

SSLS126 – Inflection of a reinforced concrete flagstone (model GLRC_DAMAGE) supported on two with dimensions: elastic mode of beam

Summary:

This test represents the calculation of a reinforced concrete flagstone, in inflection, subjected to a pressure. It makes it possible to validate modeling `DKTG` with model `GLRC_DAMAGE` for the linear elastic behavior and modeling `Q4GG` with the model `ELAS`. The flagstone is in configuration beam: simple supports on two sides opposite of the flagstone.

Four modelings are carried out:

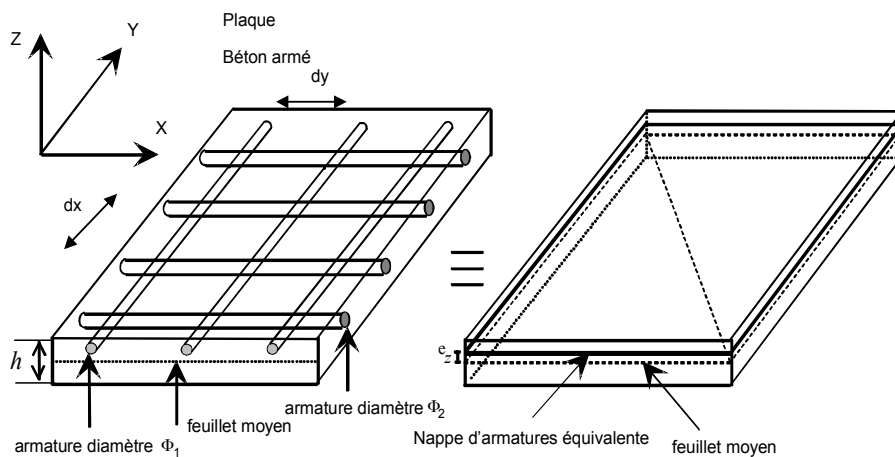
- Modeling A makes it possible to test the model `DKTG` with `TRIA3`,
- Modeling B makes it possible to test the model `DKTG` with `QUAD4`.
- Modeling C makes it possible to test the model `Q4GG` with `TRIA3`,
- Modeling D makes it possible to test the model `Q4GG` with `QUAD4`.

1 Problem of reference

1.1 Geometry

It is about a square flagstone, length $l=1.8\text{ m}$, thickness $h=0.12\text{ m}$, on bilateral simple supports. The reinforcement of inflection is parallel to the edges; it is identical on each of the two faces and in each of the two directions (dx , dy being spacings of irons in the directions x and y). The coating of the longitudinal irons closest to the faces is of 22 mm . The coating of irons compared to the side edges of the flagstone of 2 cm is neglected. The table hereafter recapitulates the data of reinforcement. Geometrical percentage of steel μ is given for a face in a direction.

Diameter of the reinforcements	Spacing	Section steel/section of the concrete	distance roasts/average surface of the flagstone
$\Phi=0,01\text{ m}$	$dx=dy=0,1\text{ m}$	$\mu=0,65$	$e_s=\pm 0,038\text{ m}$



One notes $a_x = \frac{A_x}{d_x}$, $a_y = \frac{A_y}{d_y}$ rates of reinforcement (here: $a_x = a_y = 7,854 \cdot 10^{-4}\text{ m}$), $A_x(A_y)$ being the section of an iron bar in the direction $x(y)$; e_z the distance from the tablecloths on the average surface.

1.2 Material properties

The characteristics of steels are the following ones:

Young modulus E_a	Poisson's ratio	Yield stress with 0.2% σ_y	Rupture limit σ_r	Slope of work hardening	Lengthening with rupture
210000 MPa	0.3	500 MPa	570 MPa	473 MPa	15%

Those of the concrete are the following ones:

Young modulus E_b	Poisson's ratio	Resistance in compression, σ_c	Resistance in traction, σ_t
35700 MPa	0.22	52,5 MPa	4,4 MPa

1.3 Boundary conditions and loadings

The boundary conditions are summarized in simple supports: vertical displacement blocked and free rotations on two edges in opposite, two other edges remaining free.

