SSLV100 - Hollow roll in plane deformations

Summary:

This test makes it possible to validate the elements of plane deformation on the following features:

- pressure distributed,
- matrix of rigidity,
- imposed displacements:
  - by Degré of freedom,
  - by face of element.

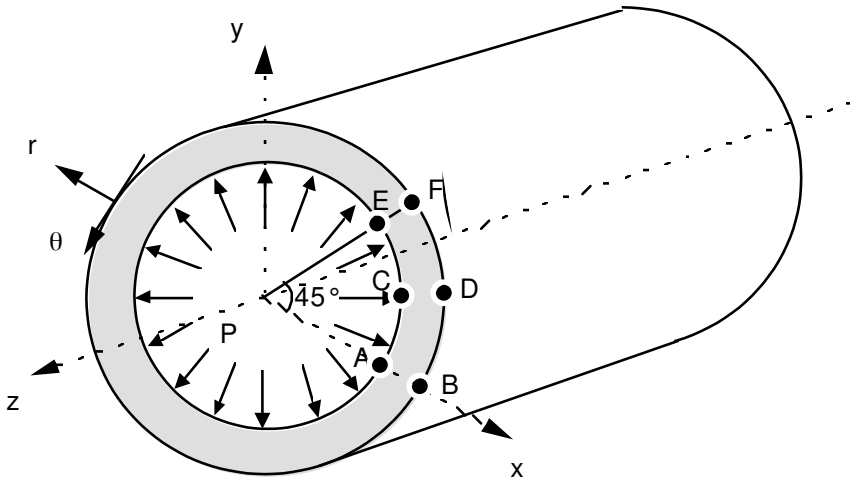
It understands 4 modelings.

The 3 first correspond to elements of the typeS differentS (linear and quadratic).

The last validates the displacements imposed by face (blocking of the normal component).

1 Problem of reference

1.1 Geometry



Rayon interne a = 0.1 m
Rayon externe b = 0.2 m

Coordinates of the points:

	A	B	C	D	E	F
x	0.100	0.200	$0.1\cos(22.5)$	$0.2\cos(22.5)$	$1/\sqrt{2}$	$\sqrt{2}$
y	0.	0.	$0.1\sin(22.5)$	$0.2\sin(22.5)$	$1/\sqrt{2}$	$\sqrt{2}$
z	0	0.	0.	0.	0.	0.

1.2 Properties of materials

$E = 2\,10^5\ Mpa$

$\nu = 0.3$

1.3 Boundary conditions and loadings

Internal pressure: $P = 60.\ MPa$

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical

$$\begin{aligned}
\sigma_{zz} &= 2\nu P\frac{a^2}{b^2-a^2}\\
\sigma_{rr} &= P\frac{a^2}{b^2-a^2}\left[1-\frac{b^2}{r^2}\right]\\
\sigma_{\theta\theta} &= P\frac{a^2}{b^2-a^2}\left[1+\frac{b^2}{r^2}\right]\\
\sigma_{r\theta} &= 0\\
u_r &= \frac{P}{E}\frac{a^2}{b^2-a^2}(1+\nu)\left[(1-2\nu)+\frac{b^2}{r^2}\right]r
\end{aligned}$$

One obtains:

For $r=0.1$ $u_r=5{,}72\,10^{-5}$ For $r=0.2$ $u_r=3{,}64\,10^{-5}$

$\sigma_{rr}=-60.$ $\sigma_{rr}=0.$

$\sigma_{\theta\theta}=100.$ $\sigma_{\theta\theta}=40.$

$\sigma_{zz}=12.$ $\sigma_{zz}=12.$

$\sigma_{r\theta}=0.$ $\sigma_{r\theta}=0.$

Passage in the system of Cartesian axes:

$$\begin{aligned}
\sigma_{xx} &= \sigma_{rr}\cos^2\theta+\sigma_{\theta\theta}\sin^2\theta-2\sigma_{r\theta}\sin\theta\cos\theta\\
\sigma_{yy} &= \sigma_{rr}\sin^2\theta+\sigma_{\theta\theta}\cos^2\theta+2\sigma_{r\theta}\sin\theta\cos\theta\\
\sigma_{xy} &= \sigma_{rr}\sin\theta\cos\theta-\sigma_{\theta\theta}\sin\theta\cos\theta-2\sigma_{r\theta}(\cos^2\theta-\sin^2\theta)
\end{aligned}$$

with:

