SSLS138 – Request of a membrane

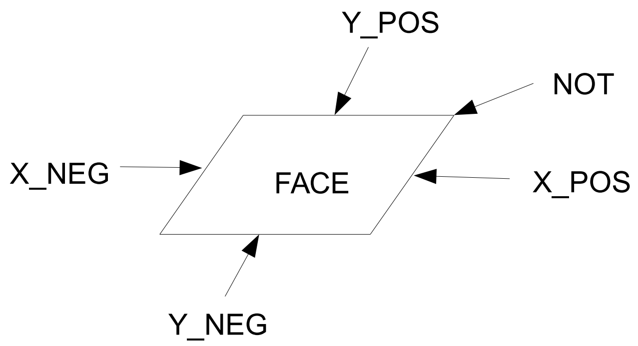
Summary:

The objective of this test is to validate the behavior of an anisotropic elastic membrane for several modes of request, as well as the calculation of the strains and the stresses in the membrane. One validates the whole of the elements of membrane available.

1 Problem of reference

1.1 Geometry

One considers a square of $1\,m$ on side, represented below:



1.2 Properties of material

The square presents an anisotropic behavior of membrane, characterized by the following coefficients (the coefficients not mentioned are worthless):

$$\begin{cases} M_{LLLL}=3 \\ M_{TTTT}=3 \\ M_{LLTT}=1 \\ M_{LTLT}=2 \end{cases}$$

These coefficients are defined in a turned reference mark of 90° around the axis (Oz) .

1.3 Boundary conditions and loadings

One carries out two calculations, corresponding to a request of traction and a request of shearing of the membrane. The conditions limits corresponding are indicated below:

- **Request of traction**

$$\begin{cases} u_Z=0 \text{ sur FACE} \\ u_X=0 \text{ sur X\_NEG} \\ u_Y=0 \text{ sur Y\_NEG} \\ F_X=1 \text{ sur X\_POS} \end{cases}$$

- **Request of shearing**

$$\begin{cases} u_Z=0 \text{ sur FACE} \\ u_Y=0 \text{ sur X\_NEG} \\ u_X=0 \text{ sur Y\_NEG} \\ u_X=u_Y \text{ sur POINT} \\ F_Y=1 \text{ sur X\_POS} \\ F_X=1 \text{ sur Y\_POS} \end{cases}$$

2 Reference solution

2.1 Method of calculating

In the two modes of request, the membrane is in a uniform state of constraint. One can thus analytically to calculate the solution of this problem in both cases.

- **Request of traction**

In the case of a simple request of traction, one shows simply that, in the total reference mark:

$$\begin{cases} \varepsilon_{XX} = F_X \dfrac{M_{LLLL}}{\left(M_{TTTT} M_{LLLL} - M_{LLTT}^2\right)} \\ \varepsilon_{YY} = -F_X \dfrac{M_{LLTT}}{\left(M_{TTTT} M_{LLLL} - M_{LLTT}^2\right)} \\ \varepsilon_{XY} = 0 \end{cases}$$

- **Request of shearing**

In the case of a request of shearing, one shows that, in the total reference mark:

$$\begin{cases} \varepsilon_{XX} = 0 \\ \varepsilon_{YY} = 0 \\ \varepsilon_{XY} = \dfrac{\sigma_{XY}}{M_{LTLT}} \end{cases}$$

2.2 Sizes and results of reference

One tests displacement, the constraint and the deformation at the top $POINT$. The sizes tested are summarized in the table below, for the two modes of request.

Size	Component	Simple traction	Shearing	Tolerance
Displacement	DX	3/8	1/2	1.E-6
	DY	-1/8	1/2	1.E-6
Membrane deformations (local reference mark)	EXX	-1/8	0	1.E-6
	EYY	3/8	0	1.E-6
	EXY	0	$\sqrt{2}/2$	1.E-6
Membrane constraints (local reference mark)	NXX	0	0	1.E-6
	NYY	1	0	1.E-6
	NXY	0	$\sqrt{2}$	1.E-6

3 Modeling A

3.1 Characteristics of the grid

Modeling A is based on a grid of 52 linear triangles (TRIA3).

4 Modeling B

4.1 Characteristics of the grid

Modeling B is based on a grid of 25 linear quadrangles (QUAD4).

5 Modeling C

5.1 Characteristics of the grid

Modeling C is based on a grid of 52 quadratic triangles (TRIA6).

6 Modeling D

6.1 Characteristics of the grid

Modeling C is based on a grid of 25 quadratic quadrangles (QUAD8).

7 Summary of the results

This test validates the behavior of an anisotropic membrane, as well as the calculation of the membrane strains and the membrane stresses.