Not $B1$	$N04$	Not $B2$	$N02$
Not $C1$	$N43$	Not $C2$	$N39$

3.2 Characteristics of the grid

Many nodes: 44 Many meshs and types: 11 SHB8
In the case of the regular grid, each element is a perfect square on side length 10m

3.3 Sizes tested and results

Regular grid:
Analytical values of reference are tested.

Thickness	Not	Size in unit	Reference	% tolerances
Case 1	$A2$	displacement $W(m)$	2.0E-9	1
$h=10\,m$	$B2$	displacement $W(m)$	2.0E-9	1
	POINT 1 MESH 1	SIEF_ELGA	0.516522512185	0,1 %
	POINT 2 NETS 1	SIEF_ELGA	0.30692750676	0.1 %
	POINT 3 NETS 1	SIEF_ELGA	0.0	1E-9 %
	POINT 4 NETS 1	SIEF_ELGA	-0.30692750676	0.1 %
	POINT 5 NETS 1	SIEF_ELGA	-0.516522512185	0.1 %
Case 2	$A2$	displacement $W(m)$	2.0E-6	1
$h=1\,m$	$B2$	displacement $W(m)$	2.0E-6	

One tests analytical values of VMIS_SG starting from the analytical values of SIEF_ELGA for case 1 like not-regression of the calculation of the fields SIEQ_ELGA and SIEQ_ELNO.

Thickness	Not	Size in unit	Reference	% tolerances
Case 3	$A2$	displacement $W(m)$	0.00199500122746	1E-6 %
$h=0{,}1\,m$	$B2$	displacement $W(m)$	0.0019950008334	1E-6 %
Case 4	$A2$	displacement $W(m)$	0.0159975015609	1E-6 %
$h=0{,}05\,m$	$B2$	displacement $W(m)$	0.0159975015627	1E-6 %

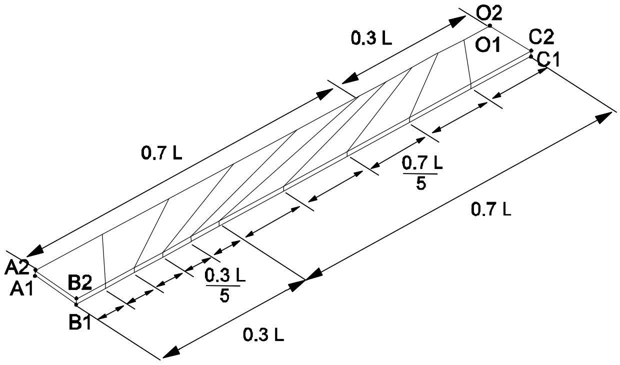
3.4 Remarks

The use of the operator STAT_NON_LINE for cases 3.4 and 5 allows to better approach the reference solution. Indeed, for the strong twinges, the matrix of rigidity becomes quasi-singular (it is necessary to increase the number of decimals lost with the resolution, using the keyword NPREC) and the precision of resolution of the linear system decreases. The non-linear solvor, by carrying out iterations, allows to converge towards the analytical solution.

4 Modeling B

4.1 Characteristics of modeling

Element SHB8



Cutting: an irregular grid is considered in this modeling.
Not-regular grid:
10 meshs SHB8 : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values thickness are considered in this modeling: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$

Boundary conditions:

- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: following blocked displacement Z

Loading:

- in $A2$: nodal force according to X : $FX=0{,}5$
- in $B2$: nodal force according to Y : $FY=0{,}5$

Names of the nodes:

Not $O1$	$N40$	Not $O2$	$N44$
Not $A1$	$N03$	Not $A2$	$N01$
Not $B1$	$N04$	Not $B2$	$N02$
Not $C1$	$N43$	Not $C2$	$N39$

4.2 Characteristics of the grid

Many nodes: 44 Many meshs and types: 11 SHB8

4.3 Sizes tested and results

Not-regular grid:

Thickness	Not	Size in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.0E-09	4
$h=10\text{m}$	$B2$	displacement $W(m)$	2.0E-09	4
Case 2	$A2$	displacement $W(m)$	2.0E-06	5
$h=1\text{m}$	$B2$	displacement $W(m)$	2.0E-06	5

One also tests the not-regression of the calculation of the fields SIEQ_ELGA and SIEQ_ELNO.