- $\theta=0°$ at the points A and B ,
- $\theta=22.5°$ at the points C and D ,
- $\theta=45°$ at the points E and F .

2.2 Results of reference

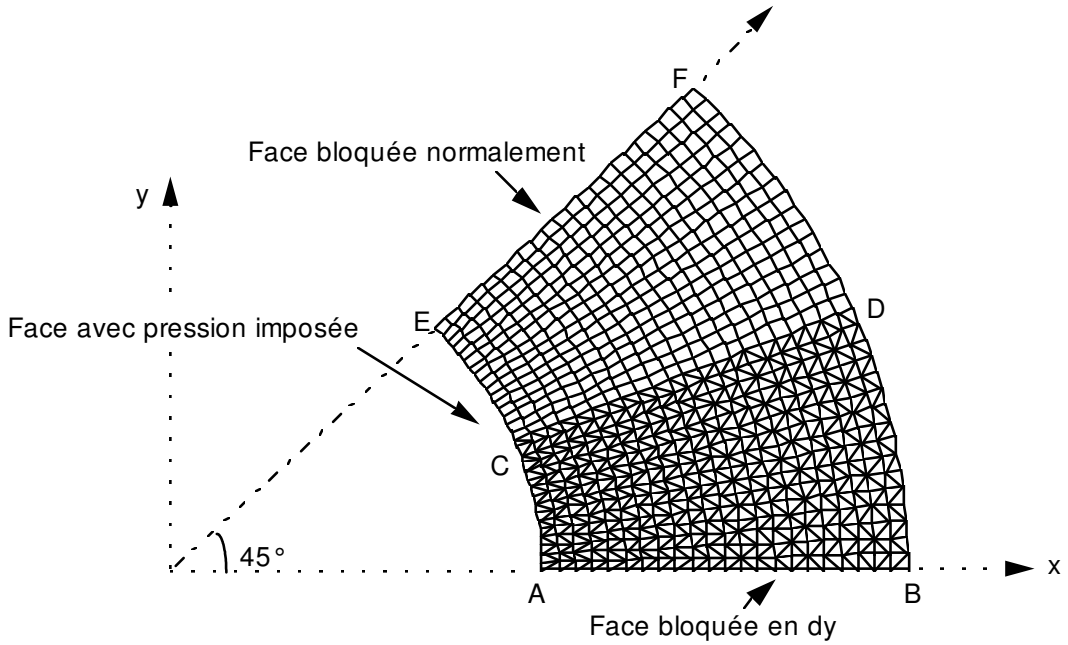
Displacements (u,v) and constraints $(\sigma_{xx},\sigma_{yy},\sigma_{zz},\sigma_{xy})$ at the points A,B,C,D,E,F .

2.3 Bibliographical references

[1] Y.C. FUNG. Fundations of solid mechanics. Prentice-hall, Inc. Englewood Cliffs. NJ. 1965 p. 243 to 245.

3 Modeling A

3.1 Characteristics of modeling: D-plan (QUAD4 + TRIA3)



Limiting conditions:

side AB	DDL_IMPO = (GROUP_NO = bordAB DY = 0.)
side EF	FACE_IMPO = (GROUP_MA = faceEF DNOR = 0.)
pressure on the face AE	PRES_REP = (GROUP_MA = faceAE CLOSE = 60.)

