SSLS505 – Composite plate subjected to mechanical deformations of purely thermal origin

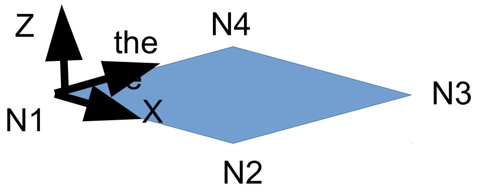
Summary:

The objective of this test is analytically to validate the mechanical deformation of thermal origin of the composite plates on a grid made up of an element quadrangle to 4 nodes.

MODELING a: plates homogeneous DKT with an orthotropic thermoelastic behavior via DEFI_COMPOSITE.

1 Problem of reference

1.1 Geometry



One considers a square of with dimensions 1mm .

1.2 Properties of material

The material is elastic orthotropic (defined via DEFI_COMPOSITE). The orthotropism is of origin thermal i.e. the longitudinal, transverse and normal properties of dilation are :

- $E_L = E_T = E_N = E = 100\text{E}9\,;\nu_{LT} = \nu_{LN} = \nu_{TN} = \nu = 0{,}3\,;G_{LT} = G_{LN} = G_{TN} = E/2(1+\nu)$
- $\alpha_{LL} = 1.\text{E}-5\,;\alpha_{TT} = 1.\text{E}-6\,;$

1.3 Boundary conditions and loadings

Blocking :
The condtions in extreme cases are such as only thermal the membrane effect of origin is authorized. Moreover, the plate is blocked so that the effects of shearing plan are negligible in front of the effects of longitudinal and transverse deflections:

- Points N1, N2, N3, N4 are such as: DZ=0.0 : what involves a worthless inflection,
- Points N1, N2 are blocked:DY=0.0 what makes negligible shearing plan,
- Points N1 is such as: DX=0 this qu embeds this point completely,
- Nodes N3, N4 are free of any movement in the plan.

Loading :
No mechanical loading is applied. The consequence is that the purely elastic deformations are worthless. One applies to all the plate a differential of loading:

$T - T_{ref} = \Delta T = 1°C$

1.4 Initial conditions

Nothing

2 Reference solution

2.1 Method of calculating

To determine the analytical solution, one bases oneself on the equation of the total deflection of a plate with taking into account of thermal dilation:

$$\varepsilon(u)=\begin{pmatrix}E_{11}\\E_{22}\\E_{12}\end{pmatrix}+x_3\begin{pmatrix}K_{11}\\K_{22}\\K_{12}\end{pmatrix}+\varepsilon^{th}\text{ et }\varepsilon^{th}=\begin{pmatrix}d_{11}\\d_{22}\\d_{12}\end{pmatrix}.\left(T(x_3)-T^{réf}\right)$$

With:

$E_{ij}=\dfrac{\partial U_i}{\partial X_j};K_{ij}=\dfrac{\partial\beta_i}{\partial X_j};i,j=1,2$ U_i being displacement in direction I and X_i the coordinate according to I. E_{ij} is the pure membrane deformation while K_{ij} represent the curve. The boundary conditions are such as $E_{ij}=K_{ij}=0$.
and

$$\boldsymbol{d}^{(m)}=\begin{pmatrix}d_{11}\\d_{22}\\d_{12}\end{pmatrix}=\boldsymbol{P}^{m^{-1}}\begin{pmatrix}\alpha_{LL}\\\alpha_{TT}\\0\end{pmatrix}_{(L,T)}=\begin{pmatrix}C^2 & S^2\\S^2 & C^2\\2CS & -2CS\end{pmatrix}\begin{pmatrix}\alpha_{LL}\\\alpha_{TT}\end{pmatrix}_{(L,T)}$$

$$\boldsymbol{P}^{(m)}=\begin{bmatrix}C^2 & S^2 & 2CS\\S^2 & C^2 & -2CS\\-CS & CS & C^2-S^2\end{bmatrix}\text{ with }\begin{matrix}C=1\\S=0\end{matrix}$$

With final, the expression of the deformation analytically is determined:

$$\varepsilon(u)=\varepsilon^{th}=\begin{pmatrix}\alpha_{LL}\\\alpha_{TT}\\0\end{pmatrix}.1$$

2.2 Sizes and results of reference

The size is tested `EPSI_ELGA` who represents the value of the deformation in a point of interior gauss to the plate. It tested on the mesh `M1`, not `1` and at the under-point `1`.