4.4 Remarks

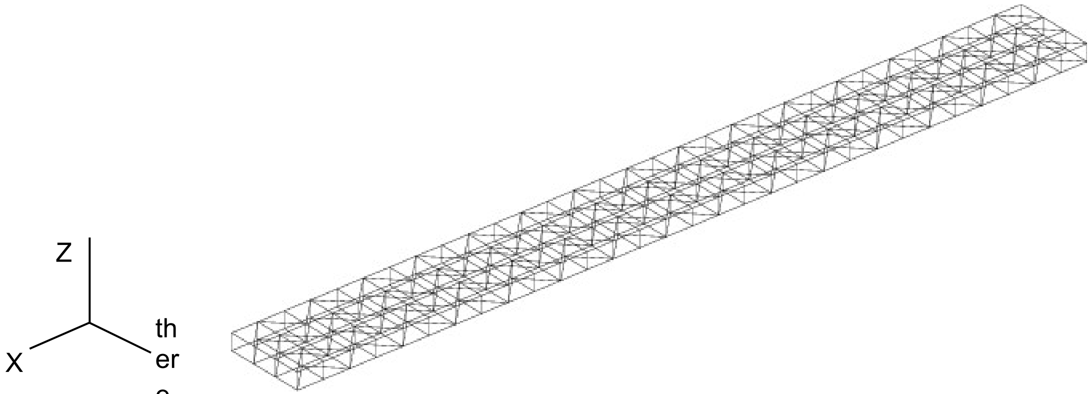
One can pass the same remark as for modeling a: when the twinge increases until becoming very important, the matrix of rigidity becomes very badly conditioned and quasi-singular. It is all the more marked in this modeling that the grid is, voluntarily, of poor quality (irregular with distorted elements). The conditioning evaluated with solvor MUMPS in case 3 exceeds 10^{12} and the solution of the linear system obtained, even after iterative refinement, has a variability between platforms of calculation. For this reason, one does not calculate a higher twinge in this modeling.

In spite of poor quality of the grid and with twinges all the same important, one notes the good behavior of the element with a good approximation of the solution in inflection.

5 Modeling C

5.1 Characteristics of modeling

Element `SHB6`



In this modeling one adapted the surface grid to the thickness considered:
(the grid above corresponds to case 2)

- nbl = many elements according to the width,
- nbL = many elements according to the length,
- 1 element according to the thickness

	h	nbL	nbl
case 1	10	10	1
case 2	5	30	3
case 3	2	100	10
case 4	1	100	10

Boundary conditions:

- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: following blocked displacement Z

Loading:

- in $A2$: nodal force according to X : $FX=0,5$
- in $B2$: nodal force according to Y : $FY=0,5$

5.2 Characteristics of the grid

case 1 Many nodes: 44 Many meshs and types: 20 `SHB6`
case 2 Many nodes: 248 Many meshs and types: 180 `SHB6`
case 3 and 4 Many nodes: 2222 Many meshs and types: 500 `SHB6`

5.3 Sizes tested and results

Regular grid:

Thickness	Not	Size in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2,00E-009	1
$h=10\text{m}$	$B2$	displacement $W(m)$	2,00E-009	1.5
Case 2	$A2$	displacement $W(m)$	1,60E-006	1
$h=5\text{m}$	$B2$	displacement $W(m)$	1,60E-006	1
Case 3	$A2$	displacement $W(m)$	2,50E-007	1
$h=2\text{m}$	$B2$	displacement $W(m)$	2,50E-007	1
Case 4	$A2$	displacement $W(m)$	2,00E-006	4
$h=1\text{m}$	$B2$	displacement $W(m)$	2,00E-006	4

One also tests the not-regression of the calculation of the fields SIEQ_ELGA and SIEQ_ELNO.

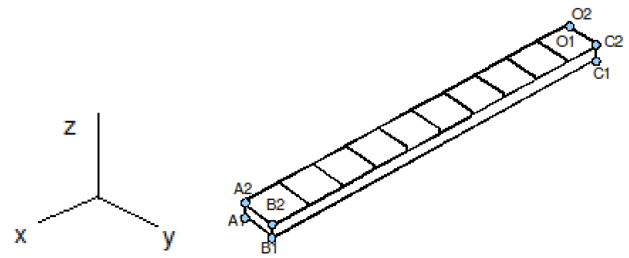
5.4 Remarks

The twinges are much weaker than for other modelings. The element SHB6 tolerate twinges less large indeed than other modelings SHB and is more sensitive to locking in shearing.

6 Modeling D

6.1 Characteristics of modeling

Element SHB20



Cutting: a regular grid is considered in this modeling.