Uniform pressure: $p = 0.01 \text{ MPa}$

1.4 Initial conditions

Without object.

2 Reference solution

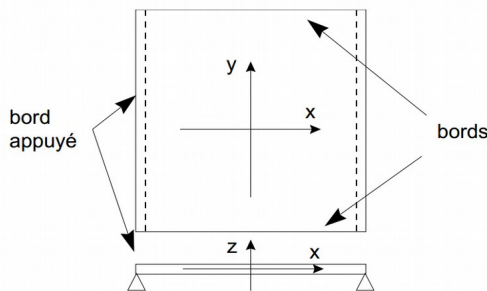
2.1 Method of calculating used for the reference solution

Elastic relations, connecting the membrane efforts N and of inflection M with the membrane deformations ε and curves κ and taking account of two symmetrical grids, are written:

$$N = \left(\frac{E_b h}{1 - \nu_b^2} \begin{bmatrix} 1 & \nu_b & 0 \\ \nu_b & 1 & 0 \\ 0 & 0 & \frac{1 - \nu_b}{2} \end{bmatrix} + 2 E_a \begin{bmatrix} a_x & 0 & 0 \\ 0 & a_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \varepsilon$$

$$M = \left(\frac{E_b h^3}{12(1 - \nu_b^2)} \begin{bmatrix} 1 & \nu_b & 0 \\ \nu_b & 1 & 0 \\ 0 & 0 & \frac{1 - \nu_b}{2} \end{bmatrix} + 2 E_a e_z^2 \begin{bmatrix} a_x & 0 & 0 \\ 0 & a_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \kappa$$

As regards a configuration beam, one assigns to the concrete a Poisson's ratio equal to 0 to cancel any inflection in the perpendicular direction. Two opposite edges of the flagstone are simply supported, two others remaining free:



The stiffness of equivalent inflection which takes account of steels is:

$$(EI)_{\acute{e}q} = \frac{E_b l h^3}{12} + 2 E_a a_x l e_z^2,$$

that is to say here:

$$(EI)_{\acute{e}q} = 10.111 \text{ MNm}^2.$$

The elastic solution is calculated in theory of beams for a value of equivalent pressure $p' = pl$. One obtains the values of the moments in configuration "plates" by division by the width of the flagstone l .

Size in the center	Expression
Arrow in the center under surface pressure	$w(l/2) = \frac{5 p' l^4}{384 (EI)_{\acute{e}q}}$
Curve	$\kappa_{xx}(l/2) = \frac{p' l^2}{8 (EI)_{\acute{e}q}}$
Deformation	$\varepsilon_{xx} = \kappa_{xx} \frac{h}{2}$
Total moment (in beam)	$M(l/2) = p' l^2 / 8$
Total moment (in plate)	$M(l/2) = p l^2 / 8$

2.2 Results of reference

For modelings A and B in which one validates the law `GLRC_DAMA` with the elements `DKTG` :

- Arrow in the center under surface pressure: $w = 2,433 \cdot 10^{-4} m$
- Curve: $k = 7,210 \cdot 10^{-4} m^{-1}$
- Deformation: $\varepsilon_{xx} = -0.4326 \cdot 10^{-4}$ on the lower skin
- Total moment (in beam): $M = 7290 Nm$
- Total moment (in plate): $M = 4050 Nm/ml$
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For modelings C and D in which one validates the law `ELAS` with the elements `Q4GG` :

- Arrow in the center under surface pressure: $w = 2,658 \cdot 10^{-4} m$
- Curve: $k = 7,878 \cdot 10^{-4} m^{-1}$
- Deformation: $\varepsilon_{xx} = -0.47269 \cdot 10^{-4}$ on the lower skin
- Total moment (in beam): $M = 7290 Nm$
- Total moment (in plate): $M = 4050 Nm/ml$

