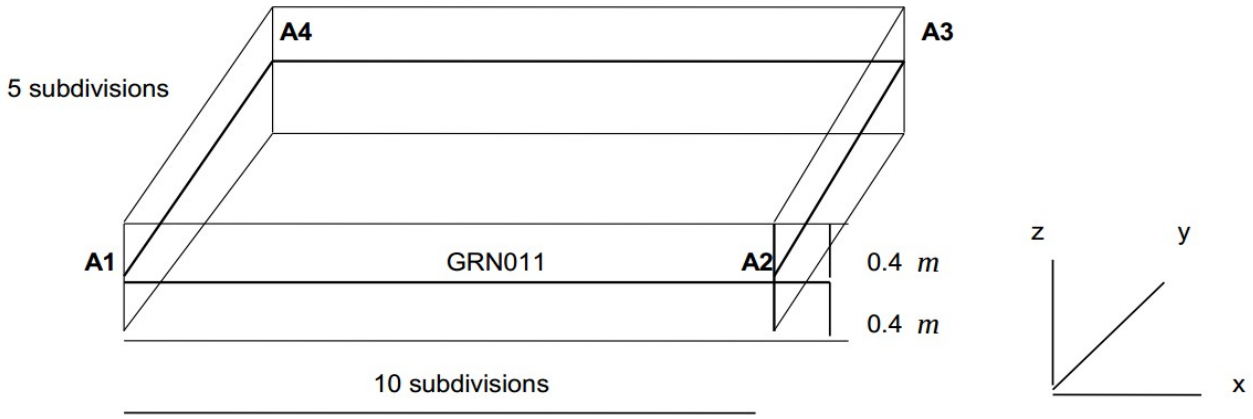
SSLS141 – Shearing and flexbeam

Summary:

The objective of this test is to validate the answer in shearing of the elements hulls DST (QUAD4) and Q4G (QUAD4).

1 Problem of reference

1.1 Geometry



The coordinates of the points are given in meters (m):

$A1(0.0,0)$	$A3(10,5)$
$A2(10.0,0)$	$A4(0,5)$

Table 1.1-1

The thickness of the plate is $e=0,8\,m$

1.2 Properties of material

The material has an isotropic elastic behavior:

Young modulus: $E=200000\,MPa$

Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

The edge $A1A4$ is embedded.
A nodal force is applied $Fz=-1000\,N$ on the edge $A2A3$.

1.4 Modelings

Various modelings are:

Modeling	Type of element	Meshs	Many elements of the grid
With	DST	QUAD4	50
B	DST	QUAD4	50

	(multi-layer, $ncou=2$)		
C	Q4G	QUAD4	800
D	Q4G (multi-layer, $ncou=2$)	QUAD4	800
E	DST	TRIA3 (regular grid)	1600
F	DST	TRIA3 (irregular grid)	1600

Table 1.4-1

2 Reference solution

2.1 Sizes and results of reference

2.1.1 Analytical model of the plate

It is considered that the answer of the plate corresponds to the answer of a beam in console (because $\nu=0$). In this case, the displacement of the plate is given by displacements of each section of plate, according to the direction $x(u)$, $z(w)$ and their rotation compared to the axis $y(\theta_y)$.

$$U_x(x,z)=u(x)+z\,\theta_y(x) \quad U_z(x,z)=w(x)$$

In any point, the deformations are:

$$\varepsilon_{xx}(x,z)=u'+z\,\theta_y' \text{ and } 2\,\varepsilon_{xz}(x,z)=\theta_y+w'=\gamma_z$$

The beam is embedded in $x=0$ and subjected to a vertical force F in $x=L$.

The resolution (principle of virtual work and the balance of the forces) give the expression of the efforts generalized in the case of an elastic linear answer:
N is the thrust load, M_y moment following the axis y and T shearing action.

$$\begin{array}{ll} N(x)=E\,A\,u'(x) & \text{and} \quad N(x)=0 \\ M_y(x)=E\,I\,\theta_y'(x) & M_y(x)=-F(L-x) \\ T(x)=G\,k\,A\,\gamma_z & T(x)=F \end{array}$$

where the parameter k is the factor of correction in shearing

with the boundary conditions, one from of deduced displacements $u, w \text{ et } \theta_y$:

$$u(x)=0\ ,\ w(x)=\frac{F}{6\,E\,I}x^2(3\,L-x)+\frac{F\,x}{G\,k\,A}\ ,\ \theta_y(x)=\frac{F}{E\,I}\frac{x(x-2\,L)}{2}$$

