SSLS134 - Reinforcement according to the method of Capra and Maury: analytical calculation

Summary:

This test relates to the analytical checking of the densities of reinforcement calculated using the operator CALC_FERRAILLAGE.
Several cases of loadings are studied to the ELS and with the ELECTED OFFICIAL.

1 Problem of reference

1.1 Geometry

One considers a square plate of with dimensions $1\,m$ and thickness $0{,}2\,m$.

1.2 Properties of material

Nothing.

1.3 Boundary conditions and loadings

There is no really mechanical calculation.
One arranges oneself so that the field of efforts in the plate corresponds to the one of the 10 following configurations:

1. compressive force of $1\,000\,000\,\mathrm{N}$ exerted along the axis X, and a shearing action of $100\,000\,\mathrm{N}$ according to Y
2. force of traction of $1\,000\,000\,\mathrm{N}$ exerted along the axis X, and a shearing action of $-20\,000\,\mathrm{N}$ according to X
3. force of traction of $1\,000\,000\,\mathrm{N}$ exerted along the axis Y, and a shearing action of $-20\,000\,\mathrm{N}$ according to X and of $80\,000\,\mathrm{N}$ according to Y
4. bending moment of $100\,000\,\mathrm{Nm}$ around the axis X.
5. bending moment of $100\,000\,\mathrm{Nm}$ around the axis Y .
6. bending moment of $100\,000\,\mathrm{Nm}$ around the axis Y and compression forces of $100\,000\,\mathrm{N}$ (with the ELECTED OFFICIAL) and $20\,000\,\mathrm{N}$ with the ELS exerted along the axis X
7. bending moment of $100\,000\,\mathrm{Nm}$ around the axis Y and traction forces of $100\,000\,\mathrm{N}$ exerted along the axis X
8. bending moment of $100\,000\,\mathrm{Nm}$ around the axis Y and traction forces of $2\,000\,000\,\mathrm{N}$ exerted along the axis X
9. bending moment of $100\,000\,\mathrm{Nm}$ around the axis Y and bending moment of $75\,000\,\mathrm{Nm}$ around the axis X
10. bending moment of $-150\,000\,\mathrm{Nm}$ (with the ELECTED OFFICIAL) and $-100\,000\,\mathrm{Nm}$ with the ELS around the axis X .
11. bending moment of $-260\,000\,\mathrm{Nm}$ (with the ELECTED OFFICIAL).
12. bending moment of $-380\,000\,\mathrm{Nm}$ (with the ELECTED OFFICIAL).

1.4 Other parameters of calculation

Coatings (higher and lower) are fixed at $e=4\,\mathrm{cm}$. Calculations are carried out with the Ultimate Absolute limit (ELU) and with the Absolute limit of Service (ELS).

The limit characteristic of elasticity of the reinforcing steel is fixed at $f_e=500\,\mathrm{MPa}$. The elastic limit used with the ELECTED OFFICIAL of the steel of the reinforcements is thus of $\sigma_s=\frac{f_e}{1.15}=435\,\mathrm{MPa}$. The working stress of traction to the ELS is thus of $\sigma_s=0.8\,f_e=400\,\mathrm{MPa}$.

Resistance characteristic in compression of the concrete is fixed at $f_{cj}=35\,\mathrm{MPa}$. Resistance in compression of the concrete used to the ELECTED OFFICIAL is thus $\sigma_b=\frac{f_{cj}}{1.5}=23.3\,\mathrm{MPa}$. The working stress of compression of the concrete to the ELS is of $\sigma_b=0.6\,f_{cj}=21\,\mathrm{MPa}$

The value of pivot A is fixed at $PIV_A = 10‰$ and the value of the pivot B with $PIV_B = 3.5‰$. The value of the coefficient of equivalence is fixed at $n = 15.1$. Moreover, one fixes the Young modulus of steel at $E_a = 210\,\mathrm{GPa}$.

2 Reference solution

2.1 Method of calculating

The densities of longitudinal steels are calculated according to the method of Capra and Maury.
Taking into account the directions of the efforts, the "dimensioning" facet is obvious.
Analytical calculation is thus summarized with a calculation of section making it possible to determine the efforts to which the 2 steel beds are subjected (higher and lower).

