SSLV104 - Beam in rotation

Summary:

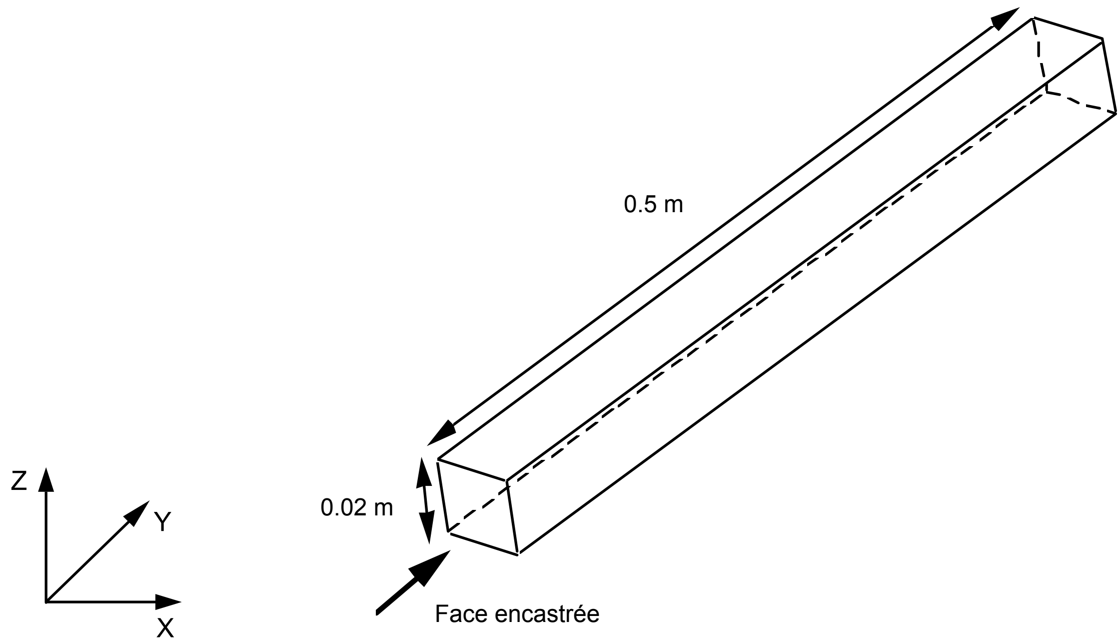
This test makes it possible to validate the linear elastic design of a slim beam subjected to a rotation of one of its ends. Four modelings are tested: elements 3D (HEXA20), elements COQUE_3D (QUAD9 and TRIA7) and elements D_PLAN (QUAD8). That tests the inertial forces of rotation, without taking account of the elementary terms of centrifugal stiffening (cf [V3.04.105]).

The reference solution is analytical (1D). The results coincide perfectly with the reference solution.

1 Problem of reference

1.1 Geometry

Directed slim beam carried in space by the axis of directing vector $(1,1,1)$.



Square section of surface: $4.0\,10^{-4}\,m^2$

Length of the beam: $0.5\,m$

1.2 Material properties

$E=2.\,10^{11}\,Pa$
$\nu=0$
$\rho=7800\,kg/m^3$
$\mathrm{A\_CIS}=0.8333$ (factor of correction of transverse shearing equal to $5/6$ for a theory of the type thin Reissner hull)

1.3 Boundary conditions and loadings

Free beam fixed in rotation around an axis perpendicular to its greater dimension and passing by the center of the embedded face.

Component of the vector rotation: $(1,0,-1)$.

Number of revolutions: $\omega=3000\,rd/s$.

The important value number of revolutions does not have anything physics.

2 Reference solution

2.1 Method of calculating used for the reference solution

In the local reference mark of the beam:

$$\frac{\partial^2 U_x}{\partial x^2}+\frac{\rho}{E}\omega^2 x=0 \text{ with } \begin{array}{l} U_x(0)=0 \\ \dfrac{\partial U_x}{\partial x}(L)=\sigma_{xx}(L)=0 \end{array}$$

By integrating the preceding differential equation one obtains, in the reference mark of the beam:

$$U_x(x)=\frac{\rho\omega^2}{2E}\left(x L^2-\frac{x^3}{3}\right) \quad U_y=U_z=0$$

