SSLS133 - Flexbeam with variable thickness

Summary:

This test represents a quasi-static calculation of a flexbeam with variable thickness. It is embedded at an end, and is subjected to a vertical force at the other end. This test makes it possible to test the elements of voluminal hull SHB8 and SHB20 to manage the variations thicknesses. Four modelings are tested:

Finite elements SHB8 for a linear variation thickness of the plate (modeling A).
Finite elements SHB20 for a linear variation thickness of the plate (modeling B).
Finite elements SHB8 for a quadratic variation thickness of the plate (modeling C).
Finite elements SHB20 for a quadratic variation thickness of the plate (modeling D).

Displacements obtained are compared with the elastic analytical solution of a beam in inflection. This test makes it possible to show the capacities and the limits of the elements SHB8 and SHB20 to manage the variations thicknesses.

1 Problem of reference

1.1 Geometry

1.1.1 Plate with thickness varying linearly

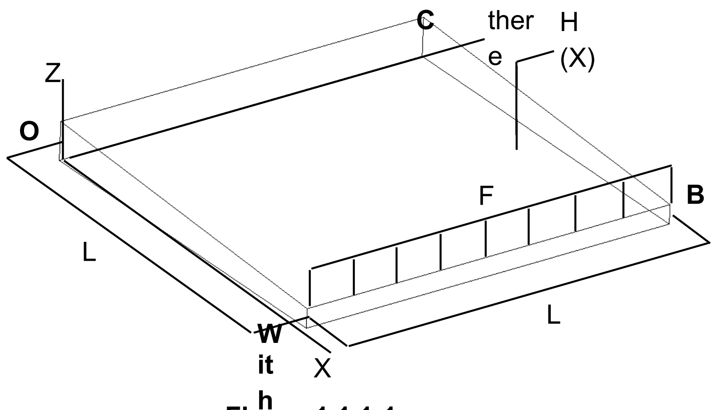

Figure 1.1.1-1 :

Length $L=100\,m$, width $l=100\,m$.

The thickness h vary linearly:

$$h(x)=ax+b$$

We pose $h(x=0)=h1=10\,m$ and $h(x=L)=h2=5\,m$ what gives us:

$$a=(h_2-h_1)/L \text{ and } b=h_1$$

1.1.2 Plate with thickness varying quadratically

The thickness h vary in a quadratic way:

$$h(x)=ax^2+bx+c$$

We pose $h(x=0)=h1=10\,m$, $h(x=L)=h2=5\,m$ and $h(x=L/2)=h_{12}=6{,}25\,m$ what gives us

$$a=(2(h_1+h_2)-4h_{12})/L^2,\ b=(4h_{12}-h_2-3h_1)/L \text{ and } c=h_1$$

1.2 Material properties

Young modulus: $E=2.10^{11}\,Pa$
Poisson's ratio: $\nu=0.0$

1.3 Boundary conditions and loadings

Boundary conditions:
Embedded on the side OC : $u=v=w=0$, $\theta_x=\theta_y=\theta_z=0$

Loading:
At the end AB , one load uniformly distributed of resultant:
Force parallel with the axis Z ; $F_z=1N$

2 Reference solution

2.1 Method of calculating used for the reference solution

The results of reference are got by the theory of the elastic beams.

In the case of a linear variation thickness, vertical displacement at the end AB is given by [1]:

$$w(x)=-\frac{FL^2}{2\,EI_{y_1}c^3}\frac{\left(2\,Lcx+c^2x^2-c^3x^2+2\mathrm{L}(L+cx)\ln\left(\frac{L}{L+cx}\right)\right)}{(L+cx)}$$

With

$$c=\left(\frac{I_{y_2}}{I_{y_1}}\right)^{\frac{1}{3}}-1 \text{ and } I_{y_i}=\frac{bh_i^3}{12}$$

In the case of a quadratic variation thickness, it is possible to find a formula exact of displacement. However its general expression is sufficiently complex not to be able to be written here. We formulated the approximate function of vertical displacement according to x of our case:

$$w(x)=3.10^{-8}\frac{2\mathrm{x}-200}{x^2-200\,x+20000}+6.10^{-10}\arctan(0.01\,x-1)-3.10^{-12}\,x+7.71238.10^{-10}\,m$$

2.2 Sizes and results of reference

Displacement of the points A and B according to Z .