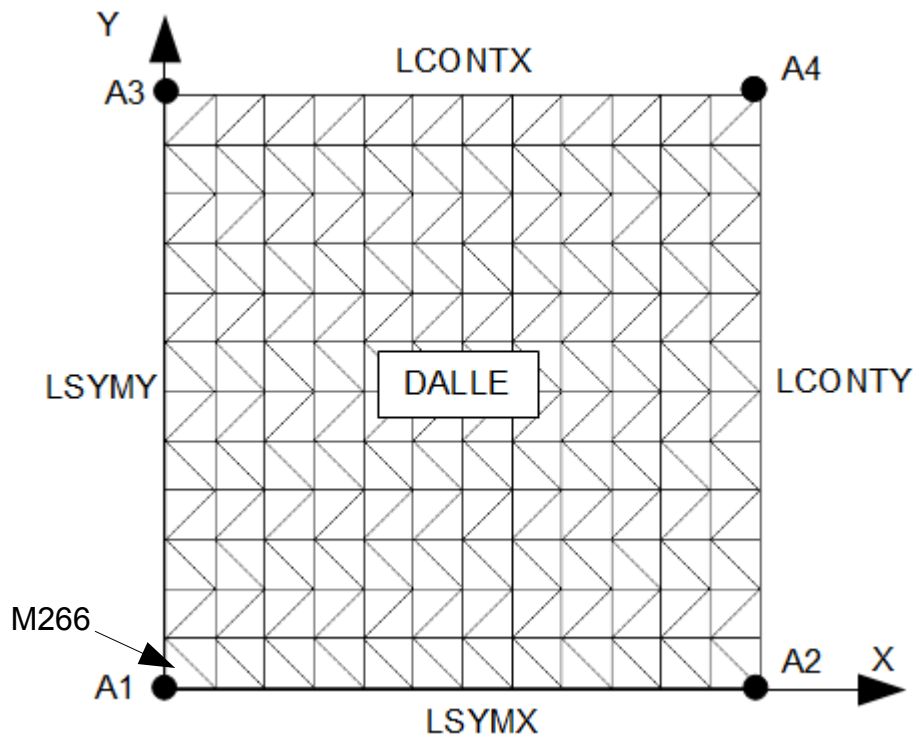
2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical reference

- [1] KOECHLIN P., MILL S., "Model total behavior of the reinforced concrete plates under dynamic loading of inflection: Law `GLRC`", Notes EDF/R & D /AMA HT-62/01/028A.

3 Modeling A



3.1 Characteristics of the grid

Many nodes: 169
Number of meshes and type: 288 TRIA3

3.2 Sizes tested and results

Identification	Type of reference	Reference	Tolerance (%)
<i>DZ</i> (A1)	'ANALYTICAL'	$2,433 \cdot 10^{-4}$	6 %
<i>MXX</i> (A1)	'ANALYTICAL'	4050.	2 %
<i>KXX</i> (A1)	'ANALYTICAL'	$7.21 \cdot 10^{-4}$	5 %

Identification	Type of reference	Reference	Tolerance (%)
<i>MXX</i> M266 Point 3	'NON_REGRESSION'	4044.16	1.e-6
<i>KXX</i> M266 Point 3	'NON_REGRESSION'	$7.1996 \cdot 10^{-4}$	1.e-6

- The sizes are expressed in the reference mark defined by the nautical angles $\alpha=33^\circ$ and $\beta=12^\circ$.

Identification	Type of reference	Reference	Tolerance (%)
<i>DZ (AI)</i>	'ANALYTICAL'	2,433 10 ⁻⁴	6 %
<i>MXX (AI)</i>	'NON_REGRESSION'	2847.47	1.e-6
<i>MYY (AI)</i>	'NON_REGRESSION'	1198.15	1.e-6
<i>MXY (AI)</i>	'NON_REGRESSION'	-1852.21	1.e-6
<i>KXX (AI)</i>	'NON_REGRESSION'	5.0692 10 ⁻⁴	1.e-6
<i>KYY (AI)</i>	'NON_REGRESSION'	2.1330 10 ⁻⁴	1.e-6
<i>KXY (AI)</i>	'NON_REGRESSION'	-3.2974 10 ⁻⁴	1.e-6

