SSLS127 – Inflection of a reinforced concrete flagstone (model `GLRC_DAMAGE`) supported on 4 with dimensions: elastic mode of plate

Summary:

This test represents the calculation of a reinforced concrete flagstone, in inflection, subjected to a pressure. It makes it possible to validate modeling `DKTG` with model `GLRC_DAMAGE` for the linear elastic behavior and modeling `Q4GG` with the model `ELAS`. The flagstone is in simple supports on its four with dimensions.

Four modelings are carried out:

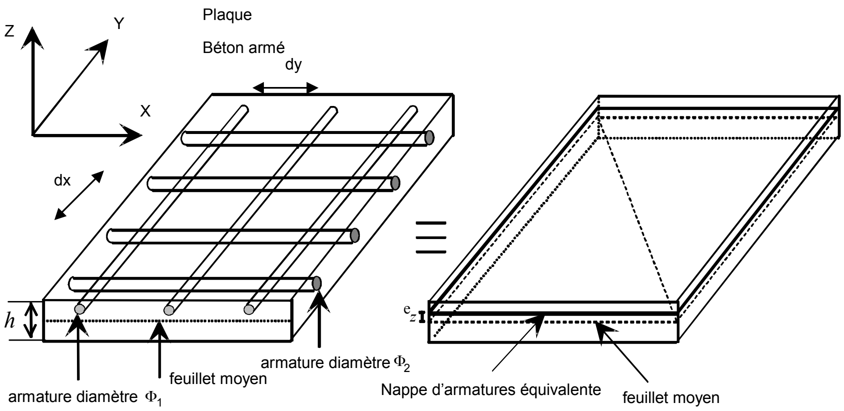
1) Modeling A makes it possible to test the model `DKTG` with `TRIA3`
2) Modeling B makes it possible to test the model `DKTG` with `QUAD4`
3) Modeling C makes it possible to test the model `Q4GG` with `TRIA3`
4) Modeling D makes it possible to test the model `Q4GG` with `QUAD4`

1 Problem of reference

1.1 Geometry

Square flagstone, length $l=1.8\,m$, thickness $h=0.12\,m$, in simple support on the four edges. The reinforcement of inflection is parallel to the edges; it is identical on each of the two faces and in each of the two directions (dx , dy being spacings of irons in the directions x and y). The coating of the longitudinal irons closest to the faces is of $22\,mm$. The coating of irons compared to the side edges of the flagstone of $2\,cm$ is neglected. The table hereafter recapitulates the data of reinforcement. Geometrical percentage of steel μ is given for a face in a direction.

Diameter of the reinforcements	Spacing	Section steel/section of the concrete	distance roasts/average surface of the flagstone
$\Phi=0{,}01\,\mathrm{m}$	$dx=dy=0{,}1\,\mathrm{m}$	$\mu=0{,}65$	$e_s=\pm 0{,}038\,\mathrm{m}$



One notes $a_x=\frac{A_x}{d_x}$ and $a_y=\frac{A_y}{d_y}$ rates of reinforcement (here: $a_x=a_y=7{,}854.10^{-4}\,\mathrm{m}$), A_x (A_y) being the surface of the section of an iron bar in the direction $x(y)$; e_s is the distance from the tablecloths on the average surface.

1.2 Material properties

The mechanical properties of steels are the following ones:

Young modulus E_a	Poisson's ratio	Yield stress with 0.2% σ_y	Rupture limit σ_r	Slope of work hardening	Lengthening with rupture
$210000\,\mathrm{MPa}$	0.3	$500\,\mathrm{MPa}$	$570\,\mathrm{MPa}$	$473\,\mathrm{MPa}$	15%

Those of the concrete are the following ones:

Young modulus E_b	Poisson's ratio	Resistance in compression σ_c	Resistance in traction σ_t
$35700\,\mathrm{MPa}$	0.22	$52{,}5\,\mathrm{MPa}$	$4{,}4\,\mathrm{MPa}$

1.3 Boundary conditions and loadings

- The boundary conditions are summarized in simple supports: vertical displacement blocked and free rotations on the four edges of the flagstone.
- Uniform pressure $p=0{,}01\,MPa$

1.4 Initial conditions

Without object.