The strains and stresses are:

$$\varepsilon_{xx}=z\frac{-F}{E\,I}(L-x) \text{ and } \sigma_{xx}=z\frac{-F}{I}(L-x)$$

$$2\,\varepsilon_{xz}=\frac{F}{G\,k\,A}=\gamma_z \text{ and } \sigma_{xz}=\frac{-F}{A}6(z^2/h^2-1/4)$$

where h is the height of the plate. The distribution of shear stresses is parabolic in the thickness of the plate to observe the conditions of free surface
($\sigma_{xz}(z=h/2)=\sigma_{xz}(z=-h/2)=0$).

The vertical answer in displacement of the point $x=L$ is:

$$U_z(L)=\frac{F}{3\,E\,I}L^3+\frac{F\,L}{G\,k\,A}$$

2.2 Uncertainties on the solution

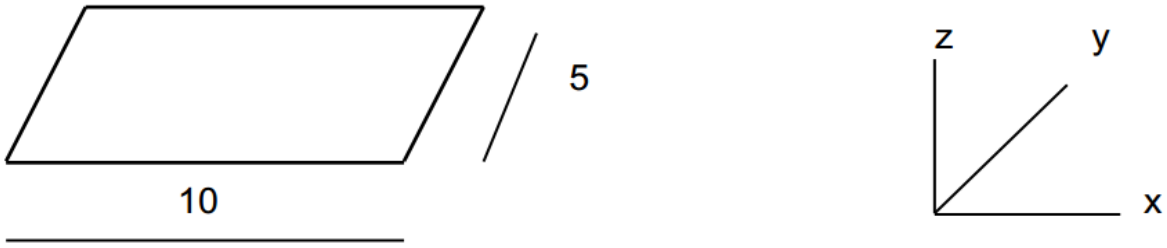
The reference solution is analytical. There is thus no uncertainty.

3 Modeling A

3.1 Characteristics of modeling

The model consists of 1 plate with a layer.
The elements used are elements of plate DST.

3.2 Characteristics of the grid



The grid is regular. There are 10 subdivisions according to x and 5 subdivisions according to y ; that is to say on the whole 50 meshs DSQ (QUAD4) and 66 nodes.

3.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
DZ ($A3$)	'ANALYTICAL'	-3.92125E-05	0,001
MXX($A1$)	'ANALYTICAL'	10000	0,001
QX($A1$)	'ANALYTICAL'	-1000	0,001
SIXX($M1, PT1, SSPT2$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M1, PT1, SSPT3$)	'ANALYTICAL'	1981.17	0,001
SIXZ($M1, PT1, SSPT2$)	'ANALYTICAL'	-1875.0	0,001
SIXZ($M1, PT1, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M50, PT2, SSPT2$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M50, PT2, SSPT3$)	'ANALYTICAL'	91768.83	0,001
SIXZ($M50, PT2, SSPT2$)	'ANALYTICAL'	-1875.0	0,001
SIXZ($M50, PT2, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
EPXX($M50, PT2, SSPT2$)	'ANALYTICAL'	0.0	1.0E-8
EPXX($M50, PT2, SSPT3$)	'ANALYTICAL'	4.58844E-07	0,001
EPXZ($M50, PT2, SSPT2$)	'ANALYTICAL'	-7.5E-09	0,001
EPXZ($M50, PT2, SSPT3$)	'ANALYTICAL'	-7.5E-09	0,001

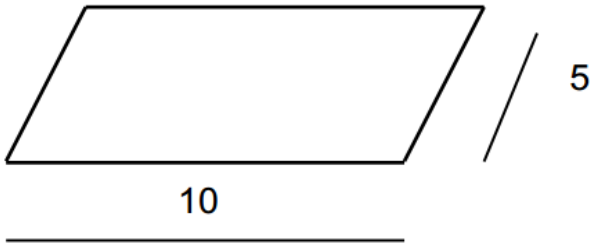
Table 3.3-1

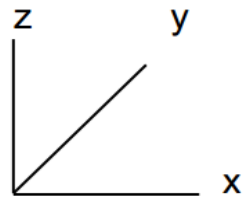
4 Modeling B

4.1 Characteristics of modeling

The model consists of 1 plate with 2 layers.
The elements used are elements of plate DST.