2.3 Bibliographical references

[1] [V3.01.400] SSLL400 – non-prismatic Beam, subjected to efforts specific or distributed.

3 Modeling A

3.1 Characteristics of modeling

Element SHB8 and thickness varying linearly

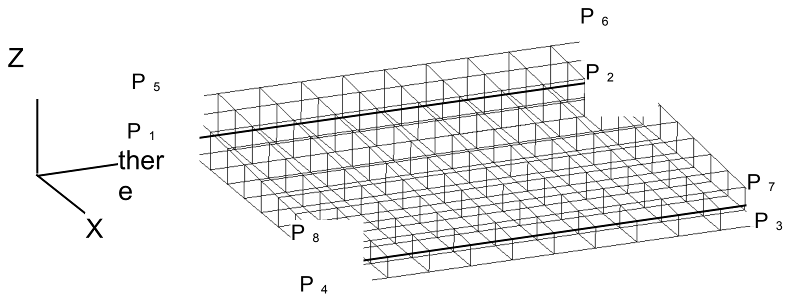

Figure 3.1-1 : Grid of modeling A

Cutting: a regular grid is considered in this modeling.

Regular grid:

100 meshs SHB8 : 10 according to the width, 10 according to the length, 1 according to the thickness

Boundary conditions:

All nodes inside the side $P_1 P_2 P_6 P_5$: following blocked displacement X
All nodes on the edge $P_1 P_5$: following blocked displacement Y
All nodes on the edge $P_2 P_1$: following blocked displacement Z

Loading:
Force linearly distributed on the edge $P_8 P_7$: F = 1

Name of the nodes:

Not P_1	N022	Not P_5	N020
Not P_2	N002	Not P_6	N001
Not P_3	N102	Not P_7	N100
Not P_4	N172	Not P_8	N171

3.2 Characteristics of the grid

Many nodes: 242

Many meshs and types: 100 SHB8

3.3 Sizes tested and results

Regular grid:

Not	Size in unit	Reference	% difference
P_7	displacement W (m)	$3.2710\ 10^{-10}$	+0,004
P_8	displacement W (m)	$3.2710\ 10^{-10}$	+0,004
POINT 1 MESH 1	SIEF_ELGA	0.0543525374822	0.1 %
POINT 2 NETS 1	SIEF_ELGA	0.0322992227094	0,1 %
POINT 3 NETS 1	SIEF_ELGA	0,0	1E-9 %
POINT 4 NETS 1	SIEF_ELGA	-0.0322734188761	0,1 %
POINT 5 NETS 1	SIEF_ELGA	-0.0542794829818	0,1 %
N22 MESH 1	IFGM_ELNO	0.0543350532544	0.1 %
N20 MESH 1	IFGM_ELNO	0.0322992227094	0,1 %

One also tests analytical fields of equivalent constraints of Von Mises.

4 Modeling B

4.1 Characteristics of modeling

Element SHB20 and thickness varying linearly

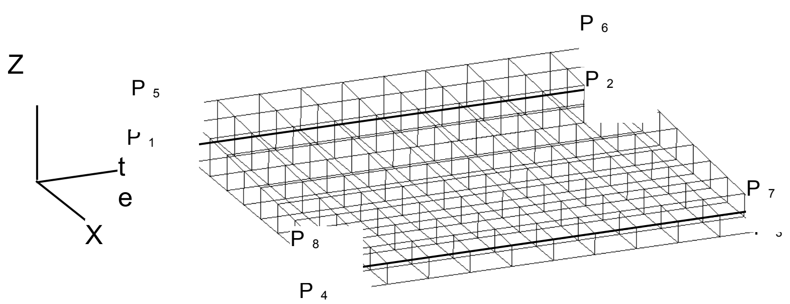

Figure 4.1-1 : Grid of modeling B

Cutting: a regular grid is considered in this modeling.