2.2 Sizes and results of reference

2.2.1 Configuration 1

The plate is subjected to a compression of $1\,000\,000\,\mathrm{N}$. Longitudinal reinforcement is thus from -1 (Pivot C not calculated, all the facets are compressed) for the ELECTED OFFICIAL and is null with the ELS in all the directions.
With the ELECTED OFFICIAL, the transverse reinforcement of with the shearing action applied $Q_y=100\,000\,\mathrm{N}$ is of:

$$A_{ST}=\frac{\sqrt{Q_x^2+Q_y^2}}{0.9\mathrm{d}(\cot\theta+\cot\alpha)\sin\alpha\,\sigma_s}=\frac{\sqrt{Q_x^2+Q_y^2}}{0.9\mathrm{d}\,\sigma_s}=15.964\,\mathrm{cm^2/m^2}$$

where $\alpha=90°$ et $\theta=45°$

2.2.2 Configuration 2

The plate is subjected to a tractive effort of $N_{xx}=1\,000\,000\,\mathrm{N}$ according to the axis X. It is about an entirely tended section in a symmetrical way.

The steel section is thus equal to $A_S=\frac{N}{\sigma_s}$.
Each bed of reinforcements thus takes again half of the effort is:

$A_{SXS}=A_{SXI}=\frac{A_S}{2}=11.494\,\mathrm{cm^2/m}$ **with the ELECTED OFFICIAL**

$A_{SXS}=A_{SXI}=\frac{A_S}{2}=12.5\,\mathrm{cm^2/m}$ **with the ELS**

The theoretical reinforcement along the axis Y is worthless taking into account the absence of effort in this direction.
With the ELECTED OFFICIAL, the transverse reinforcement of with the shearing action applied $Q_x=-20\,000\,\mathrm{N}$ is of:

$$A_{ST}=\frac{\sqrt{Q_x^2+Q_y^2}}{0.9\mathrm{d}(\cot\theta+\cot\alpha)\sin\alpha\,\sigma_s}=\frac{\sqrt{Q_x^2+Q_y^2}}{0.9\mathrm{d}\,\sigma_s}=3.193\,\mathrm{cm^2/m^2}$$

where $\alpha=90°$ et $\theta=45°$

2.2.3 Configuration 3

The theoretical results are symmetrical those of configuration 2.
$A_{SYS}=A_{SYI}=11.494\,\mathrm{cm^2/m}$ with the ELECTED OFFICIAL.
$A_{SYS}=A_{SYI}=12.5\,\mathrm{cm^2/m}$ with the ELS.

With the ELECTED OFFICIAL, the transverse reinforcement of with the efforts cutting-edges applied $Q_x=-20\,000\,\mathrm{N}$ and $Q_y=80\,000\,\mathrm{N}$ is of:

$$A_{ST}=\frac{\sqrt{Q_x^2+Q_y^2}}{0.9\mathrm{d}(\cot\theta+\cot\alpha)\sin\alpha\,\sigma_s}=\frac{\sqrt{Q_x^2+Q_y^2}}{0.9\mathrm{d}\,\sigma_s}=13.164\,\mathrm{cm^2/m^2}$$

where $\alpha=90°$ et $\theta=45°$

2.2.4 Configuration 4

The plate is subjected to one bending moment according to Y (around the axis X) equal to $M_{fx}=100\,000\,\mathrm{Nm}$. This bending moment corresponds to a tended top fibre.

Calculation is carried out here according to Eurocode 2.

With the ELECTED OFFICIAL:

Reduced ultimate moment $\mu=\frac{M_{fx}}{d^2\sigma_b}=0.167$.

The relative position of neutral fibre $\alpha=1-\sqrt{1-2\mu}=0.184$.

The arm of reduced lever $z=d\left(1-\frac{\alpha}{2}\right)=0.184$.

The section of reinforcement is thus equal to $A_{SYS}=\frac{M_{fx}}{z\,\sigma_s}=15.83\,\mathrm{cm^2/m}$ (bed Y superior).