4.2 Characteristics of the grid





The grid is regular. There are 10 subdivisions according to x and 5 subdivisions according to y ; that is to say on the whole 50 meshs DST (QUAD4) and 66 nodes.

4.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
DZ ($A3$)	'ANALYTICAL'	-3.92125E-05	0,001
MXX($A1$)	'ANALYTICAL'	10000	0,001
QX($A1$)	'ANALYTICAL'	-1000	0,001
SIXX($M1, PT1, SSPT3$)	'ANALYTICAL'	0.	1.0E-8
SIXX($M1, PT1, SSPT6$)	'ANALYTICAL'	1981.17	0,001
SIXZ($M1, PT1, SSPT3$)	'ANALYTICAL'	-1875.0	0,001
SIXZ($M1, PT1, SSPT6$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M50, PT2, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M50, PT2, SSPT6$)	'ANALYTICAL'	91768.83	0,001
SIXZ($M50, PT2, SSPT3$)	'ANALYTICAL'	-1875.0	0,001
SIXZ($M50, PT2, SSPT6$)	'ANALYTICAL'	0.0	1.0E-8
EPXX($M50, PT2, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
EPXX($M50, PT2, SSPT6$)	'ANALYTICAL'	4.58844E-07	0,001
EPXZ($M50, PT2, SSPT3$)	'ANALYTICAL'	-7.5E-09	0,001
EPXZ($M50, PT2, SSPT6$)	'ANALYTICAL'	-7.5E-09	0,001

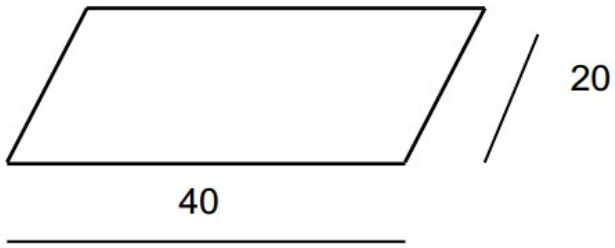
Table 4.3-1

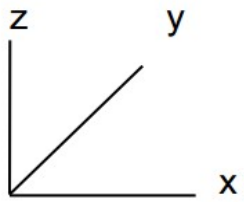
5 Modeling C

5.1 Characteristics of modeling

The model consists of 1 plate with a layer.
The elements used are elements of plate Q4G.

5.2 Characteristics of the grid





The grid is regular. There are 40 subdivisions according to x and 20 subdivisions according to y; that is to say on the whole 800 meshs Q4G (QUAD4) and 861 nodes.

5.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
DZ ($A3$)	'ANALYTICAL'	-3.92125E-05	0,001
MXX($A1$)	'ANALYTICAL'	10000	0.02
QX($A1$)	'ANALYTICAL'	-1000	0,001
SIXX($M141, PT1, SSPT2$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M141, PT1, SSPT3$)	'ANALYTICAL'	2839.04	0.3
SIXZ($M141, PT1, SSPT2$)	'ANALYTICAL'	-1875.0	0,001
SIXZ($M141, PT1, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M901, PT2, SSPT2$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M901, PT2, SSPT3$)	'ANALYTICAL'	93254.70	0.01
SIXZ($M901, PT2, SSPT2$)	'ANALYTICAL'	-1875.0	0,001
SIXZ($M901, PT2, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
EPXX($M901, PT2, SSPT2$)	'ANALYTICAL'	0.0	1.0E-8
EPXX($M901, PT2, SSPT3$)	'ANALYTICAL'	4.66273E-07	0.01
EPXZ($M901, PT2, SSPT2$)	'ANALYTICAL'	-7.5E-09	0,001
EPXZ($M901, PT2, SSPT3$)	'ANALYTICAL'	-7.5E-09	0,001

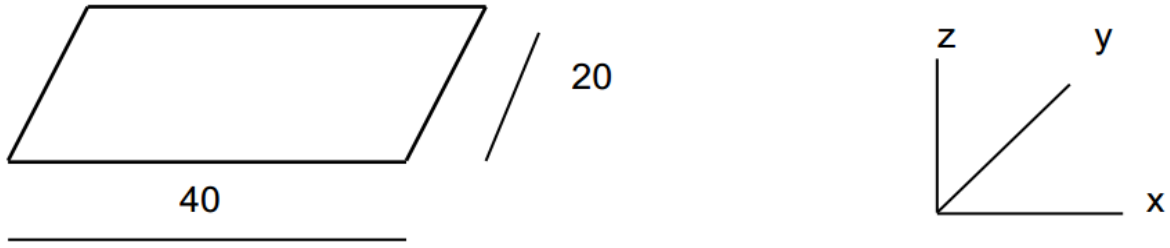
Table 5.3-1

6 Modeling D

6.1 Characteristics of modeling

The model consists of 1 plate with 2 layers.
The elements used are elements of plate Q4G.