Names of the nodes: $A = N23$ $B = N1$ $C = N391$
$D = N369$ $E = N451$ $F = 751$

3.2 Characteristics of the grid

Many nodes: 759

Many meshs and types: 704 TRIA3, 352 QUAD4

3.3 Sizes tested and results

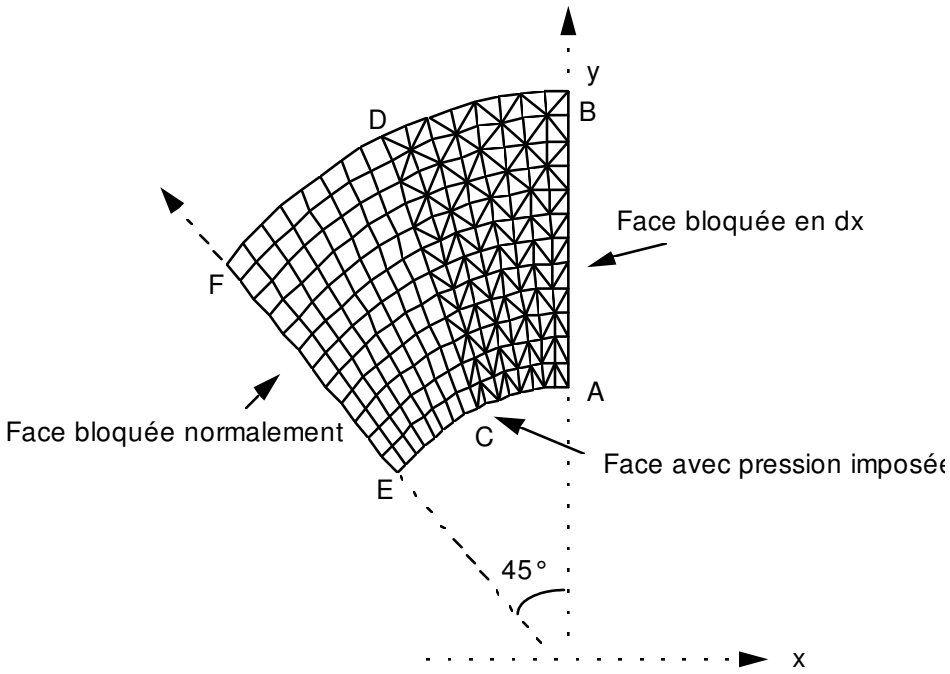
Localization	Size	Valor of Référence	Type of reference	Tolerance
not A				
	CHhasmp DEPL, comp. X	5.72 10^{-5}	'ANALYTICAL'	1 %
	Field DEPL, comp. Y	0.	'ANALYTICAL'	1E-10 (absolute)
	Field SIGM_NOEU, comp. $SIXX$	– 60.	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIYY$	100.	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIZZ$	12.	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIXY$	0.	'ANALYTICAL'	2 (absolute)
B				
	Field DEPL, comp. X	3.64 10^{-5}	'ANALYTICAL'	1 %
	Field DEPL, comp. Y	0.	'ANALYTICAL'	1E-10 (absolute)
	Field SIGM_NOEU, comp. $SIXX$	0.	'ANALYTICAL'	2 (absolute)
	Field SIGM_NOEU, comp. $SIYY$	40.	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIZZ$	12.	'ANALYTICAL'	8 %
	Field SIGM_NOEU, comp. $SIXY$	0.	'ANALYTICAL'	1 (absolute)
C				
	Field DEPL, comp. X	5.28459 10^{-5}	'ANALYTICAL'	1 %
	Field DEPL, comp. Y	2.18895 10^{-5}	'ANALYTICAL'	1 %
	Field SIGM_NOEU, comp. $SIXX$	-36.56854	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIYY$	76.56854	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIZZ$	12.	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIXY$	-56.56854	'ANALYTICAL'	5 %
D				
	Field DEPL, comp. X	3.36292 10^{-5}	'ANALYTICAL'	1 %
	Field DEPL, comp. Y	1.39297 10^{-5}	'ANALYTICAL'	1 %
	Field SIGM_NOEU, comp. $SIXX$	5.85786	'ANALYTICAL'	20 %
	Field SIGM_NOEU, comp. $SIYY$	34.14214	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIZZ$	12.	'ANALYTICAL'	1 %
	Field SIGM_NOEU, comp. $SIXY$	-14.14213	'ANALYTICAL'	8 %
E				
	Field DEPL, comp. X	4.04465 10^{-5}	'ANALYTICAL'	1 %
	Field DEPL, comp. Y	4.04465 10^{-5}	'ANALYTICAL'	1 %
	Field SIGM_NOEU, comp. $SIXX$	20.	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIYY$	20.	'ANALYTICAL'	6 %
	Field SIGM_NOEU, comp. $SIZZ$	12.	'ANALYTICAL'	1 %
	Field SIGM_NOEU, comp. $SIXY$	-80.	'ANALYTICAL'	5 %
F			'ANALYTICAL'	
	Field DEPL, comp. X	2.57387 10^{-5}	'ANALYTICAL'	1 %
	Field DEPL, comp. Y	2.57387 10^{-5}	'ANALYTICAL'	1 %
	Field SIGM_NOEU, comp. $SIXX$	20.	'ANALYTICAL'	7 %
	Field SIGM_NOEU, comp. $SIYY$	20.	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIZZ$	12.	'ANALYTICAL'	5 %
	Field SIGM_NOEU, comp. $SIXY$	-20.	'ANALYTICAL'	5 %