2.3 Uncertainties on the solution

No uncertainty on the analytical reference solution because it.

2.4 Bibliographical references

1 Theoretical documentation R4.01.01, *Pre and Postprocessing for the thin hulls out of composite materials*.

2 G. DHATT, G. TOUZOT, "Modeling of the structure finite elements", volume 2: beams and plates page 238-240, Hermès Paris, 1990.

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3 Modeling A

3.1 Characteristics of modeling

A modeling is used `DKT`.

3.2 Characteristics of the grid

The grid contains 1 elements of the type `QUAD4`.

3.3 Sizes tested and results

Identification	Type of reference	Value of reference	Tolerance
Under-point 1 M1- *EPXX*	'ANALYTICAL'	1.E-5	1, E-6
Under-point 1 M1- *EPYY*	'ANALYTICAL'	1.E-6	1, E-6

3.4 Remarks

The figure below watch that deformations according to X and Y are quite homogeneous on all the points of Gauss of the plate and that shearing plan is quasi-no one. Moreover, the deformation of inflection is worthless. It is well the expected result which validates the thermal dilation of the composite DKT.

MAILLE	POINT	SOUS_POINT	EPXX	EPYY	EPZZ	EPXY	EPXZ	EPYZ
M1	1	1	1.000000000E-05	1.000000000E-06	0.000000000E+00	3.970466940E-23	0.000000000E+00	0.000000000E+00
M1	1	2	1.000000000E-05	1.000000000E-06	0.000000000E+00	3.970466940E-23	0.000000000E+00	0.000000000E+00
M1	1	3	1.000000000E-05	1.000000000E-06	0.000000000E+00	3.970466940E-23	0.000000000E+00	0.000000000E+00
M1	2	1	1.000000000E-05	1.000000000E-06	0.000000000E+00	6.617444900E-23	0.000000000E+00	0.000000000E+00
M1	2	2	1.000000000E-05	1.000000000E-06	0.000000000E+00	6.617444900E-23	0.000000000E+00	0.000000000E+00
M1	2	3	1.000000000E-05	1.000000000E-06	0.000000000E+00	6.617444900E-23	0.000000000E+00	0.000000000E+00
M1	3	1	1.000000000E-05	1.000000000E-06	0.000000000E+00	2.646977960E-22	0.000000000E+00	0.000000000E+00
M1	3	2	1.000000000E-05	1.000000000E-06	0.000000000E+00	2.646977960E-22	0.000000000E+00	0.000000000E+00
M1	3	3	1.000000000E-05	1.000000000E-06	0.000000000E+00	2.646977960E-22	0.000000000E+00	0.000000000E+00
M1	4	1	1.000000000E-05	1.000000000E-06	0.000000000E+00	2.646977960E-22	0.000000000E+00	0.000000000E+00
M1	4	2	1.000000000E-05	1.000000000E-06	0.000000000E+00	2.646977960E-22	0.000000000E+00	0.000000000E+00
M1	4	3	1.000000000E-05	1.000000000E-06	0.000000000E+00	2.646977960E-22	0.000000000E+00	0.000000000E+00

4 Summary of the results

This test makes it possible to check that the thermal dilation of an orthotropic plate modelled with DKT is correctly calculated (with worthless elastic strain) via DEFI_COMPOSITE.

Exactly the expected analytical result is found.