Regular grid:
10 meshs SHB20 : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values thickness are considered in this modeling: **case 1** $h=10m$, **case 2** $h=1m$, **case 3** $h=0.1m$

Boundary conditions:
- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: following blocked displacement Z

Loading:
- in $A2$: nodal force according to X : $FX=0,5$
- in $B2$: nodal force according to Y : $FY=0,5$

6.2 Characteristics of the grid

Many nodes: 128 Many meshs and types: 10 SHB20
In the case of the regular grid, each element is a perfect square on side length 10m

6.3 Sizes tested and results

Regular grid:

Thickness	Not	Size in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.0E-9	1
$h=10\text{m}$	$B2$	displacement $W(m)$	2.0E-9	1
Case 2	$A2$	displacement $W(m)$	2.0E-6	1
$h=1\text{m}$	$B2$	displacement $W(m)$	2.0E-6	1

One also tests the not-regression of the calculation of the fields SIEQ_ELGA and SIEQ_ELNO.

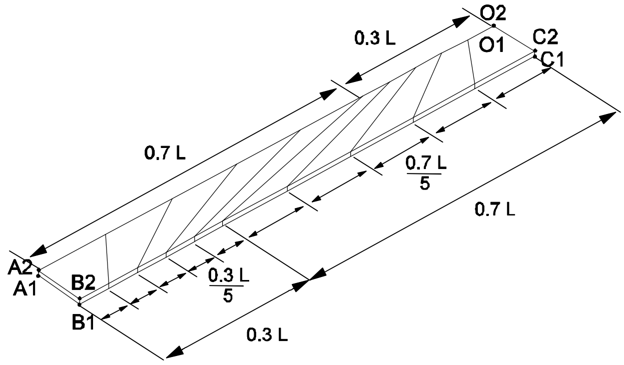
6.4 Remarks

The same remarks as for modelings A and B apply. Here in case 3, conditioning reaches already 10^{13} .

7 Modeling E

7.1 Characteristics of modeling

Element SHB20



Cutting: an irregular grid is considered in this modeling.

Not-regular grid:

10 meshs SHB20 : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values thickness are considered in this modeling: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$

Boundary conditions:

- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: following blocked displacement Z

Loading:

- in $A2$: nodal force according to X : $FX=0,5$
- in $B2$: nodal force according to Y : $FY=0,5$

7.2 Characteristics of the grid

Many nodes: 128

Many meshs and types: 10 SHB20

7.3 Sizes tested and results

Not-regular grid:

Thickness	Not	Size in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.0E-9	1
$h=10\mathrm{m}$	$B2$	displacement $W(m)$	2.0E-9	1
Case 2	$A2$	displacement $W(m)$	2.0E-6	7
$h=1\mathrm{m}$	$B2$	displacement $W(m)$	2.0E-6	7

One also tests the not-regression of the calculation of the fields SIEQ_ELGA and SIEQ_ELNO.

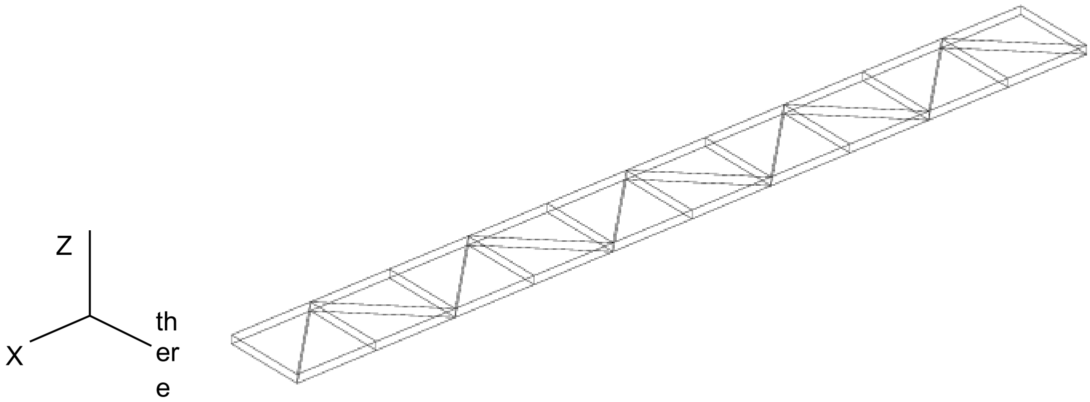
7.4 Remarks

As one can note it, the deterioration of the quality of the grid affects the quality of the result for the strong twinges but remains neutral with respect to the standard twinges.
In case 3, conditioning reaches already 10^{13} .