With the ELS:

The moment of resistance of the concrete is equal to:

$$M_{\lim}=\frac{1}{2}\sigma_b y_{\lim}\left(d-\frac{y_{\lim}}{3}\right)=101\,342\,\mathrm{Nm}$$

with $y_{\lim}=d\frac{n\sigma_b}{n\sigma_b+\sigma_s}=0.0707\,\mathrm{m}$

We are thus if $M=M_{fx}\leq M_{\lim}$. Thus, only of tended steels are necessary.

The reduced moment of service is equal to: $\mu=n\frac{M}{d^2\sigma_b}=0.148$

The coefficient α is solution of the equation: $\alpha^3-3\alpha^2-6\mu(1-\alpha)=0$

By iterative resolution, one obtains: $\alpha=0.440$

The steel section necessary is equal to: $A_{SYS}=\frac{M_{fx}}{\sigma_s d\left(1-\frac{\alpha}{3}\right)}=18.31\,\mathrm{cm^2/m}$

2.2.5 Configuration 5

The theoretical results are symmetrical those of configuration 4.
The section of reinforcement is thus equal to $A_{SXS}=15.83\,\mathrm{cm^2/m}$ (bed X superior) with the ELECTED OFFICIAL.
The section of reinforcement is thus equal to $A_{SXS}=18.31\,\mathrm{cm^2/m}$ (bed X superior) with the ELS.

2.2.6 Configuration 6

The plate is subjected to one bending moment according to X (around the axis Y) equal to $M_{fy}=100\,000\,\mathrm{Nm}$ and with a compressive force according to X equal to $N_{xx}=-100\,000\,\mathrm{N}$ with the ELECTED OFFICIAL and equal to $N_{xx}=-20\,000\,\mathrm{N}$ with the ELS.

The section is partially tended.

Calculation is carried out here according to Eurocode 2.

With the ELECTED OFFICIAL:

The moment to begin again is $M=M_{fy}+N_{xx}(d-\frac{h}{2})=106\,000\,\mathrm{Nm}$

Reduced ultimate moment $\mu=\frac{M}{d^2\sigma_b}=0.178$.

The relative position of neutral fibre $\alpha=1-\sqrt{1-2\mu}=0.197$.
The arm of reduced lever $z=d(1-\frac{\alpha}{2})=0.144$.

The section of reinforcement is thus equal to $A_{SXS}=\frac{M}{z\sigma_s}-\frac{N_{xx}}{\sigma_s}=14.596\,\mathrm{cm^2/m}$ (bed X superior).

With the ELS:

The normal effort of compression is reduced because one would exceed the working stress of compression of the concrete.

The limiting values are the same ones as calculated in configuration 4.

The moment to take into account is: $M=M_{fy}+N_{xx}(d-\frac{h}{2})=101\,200\,\mathrm{Nm}$

We are thus if $M\leqslant M_{\lim}$. Thus, only of tended steels are necessary.

The reduced moment of service is equal to: $\mu=n\frac{M}{d^2\sigma_b}=0.150$

The coefficient α is solution of the equation: $\alpha^3-3\alpha^2-6\mu(1-\alpha)=0$
By iterative resolution, one obtains: $\alpha=0.442$

The steel section necessary is equal to: $A_{SXS}=\frac{M}{\sigma_s d(1-\frac{\alpha}{3})}-\frac{N_{xx}}{\sigma_s}=18.044\,\mathrm{cm^2/m}$

2.2.7 Configuration 7

The plate is subjected to one bending moment according to X (around the axis Y) equal to $M_{fy}=100\,000\,\mathrm{Nm}$ and with a tractive effort equal to $N_{xx}=100\,000\,\mathrm{N}$ according to X .

The moment to begin again is $M=M_{fy}+N_{xx}(d-\frac{h}{2})=94\,000\,\mathrm{Nm}$.

The section thus is partially tended.
Calculation is carried out here according to Eurocode 2.

With the ELECTED OFFICIAL:

Reduced ultimate moment $\mu=\frac{M}{d^2\sigma_b}=0.157$.

The relative position of neutral fibre $\alpha=1-\sqrt{1-2\mu}=0.172$.
The arm of reduced lever $z=d(1-\frac{\alpha}{2})=0.146$.