Identification	Type of reference	Reference	Tolerance (%)
<i>MXX M266 Point 3</i>	'NON_REGRESSION'	2842.56	1.e-6
<i>MYY M266 Point 3</i>	'NON_REGRESSION'	1197.70	1.e-6
<i>MXY M266 Point 3</i>	'NON_REGRESSION'	-1849.40	1.e-6
<i>KXX M266 Point 3</i>	'NON_REGRESSION'	5.0605 10 ⁻⁴	1.e-6
<i>KYY M266 Point 3</i>	'NON_REGRESSION'	2.1322 10 ⁻⁴	1.e-6
<i>KXY M266 Point 3</i>	'NON_REGRESSION'	-3.2924 10 ⁻⁴	1.e-6

3.3 Remarks

The coefficients of the following matrices of elasticity, used during calculations, were calculated with $\nu_b=0$:

- Matrix of elasticity out of membrane:
$$\begin{pmatrix} 4614. & 0 & 0 \\ 0 & 4614. & 0 \\ 0 & 0 & 2142. \end{pmatrix} 10^6 N/m$$
- Matrix of elasticity in inflection:
$$\begin{pmatrix} 5.617 & 0 & 0 \\ 0 & 5.617 & 0 \\ 0 & 0 & 2.57 \end{pmatrix} 10^6 N/m$$

To be certain to remain in the elastic range, the yield stresses expressed in the reference mark of orthotropism, are fixed arbitrarily at a very high value:

Yield stresses in positive inflection:

- Direction *x* : $1.10^{10} MNm/ml$
Direction *y* : $1.10^{10} MNm/ml$

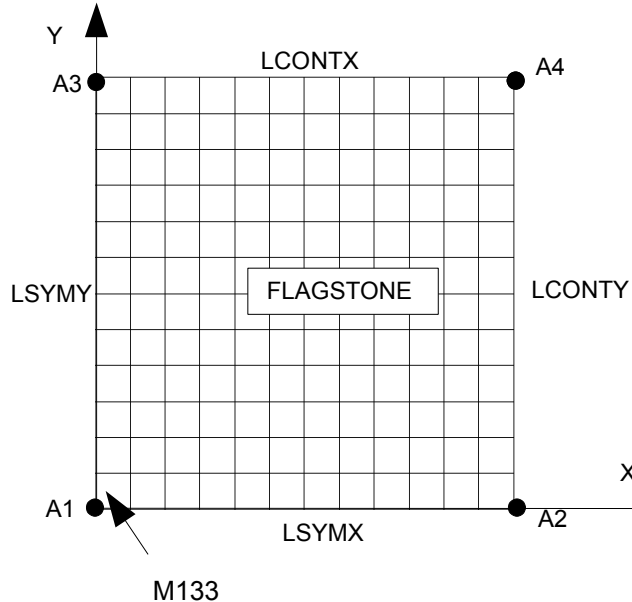
Yield stresses in negative inflection:

- Direction *x* : $-1.10^{10} MNm/ml$
Direction *y* : $-1.10^{10} MNm/ml$

As the structure remains in the elastic range, the kinematic coefficient of recall (constant of Prager) can take an unspecified value.

4 Modeling B

4.1 Characteristics of modeling



Modeling DKTG (QUAD4)

- Boundary conditions:

. Dimensioned $A2A4 : DZ = 0$

- Conditions of symmetry

. Dimensioned $A1A2 :$

$DY = DRX = 0$

. Dimensioned $A1A3 :$

$DX = DRY = 0$

The flagstone is symmetrical compared to the plans ($X=0$) and ($Y=0$), calculations are carried out on a quarter of the flagstone.

4.2 Characteristics of the grid

Many nodes: 169

Number of meshes and type: 144 QUAD4

4.3 Features tested

The macro-order `POST_COQUE` allows to extract the efforts and the deformations in an unspecified point from the hull.

4.4 Values tested

Identification	Type of reference	Reference	Tolerance (%)
$DZ(A1)$	'ANALYTICAL'	$2,433 \cdot 10^{-4}$	6 %
$MXX(A1)$	'ANALYTICAL'	4050.	2 %
$KXX(A1)$	'ANALYTICAL'	$7.21 \cdot 10^{-4}$	5 %

Identification	Type of reference	Reference	Tolerance (%)
MXX $M133$ <i>Point 4</i>	'NON_REGRESSION'	4044.05	1.e-6
KXX $M133$ <i>Point 4</i>	'NON_REGRESSION'	$7.1995 \cdot 10^{-4}$	1.e-6

- The sizes are expressed in the reference mark defined by the nautical angles $\alpha=33^\circ$ and $\beta=12^\circ$.