8 Modeling F

8.1 Characteristics of modeling

Element `SHB15`



Cutting: a regular grid is considered in this modeling.

Regular grid:
20 meshs `SHB15` : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values thickness are considered in this modeling: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$

Boundary conditions:

- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: following blocked displacement Z

Loading:

- in $A2$: nodal force according to X : $FX=0{,}5$
- in $B2$: nodal force according to Y : $FY=0{,}5$

8.2 Characteristics of the grid

Many nodes: 148

Many meshs and types: 20 `SHB15`

8.3 Sizes tested and results

Regular grid:

Thickness	Not	Size in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.0E-9	1
$h=10$m	$B2$	displacement $W(m)$	2.0E-9	1
Case 2	$A2$	displacement $W(m)$	2.0E-6	1.5
$h=1$m	$B2$	displacement $W(m)$	2.0E-6	1.5

One also tests the not-regression of the calculation of the fields `SIEQ_ELGA` and `SIEQ_ELNO`.

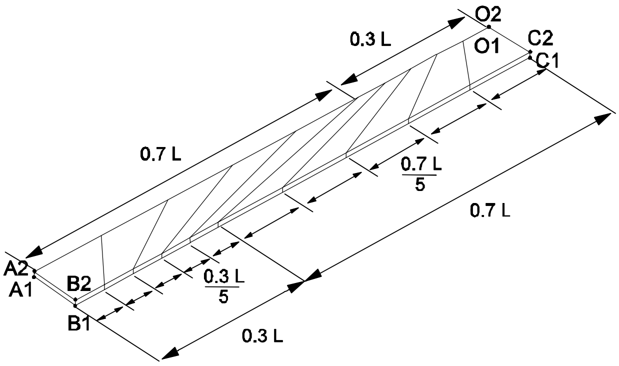
8.4 Remarks

The same remarks as for modelings A and B apply. Here in case 3, conditioning reaches already 10^{13} .

9 Modeling G

9.1 Characteristics of modeling

Element SHB15



Cutting: an irregular grid is considered in this modeling.

Not-regular grid:

20 meshs SHB15 : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values thickness are considered in this modeling: **case 1** $h=10m$, **case 2** $h=1m$, **case 3** $h=0.1m$

Boundary conditions:
In all the nodes on the side OC : following blocked displacement X

- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: following blocked displacement Z

Loading:

- in $A2$: nodal force according to X : $FX=0,5$
- in $B2$: nodal force according to Y : $FY=0,5$

9.2 Characteristics of the grid

Many nodes: 148

Many meshs and types: 20 SHB15

9.3 Sizes tested and results

Not-regular grid:

Thickness	Not	Size in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.0E-9	2.5
$h=10\text{m}$	$B2$	displacement $W(m)$	2.0E-9	2
Case 2	$A2$	displacement $W(m)$	2.0E-6	7
$h=1\text{m}$	$B2$	displacement $W(m)$	2.0E-6	7

One also tests the not-regression of the calculation of the fields SIEQ_ELGA and SIEQ_ELNO.

9.4 Remarks

As one can note it, the deterioration of the quality of the grid affects the quality of the result including for weak twinges.
In case 3, conditioning reaches already 10^{13} .

10 Summary of the results

In the case of regular grid:

- for `SHB8` (modeling A), good performances are obtained, even when the twinge of the element (side ratio/thickness) reached 500.
- for `SHB6` (modeling C), one obtains good performances on condition that refining the surface grid as the twinge increases. Nevertheless, for the twinges beyond 50, the results are degraded. The element `SHB6` present a locking (rather weak) in inflection.
- for the quadratic elements (modelings D and F), the results are very good until a twinge of 200.
- In the case of nonregular grid (modelings B , E and G), some is the twinge of the element, for `SHB8` one tends to underestimate the rigidity of the beam of approximately 4% , but the results remain good until a twinge of 500. The quadratic elements give good performances until a twinge of 200.

For the range of twinges usually met in modelings plates and hulls (from 10 to 100), elements SHB obtain good performances whatever the quality of the grid.
When one makes tighten the thickness by zero, of the phenomena of locking can return the matrix of singular rigidity and thus to prevent the resolution or slow down convergence into non-linear.
Beyond 1/1000, the plate becomes too fine and sufficient hurled. It can consequently appear a sensitivity of the results of 1.E-3%.