The section of reinforcement is thus equal to $A_{SXS}=\frac{M}{z\sigma_s}-\frac{N_{xx}}{\sigma_s}=17.079\,\mathrm{cm^2/m}$ (bed X superior).

With the ELS:

The limiting values are the same ones as calculated in configuration 4.
We are if $M\leqslant M_{\lim}$. Thus, only of tended steels are necessary.

The reduced moment of service is equal to: $\mu=n\frac{M}{d^2\sigma_b}=0.139$

The coefficient α is solution of the equation: $\alpha^3-3\alpha^2-6\mu(1-\alpha)=0$

By iterative resolution, one obtains: $\alpha=0.430$

The steel section necessary is equal to: $A_{SXS}=\frac{M}{\sigma_s d(1-\frac{\alpha}{3})}-\frac{N_{xx}}{\sigma_s}=19.642\,\mathrm{cm^2/m}$

2.2.8 Configuration 8

The plate is subjected to one bending moment according to X (around the axis Y) equal to $M_{fy}=100\,000\,\mathrm{Nm}$ and with a tractive effort equal to $N_{xx}=2\,000\,000\,\mathrm{N}$ according to X.

The section is completely tended $M=M_{fy}+N_{xx}(d-\frac{h}{2})=-20\,000\,\mathrm{Nm}<0$.

With the ELECTED OFFICIAL:
The section of reinforcement is thus equal to:

$A_{SXS}=\frac{M}{(d-e)\sigma_s}-\frac{N_{xx}}{\sigma_s}=42.145\,\mathrm{cm^2/m}$ (bed X superior).

$A_{SXI}=\frac{-M}{(d-e)\sigma_s}=3.831\,\mathrm{cm^2/m}$ (bed X inferior).

With the ELS:
The section of reinforcement is thus equal to:

$A_{SXS}=\frac{M}{(d-e)\sigma_s}-\frac{N_{xx}}{\sigma_s}=45.833\,\mathrm{cm^2/m}$ (bed X superior).

$A_{SXI}=\frac{-M}{(d-e)\sigma_s}=4.167\,\mathrm{cm^2/m}$ (bed X inferior).

2.2.9 Configuration 9

The plate is subjected to one bending moment according to X (around the axis Y) equal to $M_{fy}=100\,000\,\mathrm{Nm}$ and at one bending moment according to Y (around the axis X) equal to $M_{fx}=75\,000\,\mathrm{Nm}$.

The section is partially tended.

With the ELECTED OFFICIAL:
The section of reinforcement according to X is the same one as configuration 5.
That is to say $A_{SXS}=15.83\,\mathrm{cm^2/m}$.

According to Y :

Reduced ultimate moment $\mu=\frac{M_{fx}}{d^2\sigma_b}=0.125$.

The relative position of neutral fibre $\alpha=1-\sqrt{1-2\mu}=0.134$.

The arm of reduced lever $z=d(1-\frac{\alpha}{2})=0.149$.

The section of reinforcement is thus equal to $A_{SYS}=\frac{M_{fx}}{z\sigma_s}=11.555\,\mathrm{cm^2/m}$ (bed Y superior).

With the ELS:
The section of reinforcement according to X is the same one as configuration 5.

That is to say $A_{SXS}=18.31\,\mathrm{cm^2/m}$.
According to Y :
The limiting values are the same ones as calculated in configuration 4.
We are if $M\leq M_{\lim}$. Thus, only of tended steels are necessary.

The reduced moment of service is equal to: $\mu=n\frac{M}{d^2\sigma_b}=0.111$

The coefficient α is solution of the equation: $\alpha^3-3\alpha^2-6\mu(1-\alpha)=0$
By iterative resolution, one obtains: $\alpha=0.393$

The steel section necessary is equal to: $A_{SYS}=\frac{M}{\sigma_s d(1-\frac{\alpha}{3})}-\frac{N_{xx}}{\sigma_s}=13.486\,\mathrm{cm^2/m}$

2.2.10 Configuration 10

The plate is subjected to one bending moment according to Y (around the axis X) equal to $M_{fx}=-150\,000\,\mathrm{Nm}$ with the ELECTED OFFICIAL and with $M_{fx}=-100\,000\,\mathrm{Nm}$ with the ELS.