Identification	Type of reference	Reference	Tolerance (%)
<i>DZ (AI)</i>	'ANALYTICAL'	2,433 10 ⁻⁴	6 %
<i>MXX (AI)</i>	'NON_REGRESSION'	2848.64	1.e-6
<i>MYY (AI)</i>	'NON_REGRESSION'	1201.35	1.e-6
<i>MXY (AI)</i>	'NON_REGRESSION'	-1849.92	1.e-6
<i>KXX (AI)</i>	'NON_REGRESSION'	5.0713 10 ⁻⁴	1.e-6
<i>KYY (AI)</i>	'NON_REGRESSION'	2.1387 10 ⁻⁴	1.e-6
<i>KXY (AI)</i>	'NON_REGRESSION'	-3.2933 10 ⁻⁴	1.e-6

Identification	Type of reference	Reference	Tolerance (%)
<i>MXX MI33 Point 4</i>	'NON_REGRESSION'	2844.46	1.e-6
<i>MYY MI33 Point 4</i>	'NON_REGRESSION'	1199.59	1.e-6
<i>MXY MI33 Point 4</i>	'NON_REGRESSION'	-1847.21	1.e-6
<i>KXX MI33 Point 4</i>	'NON_REGRESSION'	5.0639 10 ⁻⁴	1.e-6
<i>KYY MI33 Point 4</i>	'NON_REGRESSION'	2.1356 10 ⁻⁴	1.e-6
<i>KXY MI33 Point 4</i>	'NON_REGRESSION'	-3.2885 10 ⁻⁴	1.e-6

4.5 Remarks

The coefficients of the matrices of following elasticities, used during calculations, were calculated with $\nu_b=0$:

- Matrix of elasticity out of membrane:
$$\begin{Bmatrix} 4614. & 0 & 0 \\ 0 & 4614. & 0 \\ 0 & 0 & 2142. \end{Bmatrix} 10^6 N/m$$
- Matrix of elasticity in inflection:
$$\begin{Bmatrix} 5.617 & 0 & 0 \\ 0 & 5.617 & 0 \\ 0 & 0 & 2.57 \end{Bmatrix} 10^6 N/m$$

To be certain to remain in the elastic range, the yield stresses expressed in the reference mark of orthotropy, are fixed arbitrarily at a very high value:

Yield stresses in positive inflection:

- Direction x : $1.10^{10} MNm/ml$
Direction y : $1.10^{10} MNm/ml$

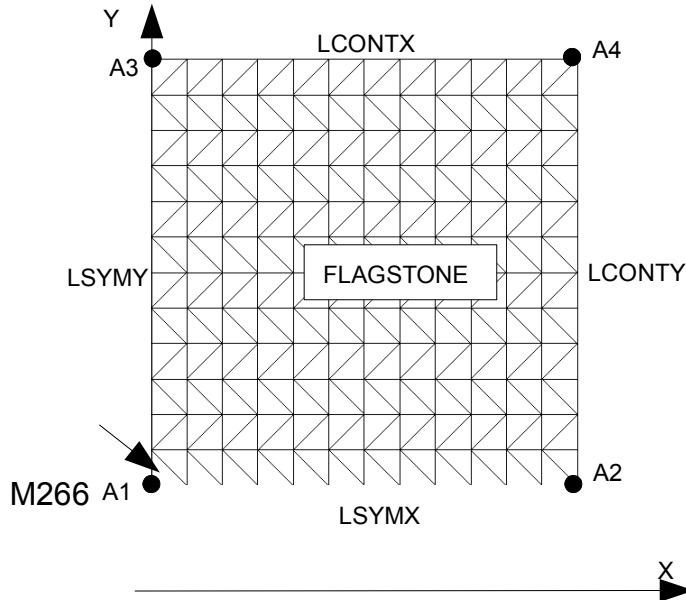
Yield stresses in negative inflection:

- Direction x : $-1.10^{10} MNm/ml$
Direction y : $-1.10^{10} MNm/ml$

As the structure remains in the elastic range, the kinematic coefficient of recall (constant of Prager) can take an unspecified value.