2 Reference solution

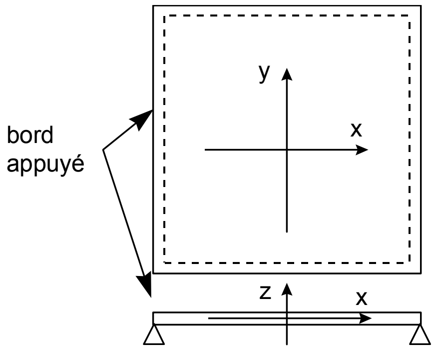
2.1 Method of calculating used for the reference solution

Elastic relations, connecting the membrane efforts N and of inflection M with the membrane deformations ε and curves κ and taking account of two symmetrical grids, are written:

$$\boldsymbol{N}=\left(\frac{E_b h}{1-\nu_b^2}\begin{bmatrix}1 & \nu_b & 0\\ \nu_b & 1 & 0\\ 0 & 0 & \dfrac{1-\nu_b}{2}\end{bmatrix}+2E_a\begin{bmatrix}a_x & 0 & 0\\ 0 & a_y & 0\\ 0 & 0 & 0\end{bmatrix}\right)\boldsymbol{\epsilon}$$

$$\boldsymbol{M}=\left(\frac{E_b h^3}{12(1-\nu_b^2)}\begin{bmatrix}1 & \nu_b & 0\\ \nu_b & 1 & 0\\ 0 & 0 & \dfrac{1-\nu_b}{2}\end{bmatrix}+2E_a e_s^2\begin{bmatrix}a_x & 0 & 0\\ 0 & a_y & 0\\ 0 & 0 & 0\end{bmatrix}\right)\boldsymbol{\kappa}$$

As regards a configuration paves, one assigns to the concrete the usual Poisson's ratio $\nu_b=0{,}22$. The flagstone is simply pressed on the four edges:



The stiffness of equivalent inflection is:

$$D_{éq}=\frac{E_b h^3}{12(1-\nu_b^2)}+2E_a e_s^2 a_x \ ,$$

that is to say here: $D_{éq}=5{,}8786\,\mathrm{MNm}$;

Moreover: $\nu_{éq}=\dfrac{\nu_b E_b h^3}{12(1-\nu_b^2)D_{éq}}$,

that is to say: $\nu_{éq}=0{,}2022$

Size	Expression
Arrow in the center under pressure [2]	$w=\dfrac{0{,}0464\,pl^4}{12(1-\nu_{éq}^2)D_{éq}}$
Curve in the center [2]	$\kappa_{xx}=\kappa_{yy}=\dfrac{0{,}04784\,pl^2}{(1+\nu_{éq})D_{éq}}$
Total moment in the center [2]	$M_{xx}=M_{yy}=0{,}04784\,pl^2$

2.2 Results of reference

For modelings A and B in which one validates law GLRC_DAMA with elements DKTG:

- Arrow in the center under pressure: $w=6{,}926.10^{-5}\,\mathrm{m}$
- Curve in the center: $\kappa_{xx}=\kappa_{yy}=2{,}193.10^{-4}\,\mathrm{m}^{-1}$
- Total moment in the center: $M_{xx}=M_{yy}=1550\,\mathrm{Nm/ml}$

For modelings C and D in which one validates law ELAS with elements DKTG:

- Arrow in the center under pressure: $w=7{,}895.10^{-5}\,\mathrm{m}$

- Curve in the center: $\kappa_{xx}=\kappa_{yy}=2{,}351.10^{-4}\,\mathrm{m}^{-1}$
- Total moment in the center: $M_{xx}=M_{yy}=1550\,\mathrm{Nm/ml}$

2.3 Uncertainty on the solution

Analytical solution.