Regular grid:

100 meshs SHB20 : 10 according to the width, 10 according to the length, 1 according to the thickness

Boundary conditions:

All nodes inside the side $P_1 P_2 P_6 P_5$: following blocked displacement X
All nodes on the edge $P_1 P_5$: following blocked displacement Y
All nodes on the edge $P_2 P_1$: following blocked displacement Z

Loading:
Force linearly distributed on the edge $P_8 P_7$: $F=1$

Name of the nodes:

Not P_1	N347	Not P_5	N340
Not P_2	N579	Not P_6	N572
Not P_3	N006	Not P_7	N002
Not P_4	N074	Not P_8	N067

4.2 Characteristics of the grid

Many nodes: 803 Many meshs and types: 100 `SHB20`

4.3 Sizes tested and results

Regular grid:

Not	Size in unit	Reference	*Aster*	% difference
P_7	displacement W (m)	$3.2710\ 10^{-10}$	$3.2866\ 10^{-10}$	+0,476
P_8	displacement W (m)	$3.2710\ 10^{-10}$	$3.2866\ 10^{-10}$	+0,476

5 Modeling C

5.1 Characteristics of modeling

Element `SHB8` and thickness varying quadratically

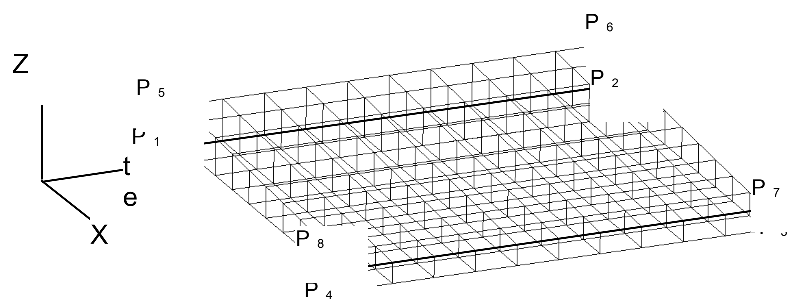

Figure 5.1-1 : Grid of modeling C

The characteristics are the same ones as for modeling A

Name of the nodes:

Not P_1	N005	Not P_5	N003
Not P_2	N006	Not P_6	N004
Not P_3	N008	Not P_7	N002
Not P_4	N007	Not P_8	N001

5.2 Characteristics of the grid

The grid is the same one as modeling A except for the thickness which varies here in a quadratic way.

5.3 Sizes tested and results

Regular grid:

Not	Size in unit	Reference	*Aster*	% difference
P_7	displacement W (m)	$4.7124\ 10^{-10}$	$5.1212\ 10^{-10}$	9 %
P_8	displacement W (m)	$4.7124\ 10^{-10}$	$5.1212\ 10^{-10}$	9 %

6 Modeling D

6.1 Characteristics of modeling

Element SHB20 and thickness varying quadratically

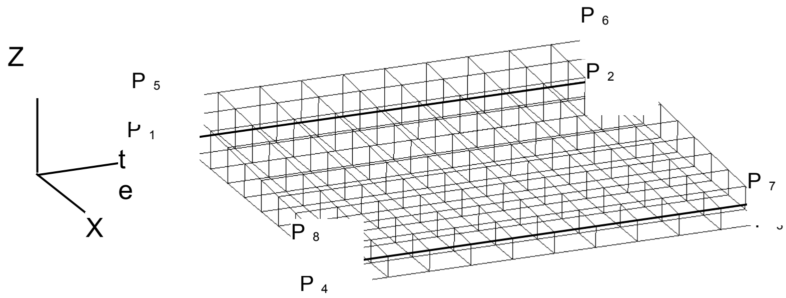

Figure 6.1-1 : Grid of modeling D

The characteristics are the same ones as for modeling B

Name of the nodes:

Not P_1	N005	Not P_5	N003
Not P_2	N006	Not P_6	N004
Not P_3	N008	Not P_7	N002
Not P_4	N007	Not P_8	N001

6.2 Characteristics of the grid

The grid is the same one as modeling B except for the thickness which varies here in a quadratic way.

6.3 Sizes tested and results

Regular grid:

Not	Size in unit	Reference	*Aster*	% difference
P_7	displacement W (m)	$4.7124\ 10^{-10}$	$4.6754\ 10^{-10}$	-0,784
P_8	displacement W (m)	$4.7124\ 10^{-10}$	$4.6754\ 10^{-10}$	-0,784

7 Summary of the results

In the case of a variation linaire thickness of the plate, good solutions are obtained some is the finite element used (SHB8 or SHB20).

When the geometrical variation is of a quadratic nature, elements SHB20 provide more precise results (error $<1\%$) that elements SHB8 (error $<9\%$).