3.4 Remarks

Increase in the error, when one passes from AB with CD then EF , is ascribable with the grid (density in elements QUAD4 lower than that in TRIA3).

4 Modeling B

4.1 Characteristics of modeling: D-plan (QUAD8 + TRIA6)



Limiting conditions:

side AB	DDL_IMPO = (GROUP_NO = bordAB DY = 0.)
side EF	FACE_IMPO = (GROUP_MA = faceEF DNOR = 0.)
pressure on AE	PRES_REP = (GROUP_MA = faceAE CLOSE = 60.)

Names of the nodes: $A = N2$ $B = N48$ $C = N401$
$D = N424$ $E = N606$ $F = N494$

4.2 Characteristics of the grid

Many nodes: 729

Many meshs and types: 192 TRIA6, 96 QUAD8

4.3 Sizes tested and results

Localization	Size	Valor of Référence	Type of reference	Tolerance
not A				
-	CHhasmp DEPL, comp. Y	5.72 10^{-5}	'ANALYTICAL'	1 %
-	Field DEPL, comp. X	0.	'ANALYTICAL'	1E-10 (absolute)
M1 mesh	Field SIGM_ELNO, comp. $SIXX$	100.	'ANALYTICAL'	1 %
Mesh M2	Field SIGM_ELNO, comp. $SIXX$	100.	'ANALYTICAL'	1 %
M1 mesh	Field SIGM_ELNO, comp. $SIYY$	-60.	'ANALYTICAL'	1 %
Mesh M2	Field SIGM_ELNO, comp. $SIYY$	-60.	'ANALYTICAL'	1 %
M1 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	2 %
Mesh M2	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	2 %
M1 mesh	Field SIGM_ELNO, comp. $SIXY$	0.	'ANALYTICAL'	0.5 (absolute)
Mesh M2	Field SIGM_ELNO, comp. $SIXY$	0.	'ANALYTICAL'	0.5 (absolute)
B				
-	CHhasmp DEPL, comp. Y	3.64 10^{-5}	'ANALYTICAL'	1 %
-	Field DEPL, comp. X	0.	'ANALYTICAL'	1E-10 (absolute)
M23 mesh	Field SIGM_ELNO, comp. $SIXX$	40.	'ANALYTICAL'	1 %
M24 mesh	Field SIGM_ELNO, comp. $SIXX$	40.	'ANALYTICAL'	1 %
M23 mesh	Field SIGM_ELNO, comp. $SIYY$	0.	'ANALYTICAL'	0.5 (absolute)
M24 mesh	Field SIGM_ELNO, comp. $SIYY$	0.	'ANALYTICAL'	0.5 (absolute)
M23 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	1 %
M24 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	1 %
M23 mesh	Field SIGM_ELNO, comp. $SIXY$	0.	'ANALYTICAL'	0.5 (absolute)
M24 mesh	Field SIGM_ELNO, comp. $SIXY$	0.	'ANALYTICAL'	0.5 (absolute)
C				
	CHhasmp DEPL, comp. Y	5.28459 10^{-5}	'ANALYTICAL'	1 %
	Field DEPL, comp. X	2.18895 10^{-5}	'ANALYTICAL'	1 %
Send a M169 e-mail	Field SIGM_ELNO, comp. $SIXX$	76.56854	'ANALYTICAL'	1 %
Send a M170 e-mail	Field SIGM_ELNO, comp. $SIXX$	76.56854	'ANALYTICAL'	1 %
Send a M193 e-mail	Field SIGM_ELNO, comp. $SIXX$	76.56854	'ANALYTICAL'	1 %