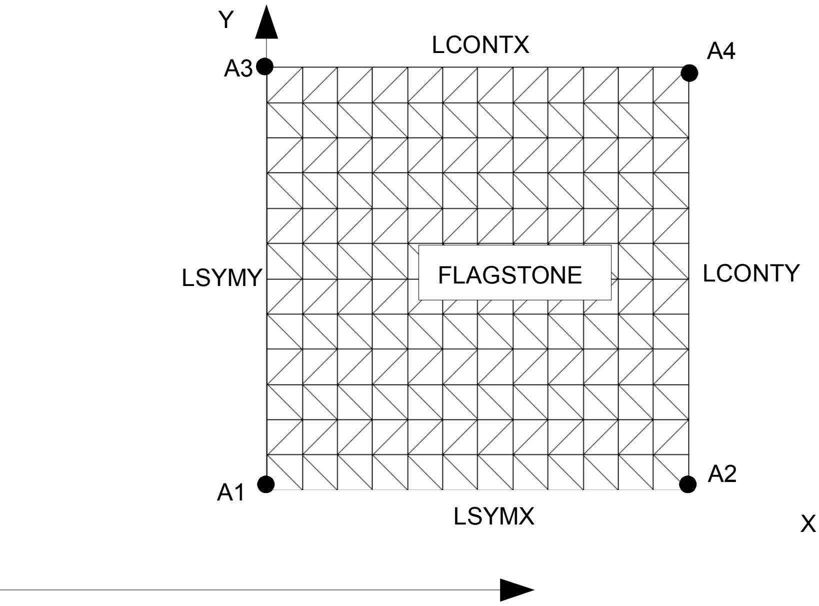
2.4 Bibliographical references

[1] KOECHLIN P., MILL S., “Model total behavior of the reinforced concrete plates under dynamic loading of inflection: Law GLRC”, Notes EDF/R & D /AMA HT-62/01/028A.

[2] J.Dulac, “elastoplastic dynamic Behavior of the reinforced concrete flagstones. Tests CEMETE – December 1979 – Flagstones 8 to 12”: EDF note: ESE/GC/82/13/A

3 Modeling A

3.1 Characteristics of modeling



Modeling Q4GG (TRIA3)

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

The flagstone is symmetrical compared to the plans $(X=0)$ and $(Y=0)$, calculations are carried out on a quarter of the flagstone.

3.2 Characteristics of the grid

Many nodes: 169
Number of meshs and type: 288 TRIA3

3.3 Sizes tested and results

Identification	**Type of reference**	***Reference***	***Tolerance (%)***
$DZ(A1)$	'ANALYTICAL'	$6,926.10^{-5}$	5 %
$MXX(A1)$	'ANALYTICAL'	1550	8 %
$MYY(A1)$	'ANALYTICAL'	1550	8 %
$KXX(A1)$	'ANALYTICAL'	$2,193.10^{-4}$	8 %
$KYY(A1)$	'ANALYTICAL'	$2,193.10^{-4}$	8 %

The sizes are expressed in the reference mark defined by the nautical angles $\alpha=33°$ and $\beta=12°$

Identification	**Type of reference**	**Reference**	**Tolerance**
$DZ(A1)$	'ANALYTICAL'	$6,926.10^{-5}$	5 %
$MXX(A1)$	'ANALYTICAL'	1550.0	8 %
$MYY(A1)$	'ANALYTICAL'	1550.0	8 %
$MXY(A1)$	'ANALYTICAL'	0.	2.
$KXX(A1)$	'ANALYTICAL'	$2,193\ 10^{-4}$	8 %
$KYY(A1)$	'ANALYTICAL'	$2,193\ 10^{-4}$	8 %

$KXY(A1)$	'ANALYTICAL'	0.	0,001

Identification			Type of reference	Reference	Tolérance%
MXX	$M266$	$Point\,3$	'NON_REGRESSION'	1445.794	1.e-6
MYY	$M266$	$Point\,3$	'NON_REGRESSION'	1447.847	1.e-6
MXY	$M266$	$Point\,3$	'NON_REGRESSION'	0,526	1.e-6
KXX	$M266$	$Point\,3$	'NON_REGRESSION'	$2.14096\ 10^{-4}$	1.e-6
KYY	$M266$	$Point\,3$	'NON_REGRESSION'	$2.14565\ 10^{-4}$	1.e-6
KXY	$M266$	$Point\,3$	'NON_REGRESSION'	$1.2018\ 10^{-7}$	1.e-6

3.4 Remarks

The coefficients of the following matrices of elasticity, used during calculations, were calculated with $\nu_b=0,22$:

1) Matrix of elasticity out of membrane: $\begin{bmatrix} 4832 & 990,4 & 0 \\ 990,4 & 4832 & 0 \\ 0 & 0 & 1756 \end{bmatrix} 10^6\,\mathrm{N/m}$

2) Matrix of elasticity in inflection: $\begin{bmatrix} 5,879 & 1,188 & 0 \\ 1,188 & 5,879 & 0 \\ 0 & 0 & 2,107 \end{bmatrix} 10^6\,\mathrm{N/m}$