The section is partially tended.

With the ELECTED OFFICIAL:

Reduced ultimate moment $\mu=\frac{M_{fx}}{d^2\sigma_b}=0.251$.

The relative position of neutral fibre $\alpha=1-\sqrt{1-2\mu}=0.295$.
The arm of reduced lever $z=d(1-\frac{\alpha}{2})=0.136$.

One is in the case of the pivot B ($\alpha>\alpha_{AB}=\frac{PIV_B}{(PIV_B+PIV_A)}=0.259$). The deformation in the concrete is thus maximum $\varepsilon_B=PIV_B$ and the deformation in steel is $\varepsilon_A=\frac{\varepsilon_B(1-\alpha)}{\alpha}=0.0084$. One is if $\varepsilon_A>\varepsilon_y=\sigma_S/E_A=0.0021$.

The section of reinforcement is thus equal to $A_{SYI}=\frac{M_{fx}}{z\sigma_s}=25.28\,\mathrm{cm^2/m}$ (bed Y inferior).

With the ELS:
The bending moment is reduced because one would exceed the working stress of compression of the concrete.
The limiting values are the same ones as calculated in configuration 4.
We are if $M\leq M_{\lim}$. Thus, only of tended steels are necessary.
The configuration is the same one as configuration 5 but for lower steels.
The steel section necessary is thus equal to: $A_{SYI}=18.31\,\mathrm{cm^2/m}$

2.2.11 Configuration 11

The plate is subjected to one bending moment according to Y (around the axis X) equal to $M_{fx}=-260\,000\,\mathrm{Nm}$. Is calculated only to the ELECTED OFFICIAL.

The section is partially tended.

Reduced ultimate moment $\mu=\frac{M_{fx}}{d^2\sigma_b}=0.435$.

The relative position of neutral fibre $\alpha=1-\sqrt{1-2\mu}=0.642$.
The arm of reduced lever $z=d(1-\frac{\alpha}{2})=0.109$.

One is in the case of the pivot B ($\alpha>\alpha_{AB}=\frac{PIV_B}{(PIV_B+PIV_A)}=0.259$). The deformation in the concrete is thus maximum $\varepsilon_B=PIV_B$ and the deformation in steel is $\varepsilon_A=\varepsilon_B\frac{(1-\alpha)}{\alpha}=0.00195$ · One is if $\varepsilon_A<\varepsilon_y=\sigma_S/E_A=0.0021$.

The section of reinforcement is thus equal to $A_{SYI}=\frac{M_{fx}}{z\,E_A\varepsilon_A}=58.368\,\mathrm{cm^2/m}$ (bed Y inferior).

2.2.12 Configuration 12

The plate is subjected to one bending moment according to Y (around the axis X) equal to $M_{fx}=-380\,000\,\mathrm{Nm}$. Is calculated only to the ELECTED OFFICIAL.

Reduced ultimate moment $\mu=\frac{M_{fx}}{d^2\sigma_b}=0.63>0.48$. Steels are completely compressed, the steel sections are put at -1. Attention, in this case, certain facets of Capra-Maury are partially tended (see R7.04.05).

2.3 Uncertainties on the solution

None.

3 Modeling A

3.1 Characteristics of modeling

A modeling is used DKT.

3.2 Characteristics of the grid

The grid contains 1 element of the type QUAD4.

3.3 Sizes tested and results

Configuration	Absolute limit considered	Identification	Type of reference	Value of reference cm^2/m	Relative error
1	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	-1	-1
1	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	-1	-1
1	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	-1	-1
1	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	-1	-1
1	ELECTED OFFICIAL	DNST	'ANALYTICAL'	15,964	0,001
2	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	11,494	0,002
2	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	11,494	0,002
2	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
2	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	0	0
2	ELECTED OFFICIAL	DNST	'ANALYTICAL'	3,193	0,004
3	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	0	0
3	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	0	0
3	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	11,494	0,002
3	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	11,494	0,002
3	ELECTED OFFICIAL	DNST	'ANALYTICAL'	13,164	0,003
4	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	0	0