Displacements of all points of the beam are thus written in the total reference mark:

$$U_x(X,Y,Z)=\frac{\rho\omega^2}{2\sqrt{3}E}\left(r L^2-\frac{r^3}{3}\right)$$
$$U_y(X,Y,Z)=\frac{\rho\omega^2}{2\sqrt{3}E}\left(r L^2-\frac{r^3}{3}\right)$$
$$U_z(X,Y,Z)=\frac{\rho\omega^2}{2\sqrt{3}E}\left(r L^2-\frac{r^3}{3}\right)$$
$$\text{with } r=\sqrt{X^2+Y^2+Z^2}$$

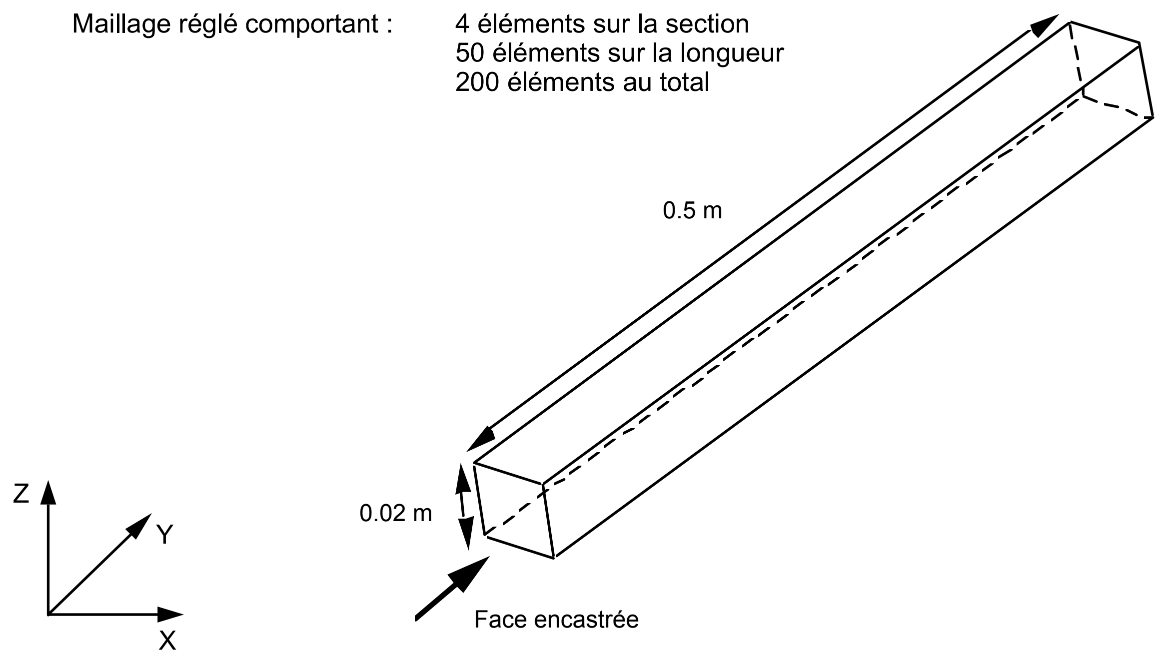
2.2 Results of reference

Values of three displacements in the center of the section furthest away from the axis of rotation.

3 Modeling A

3.1 Characteristics of modeling

Elements 3D (HEXA20)



3.2 Characteristics of the grid

Many nodes: 1521

Many meshs and types: 200 HEXA20

3.3 Sizes tested and results

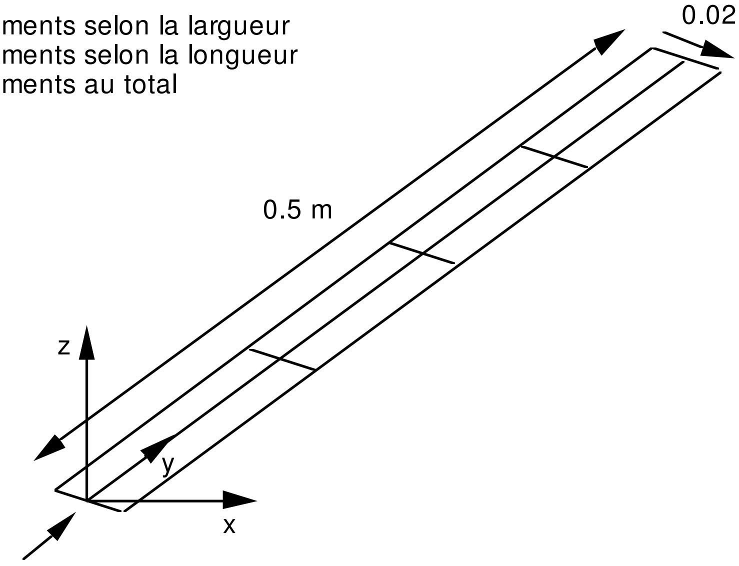
Identification	Reference
DX in L	8.44 10^{-3}
DY in L	8.44 10^{-3}
DZ in L	8.44 10^{-3}