Send a M169 e-mail	Field SIGM_ELNO, comp. $SIYY$	-36.56854	'ANALYTICAL'	2 %
Send a M170 e-mail	Field SIGM_ELNO, comp. $SIYY$	-36.56854	'ANALYTICAL'	2 %
Send a M193 e-mail	Field SIGM_ELNO, comp. $SIYY$	-36.56854	'ANALYTICAL'	2 %
Send a M169 e-mail	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	5 %
Send a M170 e-mail	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	5 %
Send a M193 e-mail	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	5 %
Send a M169 e-mail	Field SIGM_ELNO, comp. $SIXY$	56.56854	'ANALYTICAL'	1 %

Send a M170 e-mail	Field SIGM_ELNO, comp.	$SIXY$	56.56854	'ANALYTICAL'	1 %
Send a M193 e-mail	Field SIGM_ELNO, comp.	$SIXY$	56.56854	'ANALYTICAL'	1 %
D					
-	CHhasmp DEPL, comp.	Y	$3.36292\ 10^{-5}$	'ANALYTICAL'	1 %
-	Field DEPL, comp.	X	$1.39297\ 10^{-5}$	'ANALYTICAL'	1 %
M190 mesh	Field SIGM_ELNO, comp.	$SIXX$	34.14214	'ANALYTICAL'	1 %
M192 mesh	Field SIGM_ELNO, comp.	$SIXX$	34.14214	'ANALYTICAL'	1 %
M204 mesh	Field SIGM_ELNO, comp.	$SIXX$	34.14214	'ANALYTICAL'	1 %
M190 mesh	Field SIGM_ELNO, comp.	$SIYY$	5.85786	'ANALYTICAL'	5 %
M192 mesh	Field SIGM_ELNO, comp.	$SIYY$	5.85786	'ANALYTICAL'	5 %
M204 mesh	Field SIGM_ELNO, comp.	$SIYY$	5.85786	'ANALYTICAL'	5 %
M190 mesh	Field SIGM_ELNO, comp.	$SIZZ$	12.	'ANALYTICAL'	1 %
M192 mesh	Field SIGM_ELNO, comp.	$SIZZ$	12.	'ANALYTICAL'	1 %
M204 mesh	Field SIGM_ELNO, comp.	$SIZZ$	12.	'ANALYTICAL'	1 %
M190 mesh	Field SIGM_ELNO, comp.	$SIXY$	14.14213	'ANALYTICAL'	1 %
M192 mesh	Field SIGM_ELNO, comp.	$SIXY$	14.14213	'ANALYTICAL'	1 %
M204 mesh	Field SIGM_ELNO, comp.	$SIXY$	14.14213	'ANALYTICAL'	1 %
E					
-	CHhasmp DEPL, comp.	Y	$4.04465\ 10^{-5}$	'ANALYTICAL'	1 %
-	Field DEPL, comp.	X	$-4.04465\ 10^{-5}$	'ANALYTICAL'	1 %
M256 mesh	Field SIGM_ELNO, comp.	$SIXX$	20.	'ANALYTICAL'	5 %
M256 mesh	Field SIGM_ELNO, comp.	$SIYY$	20.	'ANALYTICAL'	5 %
M256 mesh	Field SIGM_ELNO, comp.	$SIZZ$	12.	'ANALYTICAL'	5 %
M256 mesh	Field SIGM_ELNO, comp.	$SIXY$	80.	'ANALYTICAL'	1 %
F				'ANALYTICAL'	
-	CHhasmp DEPL, comp.	Y	$2.57387\ 10^{-5}$	'ANALYTICAL'	1 %
-	Field DEPL, comp.	X	$-2.57387\ 10^{-5}$	'ANALYTICAL'	1 %
M222 mesh	Field SIGM_ELNO, comp.	$SIXX$	20.	'ANALYTICAL'	1 %
M222 mesh	Field SIGM_ELNO, comp.	$SIYY$	20.	'ANALYTICAL'	1 %
M222 mesh	Field SIGM_ELNO, comp.	$SIZZ$	12.	'ANALYTICAL'	1 %
M222 mesh	Field SIGM_ELNO, comp.	$SIXY$	20.	'ANALYTICAL'	1 %