4	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	0	0
4	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
4	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	15.83	0,001
5	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	0	0
5	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	15.83	0,001
5	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
5	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	0	0
6	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	0	0
6	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	15,582	0,003
6	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
6	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	0	0
7	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	0	0
7	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	17,079	0.0004
7	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
7	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	0	0
8	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	3,831	0,011
8	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	42,146	0.0009
8	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
9	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	0	0
9	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	15.83	0,001
9	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
9	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	11,555	0,002
10	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	25.28	0,001
10	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	0	0

10	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
10	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	0	0
11	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	58,368	0,001
11	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	0	0
11	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	0	0
11	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	0	0
12	ELECTED OFFICIAL	DNSXI	'ANALYTICAL'	-1	-1
12	ELECTED OFFICIAL	DNSXS	'ANALYTICAL'	-1	-1
12	ELECTED OFFICIAL	DNSYI	'ANALYTICAL'	-1	-1
12	ELECTED OFFICIAL	DNSYS	'ANALYTICAL'	-1	-1

Configuration	Absolute limit considered	Identification	Type of reference	Value of reference cm^2/m	Relative error
1	ELS	DNSXI	'ANALYTICAL'	0	0
1	ELS	DNSXS	'ANALYTICAL'	0	0
1	ELS	DNSYI	'ANALYTICAL'	0	0
1	ELS	DNSYS	'ANALYTICAL'	0	0
2	ELS	DNSXI	'ANALYTICAL'	12.5	0
2	ELS	DNSXS	'ANALYTICAL'	12.5	0
2	ELS	DNSYI	'ANALYTICAL'	0	0
2	ELS	DNSYS	'ANALYTICAL'	0	0
3	ELS	DNSXI	'ANALYTICAL'	0	0
3	ELS	DNSXS	'ANALYTICAL'	0	0
3	ELS	DNSYI	'ANALYTICAL'	12.5	0
3	ELS	DNSYS	'ANALYTICAL'	12.5	0
4	ELS	DNSXI	'ANALYTICAL'	0	0
4	ELS	DNSXS	'ANALYTICAL'	0	0
4	ELS	DNSYI	'ANALYTICAL'	0	0
4	ELS	DNSYS	'ANALYTICAL'	18.31	0,001
5	ELS	DNSXI	'ANALYTICAL'	0	0
5	ELS	DNSXS	'ANALYTICAL'	18.31	0,001
5	ELS	DNSYI	'ANALYTICAL'	0	0

5	ELS	DNSYS	'ANALYTICAL'	0	0
6	ELS	DNSXI	'ANALYTICAL'	0	0
6	ELS	DNSXS	'ANALYTICAL'	18,044	0,002
6	ELS	DNSYI	'ANALYTICAL'	0	0
6	ELS	DNSYS	'ANALYTICAL'	0	0
7	ELS	DNSXI	'ANALYTICAL'	0	0
7	ELS	DNSXS	'ANALYTICAL'	19,642	0.0001
7	ELS	DNSYI	'ANALYTICAL'	0	0
7	ELS	DNSYS	'ANALYTICAL'	0	0
8	ELS	DNSXI	'ANALYTICAL'	4,167	0,008
8	ELS	DNSXS	'ANALYTICAL'	45,833	0.0007
8	ELS	DNSYI	'ANALYTICAL'	0	0
9	ELS	DNSXI	'ANALYTICAL'	0	0
9	ELS	DNSXS	'ANALYTICAL'	18.31	0,001
9	ELS	DNSYI	'ANALYTICAL'	0	0
9	ELS	DNSYS	'ANALYTICAL'	13,486	0,003
10	ELS	DNSXI	'ANALYTICAL'	18.31	0,001
10	ELS	DNSXS	'ANALYTICAL'	0	0
10	ELS	DNSYI	'ANALYTICAL'	0	0
10	ELS	DNSYS	'ANALYTICAL'	0	0

4 Summary of the results

This test makes it possible to highlight the validity of calculations of density of reinforcement on simple cases. The results got with the model are indeed in conformity with the given values in an analytical way.