5 Modeling C

5.1 Characteristics of modeling



Modeling Q4GG (TRIA3)

- Boundary conditions:
 - . Dimensioned A2A4 : $DZ = 0$
- Conditions of symmetry
 - . Dimensioned A1A2 : $DY = DRX = 0$
 - . Dimensioned A1A3 : $DX = DRY = 0$

The flagstone is symmetrical compared to the plans ($X=0$) and ($Y=0$), calculations are carried out on a quarter of the flagstone.

5.2 Characteristics of the grid

Many nodes: 169
Number of meshes and type: 288 TRIA3

5.3 Sizes tested and results

Identification	Type of reference	Reference	Tolerance (%)
$DZ (A1)$	'ANALYTICAL'	$2,658 \cdot 10^{-4}$	1 %
$MXX (A1)$	'ANALYTICAL'	4050.	0.5 %
$KXX (A1)$	'ANALYTICAL'	$7,878 \cdot 10^{-4}$	0.5 %

Identification			Type of reference	Reference	Tolerance (%)
MXX	M266	Point 1	'NON_REGRESSION'	4032.58	1.e-6
KXX	M266	Point 1	'NON_REGRESSION'	$7.8442 \cdot 10^{-4}$	1.e-6

- The sizes are expressed in the reference mark defined by the nautical angles $\alpha=33^\circ$ and $\beta=12^\circ$.

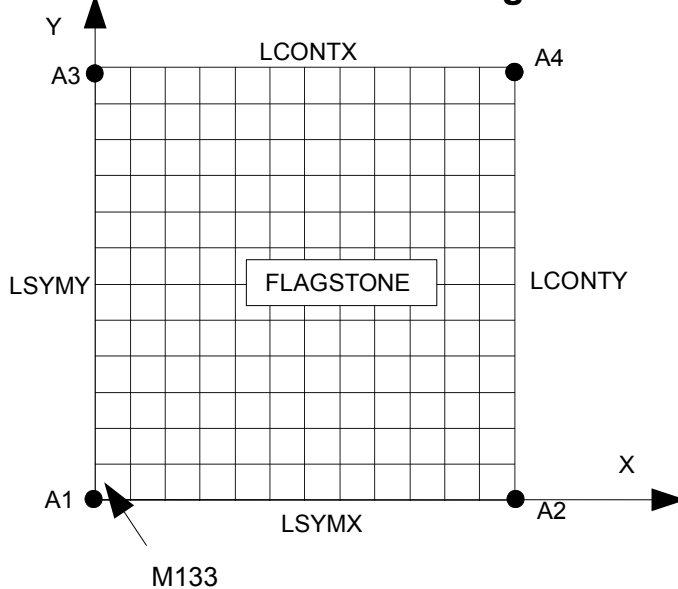
Identification	Type of reference	Reference	Tolerance (%)
<i>DZ (AI)</i>	'ANALYTICAL'	2,658 10 ⁻⁴	1 %
<i>MXX (AI)</i>	'NON_REGRESSION'	2836.04	1.e-6
<i>MYY (AI)</i>	'NON_REGRESSION'	1195.36	1.e-6
<i>MXY (AI)</i>	'NON_REGRESSION'	-1842.50	1.e-6
<i>KXX (AI)</i>	'NON_REGRESSION'	5.5167 10 ⁻⁴	1.e-6
<i>KYY (AI)</i>	'NON_REGRESSION'	2.3252 10 ⁻⁴	1.e-6
<i>KXY (AI)</i>	'NON_REGRESSION'	-3.5840 10 ⁻⁴	1.e-6