6.2 Characteristics of the grid



The grid is regular. There are 40 subdivisions according to x and 20 subdivisions according to y; that is to say on the whole 800 meshs Q4G (QUAD4) and 861 nodes.

6.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
DZ ($A3$)	'ANALYTICAL'	-3.92125E-05	0,001
MXX($A1$)	'ANALYTICAL'	10000	0.02
QX($A1$)	'ANALYTICAL'	-1000	0,001
SIXX ($M141, PT1, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
SIXX ($M141, PT1, SSPT6$)	'ANALYTICAL'	2839.04	0.3
SIXZ($M141, PT1, SSPT3$)	'ANALYTICAL'	-1875.0	0,001
SIXZ($M141, PT1, SSPT6$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M901, PT2, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
SIXX($M901, PT2, SSPT6$)	'ANALYTICAL'	93254.70	0.01
SIXZ($M901, PT2, SSPT3$)	'ANALYTICAL'	-1875.0	0,001
SIXZ($M901, PT2, SSPT6$)	'ANALYTICAL'	0.0	1.0E-8
EPXX($M901, PT2, SSPT3$)	'ANALYTICAL'	0.0	1.0E-8
EPXX($M901, PT2, SSPT6$)	'ANALYTICAL'	4.66273E-07	0.01
EPXZ($M901, PT2, SSPT3$)	'ANALYTICAL'	-7.5E-09	0,001
EPXZ($M901, PT2, SSPT6$)	'ANALYTICAL'	-7.5E-09	0,001

Table 6.3-1

7 Modeling E

7.1 Characteristics of modeling

The model consists of 1 plate with 1 layer.
The elements used are elements of plate DST.

7.2 Characteristics of the grid

The grid is regular. There are 40 subdivisions according to x and 20 subdivisions according to y; that is to say on the whole 1600 meshs DST (TRIA3) and 861 nodes.

7.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
DZ ($A3$)	'ANALYTICAL'	-3.92125E-05	0.13
DZ ($A2$)	'ANALYTICAL'	-3.92125E-05	0.13
MXX($A4$)	'ANALYTICAL'	10000	0.33
MXX($A1$)	'ANALYTICAL'	10000	0.02

Table 7.3-1

8 Modeling F

8.1 Characteristics of modeling

The model consists of 1 plate with 1 layer.
The elements used are elements of plate DST.

8.2 Characteristics of the grid

The grid is free. There are 1600 meshs DST (TRIA3) and 853 nodes.

8.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
DZ (*A3*)	'ANALYTICAL'	-3.92125E-05	0,011
DZ (*A2*)	'ANALYTICAL'	-3.92125E-05	0,009
MXX(*A4*)	'ANALYTICAL'	10000	0,053
MXX(*A1*)	'ANALYTICAL'	10000	0,022

Table 8.3-1

9 Summary of the results

- For a plate with one or more layers, elements DST (QUAD4) give good estimates of displacements, forces generalized, deformations and forced (Modeling A and B).
- For a finer grid, elements Q4G (QUAD4) lead to errors a little more important, in particular for axial stresses SIXX (Modeling C and D).
- Two types of elements DST (QUAD4) and Q4G (QUAD4) the same results with one give respectively (Modeling A and C) or two layers (Modeling B and D).
- Elements DST (TRIA3) results much less good (Modelings E and F) give. In particular one highlights important effects of grid: modelings E and F have the same number of elements but the grid E is regulated whereas the grid F is free. The error on the arrow is of 13 % for E and 1 % for F. One notes similar variations at the time. There is also a strong dissymmetry at the time between the points $A1$ and $A4$ for the 2 grids. This is with the fact that the grids are not symmetrical compared to the loading. On a symmetrical grid the problem does not appear any more. This element thus has a behavior in inflection very depend on the grid, which undoubtedly deserves a thorough study.