4 Modeling B

4.1 Characteristics of modeling

Elements of hull `MEC3QU9H`

Maillage comportant : 2 éléments selon la largeur
4 éléments selon la longueur
8 éléments au total



4.2 Characteristics of the grid

Many nodes: 45

Many meshs and types: 8 QUAD9

4.3 Sizes tested and results

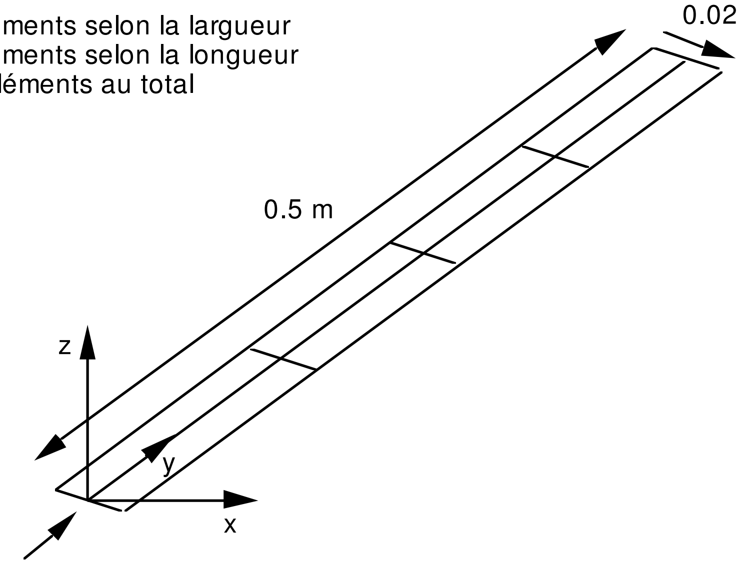
Identification	Reference
DX in L	$8.44\ 10^{-3}$
DY in L	$8.44\ 10^{-3}$
DZ in L	$8.44\ 10^{-3}$

5 Modeling C

5.1 Characteristics of modeling

Elements of hull MEC3TR7H

Maillage comportant : 4 éléments selon la largeur
8 éléments selon la longueur
64 éléments au total



5.2 Characteristics of the grid

Many nodes: 217

Many meshs and types: 64 TRIA7

5.3 Sizes tested and results

Identification	Reference
DX in L	$8.44\ 10^{-3}$
DY in L	$8.44\ 10^{-3}$
DZ in L	$8.44\ 10^{-3}$

6 Modeling D

6.1 Characteristics of modeling

Elements D_PLAN **MEDPQU8**

Grid comprising 2 elements according to the width, 50 elements according to the length.
100 elements on the whole.
In 2D, displacements of the beam are written :

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$$U_x(X,Y)=\frac{\rho\omega^2}{2\sqrt{2}E}\left(rL^2-\frac{r^3}{3}\right)$$

$$U_y(X,Y)=\frac{\rho\omega^2}{2\sqrt{2}E}\left(rL^2-\frac{r^3}{3}\right)$$

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with $r=\sqrt{X^2+Y^2}$

6.2 Characteristics of the grid

Many nodes: 405

Many meshs and types: 100 QUAD8

6.3 Sizes tested and results

Identification	Reference
DX in L	10,341 10^{-3}
DY in L	10,341 10^{-3}

7 Summary of the results

The coincidence of the results with the analytical solution makes it possible to validate the loading due to the inertial forces of rotation.

Modeling COQUE_3D with MEC3QU9H give the solution with very few elements.

One will refer to test SSLV105 [V3.04.105] to evaluate the effect of the centrifugal stiffening for the element 3D, HEXA20.