Identification	Type of reference	Reference	Tolerance (%)
<i>MXX M266 Point 1</i>	'NON_REGRESSION'	2836.04	1.e-6
<i>MYY M266 Point 1</i>	'NON_REGRESSION'	1195.36	1.e-6
<i>MXY M266 Point 1</i>	'NON_REGRESSION'	-1842.50	1.e-6
<i>KXX M266 Point 1</i>	'NON_REGRESSION'	5.5167 10 ⁻⁴	1.e-6
<i>KYY M266 Point 1</i>	'NON_REGRESSION'	2.3252 10 ⁻⁴	1.e-6
<i>KXY M266 Point 1</i>	'NON_REGRESSION'	-3.5840 10 ⁻⁴	1.e-6

5.4 Remarks

6 Modeling D

6.1 Characteristics of modeling



Modeling Q4GG (QUAD4)

- Boundary conditions:
 - . Dimensioned A2A4 : $DZ = 0$
- Conditions of symmetry
 - . Dimensioned A1A2 : $DY = DRX = 0$
 - . Dimensioned A1A3 : $DX = DRY = 0$

The flagstone is symmetrical compared to the plans ($X=0$) and ($Y=0$), calculations are carried out on a quarter of the flagstone.

6.2 Characteristics of the grid

Many nodes: 169
Number of meshes and type: 144 QUAD4

6.3 Features tested

The macro-order `POST_COQUE` allows to extract the efforts and the deformations in an unspecified point from the hull.

6.4 Values tested

Identification	Type of reference	Reference	Tolerance (%)
$DZ(AI)$	'ANALYTICAL'	$2,658 \cdot 10^{-4}$	1 %
$MXX(AI)$	'ANALYTICAL'	4050.	0.5 %
$KXX(AI)$	'ANALYTICAL'	$7,878 \cdot 10^{-4}$	0.5 %

Identification	Type of reference	Reference	Tolerance (%)
MXX M133 Point 4	'NON_REGRESSION'	4035.93	1.e-6
KXX M133 Point 4	'NON_REGRESSION'	$7.8507 \cdot 10^{-4}$	1.e-6

- The sizes are expressed in the reference mark defined by the nautical angles $\alpha = 33^\circ$ and $\beta = 12^\circ$.

Identification	Type of reference	Reference	Tolerance (%)
$DZ(AI)$	'ANALYTICAL'	$2,658 \cdot 10^{-4}$	1 %
$MXX(AI)$	'NON_REGRESSION'	2838.75	1.e-6
$MYY(AI)$	'NON_REGRESSION'	1197.18	1.e-6
$MXY(AI)$	'NON_REGRESSION'	-1843.50	1.e-6
$KXX(AI)$	'NON_REGRESSION'	$5.5220 \cdot 10^{-4}$	1.e-6

<i>KYY (A1)</i>	'NON_REGRESSION'	2.3287 10 ⁻⁴	1.e-6
<i>KXY (A1)</i>	'NON_REGRESSION'	-3.5860 10 ⁻⁴	1.e-6

Identification			Type of reference	Reference	Tolerance (%)
<i>MXX</i>	<i>M133</i>	<i>Point 4</i>	'NON_REGRESSION'	2838.75	1.e-6
<i>MYX</i>	<i>M133</i>	<i>Point 4</i>	'NON_REGRESSION'	1197.18	1.e-6
<i>MYX</i>	<i>M133</i>	<i>Point 4</i>	'NON_REGRESSION'	-1843.50	1.e-6
<i>KXX</i>	<i>M133</i>	<i>Point 4</i>	'NON_REGRESSION'	5.5220 10 ⁻⁴	1.e-6
<i>KYY</i>	<i>M133</i>	<i>Point 4</i>	'NON_REGRESSION'	2.3287 10 ⁻⁴	1.e-6
<i>KXY</i>	<i>M133</i>	<i>Point 4</i>	'NON_REGRESSION'	-3.5860 10 ⁻⁴	1.e-6

6.5 Remarks

7 Summary of the results

The results of four modelings are close to the analytical solution:

- DKTG : to the maximum 6% of variation for displacements, and 5% for the moment and the curve.
- Q4GG : to the maximum 1% of variation for displacements, and 0.5% for the moment and the curve.

These modelings validate:

- modeling DKTG with the model GLRC_DAMAGE in elastic behavior.
- modeling Q4GG with the model ELAS.