4.4 Remarks

Evolution of the error induced by the following grid AB , CD or EF , is clearly attenuated compared to modeling A.

5 Modeling C

5.1 Characteristics of modeling: D-plan (QUAD9)

Limiting conditions:

side AB	DDL_IMPO =	(GROUP_NO = bordAB	DY = 0.)
side EF	FACE_IMPO =	(GROUP_MA = faceEF	DNOR = 0.)
pressure on AE	PRES_REP =	(GROUP_MA = faceAE	CLOSE = 60.)

Names of the nodes: $A = N1$ $B = N47$ $C = N351$
$D = N374$ $E = N569$ $F = N423$

5.2 Characteristics of the grid

Many nodes: 725

Many meshs and types: 168 QUAD9

5.3 Sizes tested and results

Localization	Size	Valor of Référence	Type of reference	Tolerance
not A				
-	CHhasmp DEPL, comp. X	5.72 10^{-5}	'ANALYTICAL'	1 %
-	Field DEPL, comp. Y	0.	'ANALYTICAL'	1E-10 (absolute)
M1 mesh	Field SIGM_ELNO, comp. $SIXX$	– 60.	'ANALYTICAL'	1 %
M1 mesh	Field SIGM_ELNO, comp. $SIYY$	100.	'ANALYTICAL'	1 %
M1 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	5 %
M1 mesh	Field SIGM_ELNO, comp. $SIXY$	0.	'ANALYTICAL'	1E-2 (absolute)
B				
-	Field DEPL, comp. X	3.64 10^{-5}	'ANALYTICAL'	1 %
-	Field DEPL, comp. Y	0.	'ANALYTICAL'	1E-10 (absolute)
M12 mesh	Field SIGM_ELNO, comp. $SIXX$	0.	'ANALYTICAL'	0.1 (absolute)
M12 mesh	Field SIGM_ELNO, comp. $SIYY$	40.	'ANALYTICAL'	1 %
M12 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	1 %
M12 mesh	Field SIGM_ELNO, comp. $SIXY$	0.	'ANALYTICAL'	1E-2 (absolute)
C				
-	Field DEPL, comp. X	5.28459 10^{-5}	'ANALYTICAL'	1 %
-	Field DEPL, comp. Y	2.18895 10^{-5}	'ANALYTICAL'	1 %
M73 mesh	Field SIGM_ELNO, comp. $SIXX$	-36.56854	'ANALYTICAL'	2 %
M85 mesh	Field SIGM_ELNO, comp. $SIXX$	-36.56854	'ANALYTICAL'	2 %
M73 mesh	Field SIGM_ELNO, comp. $SIYY$	76.56854	'ANALYTICAL'	1 %