To be certain to remain in the elastic range, the yield stresses expressed in the reference mark of orthotropism, are fixed arbitrarily at a very high value:

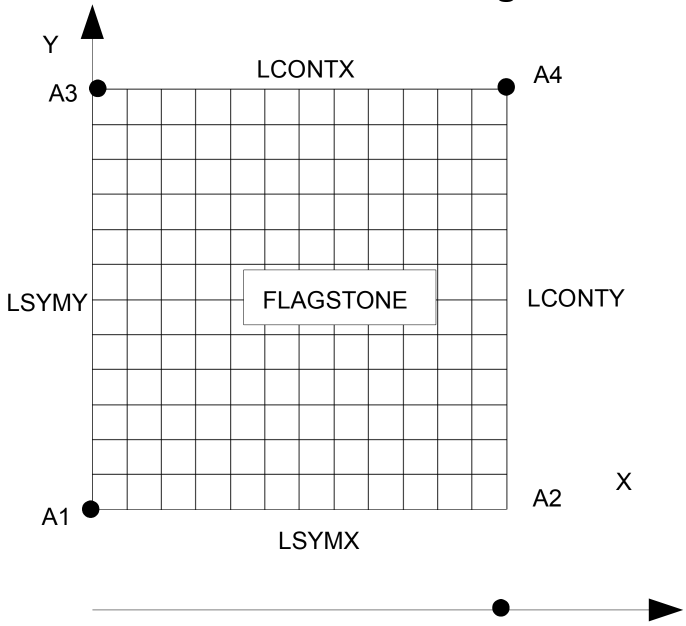
1. Yield stresses in positive inflection:
   Directorate X: $1.10^{10}\,\mathrm{MNm/ml}$
   Direction there: $1.10^{10}\,\mathrm{MNm/ml}$

2. Yield stresses in negative inflection:
   Directorate X: $-1.10^{10}\,\mathrm{MNm/ml}$
   Direction there: $-1.10^{10}\,\mathrm{MNm/ml}$

As the structure remains in the elastic range, the kinematic coefficient of recall (constant of Prager) can take an unspecified value.

4 Modeling B

4.1 Characteristics of modeling



Modeling Q4GG (QUAD4)

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
  . Dimensioned $A3A4$: $DZ=0$

- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

The flagstone is symmetrical compared to the plans $(X=0)$ and $(Y=0)$, calculations are carried out on a quarter of the flagstone.

4.2 Characteristics of the grid

Many nodes: 169

Number of meshs and type: 144 QUAD4

4.3 Sizes tested and results

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Identification	**Type of reference**	***Reference***	***Tolerance (%)***
$DZ(A1)$	'ANALYTICAL'	$6,926.10^{-5}$	5 %
$MXX(A1)$	'ANALYTICAL'	1550	8 %
$MYY(A1)$	'ANALYTICAL'	1550	8 %
$KXX(A1)$	'ANALYTICAL'	$2,193.10^{-4}$	8 %
$KYY(A1)$	'ANALYTICAL'	$2,193.10^{-4}$	8 %

The sizes are expressed in the reference mark defined by the nautical angles $\alpha=33°$ and $\beta=12°$.

Identification	**Type of reference**	**Reference**	**Tolerance**
$DZ(A1)$	'ANALYTICAL'	$6,926.10^{-5}$	5 %
$MXX(A1)$	'ANALYTICAL'	1550.0	8 %
$MYY(A1)$	'ANALYTICAL'	1550.0	8 %
$MXY(A1)$	'ANALYTICAL'	0.	2.
$KXX(A1)$	'ANALYTICAL'	$2,193\ 10^{-4}$	8 %
$KYY(A1)$	'ANALYTICAL'	$2,193\ 10^{-4}$	8 %
$KXY(A1)$	'ANALYTICAL'	0.	0,001