M85 mesh	Field SIGM_ELNO, comp. $SIYY$	76.56854	'ANALYTICAL'	1 %
M73 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	3 %
M85 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	3 %
M73 mesh	Field SIGM_ELNO, comp. $SIXY$	-56.56854	'ANALYTICAL'	1 %
M85 mesh	Field SIGM_ELNO, comp. $SIXY$	-56.56854	'ANALYTICAL'	1 %
D				
-	Field DEPL, comp. X	3.36292 10^{-5}	'ANALYTICAL'	1 %
-	Field DEPL, comp. Y	1.39297 10^{-5}	'ANALYTICAL'	1 %
M84 mesh	Field SIGM_ELNO, comp. $SIXX$	5.85786	'ANALYTICAL'	2 %
M96 mesh	Field SIGM_ELNO, comp. $SIXX$	5.85786	'ANALYTICAL'	2 %
M84 mesh	Field SIGM_ELNO, comp. $SIYY$	34.14214	'ANALYTICAL'	1 %
M96 mesh	Field SIGM_ELNO, comp. $SIYY$	34.14214	'ANALYTICAL'	1 %
M84 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	1 %
M96 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	1 %
M84 mesh	Field SIGM_ELNO, comp. $SIXY$	-14.14213	'ANALYTICAL'	1 %
M96 mesh	Field SIGM_ELNO, comp. $SIXY$	-14.14213	'ANALYTICAL'	1 %
E				
-	Field DEPL, comp. X	4.04465 10^{-5}	'ANALYTICAL'	1 %
-	Field DEPL, comp. Y	4.04465 10^{-5}	'ANALYTICAL'	1 %
M136 mesh	Field SIGM_ELNO, comp. $SIXX$	20.	'ANALYTICAL'	3 %
M136 mesh	Field SIGM_ELNO, comp. $SIYY$	20.	'ANALYTICAL'	3 %
M136 mesh	Field SIGM_ELNO, comp. $SIZZ$	12.	'ANALYTICAL'	3 %
M136 mesh	Field SIGM_ELNO, comp. $SIXY$	-80.	'ANALYTICAL'	1 %
F			'ANALYTICAL'	
-	Field DEPL, comp. X	2.57387 10^{-5}	'ANALYTICAL'	1 %

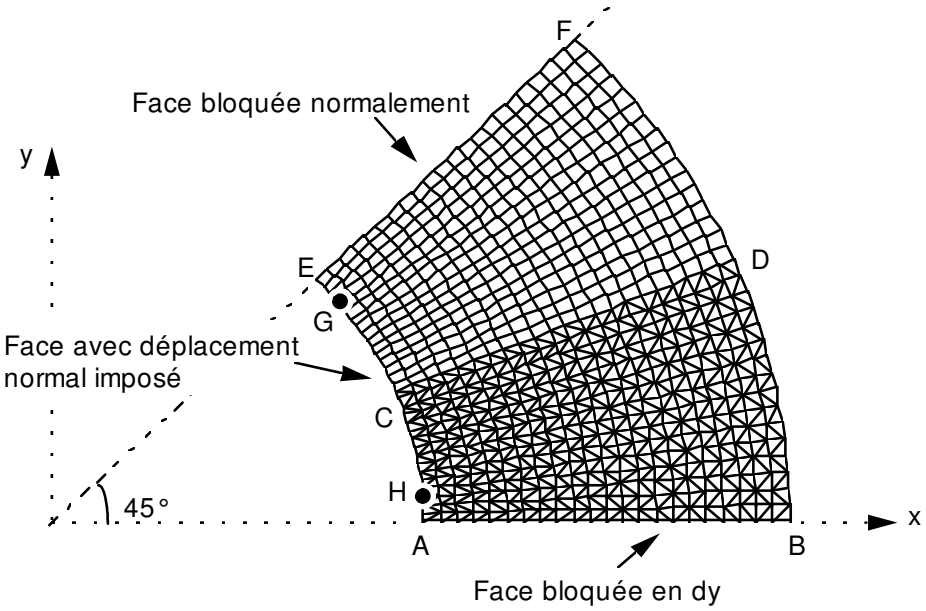
-	Field DEPL, comp.	Y	$2.57387\ 10^{-5}$	'ANALYTICAL'	1 %
M102 mesh	Field SIGM_ELNO, comp.	$SIXX$	20.	'ANALYTICAL'	1 %
M102 mesh	Field SIGM_ELNO, comp.	$SIYY$	20.	'ANALYTICAL'	1 %
M102 mesh	Field SIGM_ELNO, comp.	$SIZZ$	12.	'ANALYTICAL'	1 %
M102 mesh	Field SIGM_ELNO, comp.	$SIXY$	-20.	'ANALYTICAL'	1 %