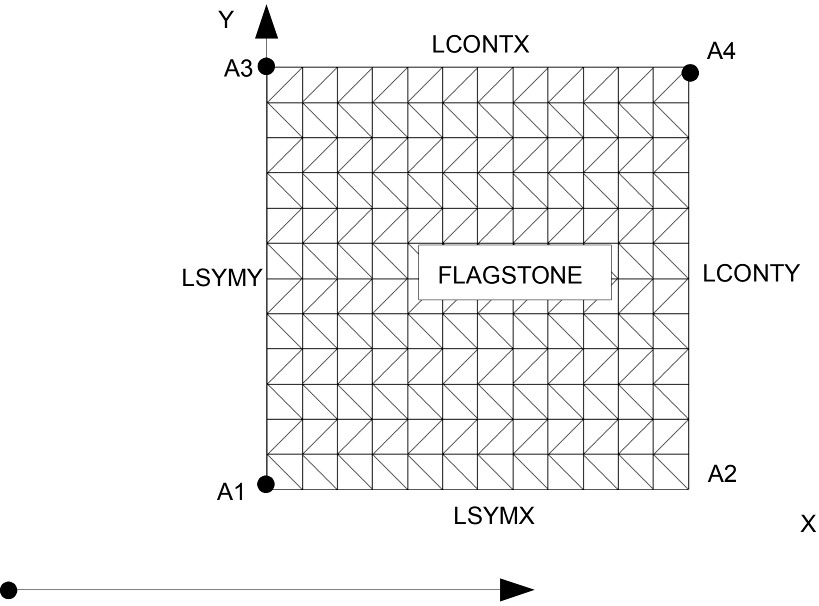
Identification			Type of reference	Reference	Tolérance %
MXX	*M133*	*Point* 4	'NON_REGRESSION'	1444.999	1.e-6
MYY	*M133*	*Point* 4	'NON_REGRESSION'	1447.976	1.e-6
MXY	*M133*	*Point* 4	'NON_REGRESSION'	-0.6626	1.e-6
KXX	*M133*	*Point* 4	'NON_REGRESSION'	$2.1394\ 10^{-4}$	1.e-6
KYY	*M133*	*Point* 4	'NON_REGRESSION'	$2.1462\ 10^{-4}$	1.e-6
KXY	*M133*	*Point* 4	'NON_REGRESSION'	$-1.5151\ 10^{-7}$	1.e-6

4.4 Remarks

See remarks of modeling A

5 Modeling C

5.1 Characteristics of modeling



Modeling `Q4GG (TRIA3)`

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

The flagstone is symmetrical compared to the plans $(X=0)$ and $(Y=0)$, calculations are carried out on a quarter of the flagstone.

5.2 Characteristics of the grid

Many nodes: 169
Number of meshs and type: 288 `TRIA3`

5.3 Sizes tested and results

Identification	Type of Reference	Reference	% Tolerance
$DZ(A1)$	'ANALYTICAL'	$7{,}895.10^{-5}$	8 %
$MXX(A1)$	'ANALYTICAL'	1550	3 %
$MYY(A1)$	'ANALYTICAL'	1550	3 %
$KXX(A1)$	'ANALYTICAL'	$2{,}351.10^{-4}$	3 %
$KYY(A1)$	'ANALYTICAL'	$2{,}351.10^{-4}$	3 %

The sizes are expressed in the reference mark defined by the nautical angles $\alpha=33°$ and $\beta=12°$

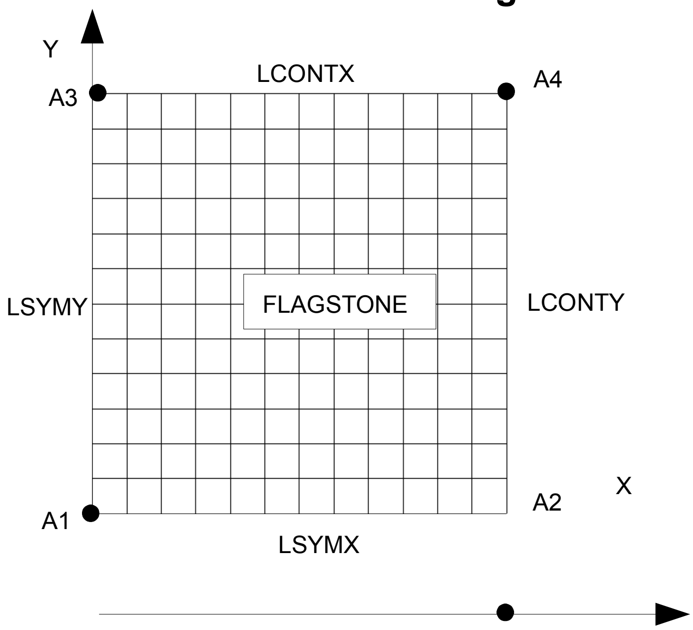
Identification	Type of reference	Reference	Tolerance
$DZ(A1)$	'ANALYTICAL'	$7{,}895.10^{-5}$	8 %
$MXX(A1)$	'ANALYTICAL'	1550.0	3 %
$MYY(A1)$	'ANALYTICAL'	1550.0	3 %
$MXY(A1)$	'ANALYTICAL'	0.	0.1
$KXX(A1)$	'ANALYTICAL'	$2{,}351.10^{-4}$	3 %
$KYY(A1)$	'ANALYTICAL'	$2{,}351.10^{-4}$	3 %
$KXY(A1)$	'ANALYTICAL'	0.	0,001