5.4 Remarks

Evolution of the error induced by the following grid AB , CD or EF , is clearly attenuated compared to modeling A.

6 Modeling D

6.1 Characteristics of modeling: D-plan (QUAD4 + TRIA3)



Limiting conditions:

side AB D DL_IMPO: (GROUP_NO = bordAB DY = 0.)

side EF FACE_IMPO: (GROUP_MA = faceEF DNOR = 0.)

on normal displacement imposed on $5.72E-5m$

FACE_IMPO: (GROUP_MA = faceAE DNOR = -5.72 E-5)

Names of the nodes:

$A=N23$ $B=N1$ $C=N391$
$D=N369$ $E=N451$ $F=N751$
$H=N92$ $G=N447$

6.2 Characteristics of the grid

Many nodes: 759

Many meshs and types: 704 TRIA3, 352 QUAD4

6.3 Sizes tested and results

Localization	Size	Value of Référence	Type of reference	Tolerance
C	Field REAC_NODA, comp FX	0.1360	'NON_DEFINI'	3.5 %
	Field REAC_NODA , comp F Y	0,056	'NON_DEFINI'	4.1 %
H	Field REAC_NODA, comp FX	0.14686	'NON_DEFINI'	7.4 %
	Field REAC_NODA , comp F Y	0.0108	'NON_DEFINI'	7.1 %
G	Field REAC_NODA, comp FX	0.1138	'NON_DEFINI'	0.3 %
	Field REAC_NODA , comp F Y	0,093	'NON_DEFINI'	0.7 %

6.4 Remarks

One checks that the nodal forces of reaction are worthless in all the nodes, except on the nodes of the line AE , EF and AB .

7 Summary of the results

D_plan

		modeling		
Summary of the errors max in %		**With**	**B**	**C**
Displacements	WITH, B	0.08	0.04	0.05
	C, D	0.51	0.04	0.05
	E, F	0.11	0.04	0.05
Constraints σ_{xx}	WITH, B	6.04	0.29	0.27
	C, D	10.84	0.17	0.32
	E, F	17.46	4.10^{-4}	2.10^{-4}
Constraints σ_{yy}	WITH, B	3.61	0.38	0.16
	C, D	0.72	0.63	0.14
	E, F	27.07	2.10^{-5}	$5.5.10^{-4}$
Constraints σ_{zz}	WITH, B	1.33	0.16	0.02
	C, D	8.51	0.63	0.02
	E, F	22.27	2.10^{-4}	2.10^{-4}
Constraints σ_{xy}	WITH, B	-	-	-
	C, D	4.99	0.50	0.2
	E, F	2.11	0.23	0.2

These 3 modelings appreciably have the same number of nodes; the results got with elements of order 1 (modeling A in TRIA3 and QUAD4) are definitely less precise, in particular on the internal wall.