Identification			Type of reference	Reference	Tolérance%
MXX	$M266$	$Point\,1$	'NON_REGRESSION'	1506.61	1.e-6
MYY	$M266$	$Point\,1$	'NON_REGRESSION'	1506.70	1.e-6
KXX	$M266$	$Point\,1$	'NON_REGRESSION'	2,228 10^{-4}	1.e-6
KYY	$M266$	$Point\,1$	'NON_REGRESSION'	2,228 10^{-4}	1.e-6
KXY	$M266$	$Point\,1$	'NON_REGRESSION'	2.31 10^{-8}	1.e-6

5.4 Remarks

6 Modeling D

6.1 Characteristics of modeling



Modeling Q4GG (QUAD4)

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
  . Dimensioned $A3A4$: $DZ=0$

- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

The flagstone is symmetrical compared to the plans $(X=0)$ and $(Y=0)$, calculations are carried out on a quarter of the flagstone.

6.2 Characteristics of the grid

Many nodes: 169

Number of meshs and type: 144 QUAD4

6.3 Sizes tested and results

Identification	Type of Reference	Reference	% Tolerance
$DZ(A1)$	'ANALYTICAL'	$7{,}895.10^{-5}$	8 %
$MXX(A1)$	'ANALYTICAL'	1550	3 %
$MYY(A1)$	'ANALYTICAL'	1550	3 %
$KXX(A1)$	'ANALYTICAL'	$2{,}351.10^{-4}$	3 %
$KYY(A1)$	'ANALYTICAL'	$2{,}351.10^{-4}$	3 %

The sizes are expressed in the reference mark defined by the nautical angles $\alpha=33°$ and $\beta=12°$.

Identification	Type of reference	Reference	Tolerance
$DZ(A1)$	'ANALYTICAL'	$7{,}895.10^{-5}$	8 %
$MXX(A1)$	'ANALYTICAL'	1550.0	3 %
$MYY(A1)$	'ANALYTICAL'	1550.0	3 %
$MXY(A1)$	'ANALYTICAL'	0.	0.1
$KXX(A1)$	'ANALYTICAL'	$2{,}351.10^{-4}$	3 %
$KYY(A1)$	'ANALYTICAL'	$2{,}351.10^{-4}$	3 %
$KXY(A1)$	'ANALYTICAL'	0.	0,001

Identification			Type of reference	Reference	Tolérance%
MXX	$M266$	$Point\,1$	'NON_REGRESSION'	1512.79	1.e-6
MYY	$M266$	$Point\,1$	'NON_REGRESSION'	1515.82	1.e-6
MXY	$M266$	$Point\,1$	'NON_REGRESSION'	-0.6749	1.e-6
KXX	$M266$	$Point\,1$	'NON_REGRESSION'	2,294 10^{-4}	1.e-6
KYY	$M266$	$Point\,1$	'NON_REGRESSION'	2,301 10^{-4}	1.e-6
KXY	$M266$	$Point\,1$	'NON_REGRESSION'	-1,601 10^{-7}	1.e-6

6.4 Remarks

7 Summary of the results

By comparing the results of four modelings with the analytical solution, one observes:

- DKTG : to the maximum 5% of variation for displacements, and 8% for the moment and the curve.
- Q4GG : to the maximum 8% of variation for displacements, and 3% for the moment and the curve.

One can thus estimate that these modelings validate modeling DKTG and the model GLRC in elastic behavior, modeling Q4GG and the model ELAS.