

SSLV109 - Full cylinder in nonuniform pressure mode 1

Summary:

This test validates all the elements of Fourier (triangles and quadrangles of degrees 1 and 2) in elasticity. The features are the following ones:

- variable pressure spaces some,
- imposed displacements,
- matrices of rigidity Fourier mode 1,
- constraints with the nodes Fourier mode 1,
- recombination of Fourier on displacements and constraints (modeling A),
- transverse isotropic material (modeling F).

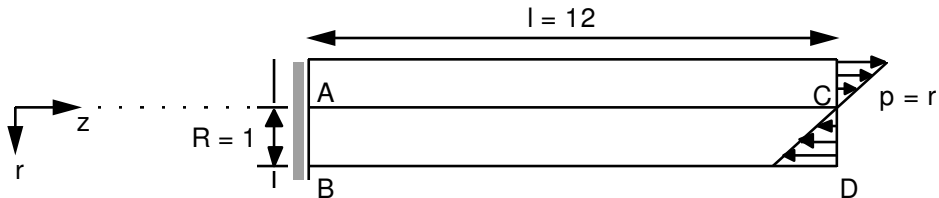
The test has a quadratic analytical solution in displacements.

The interest of the test lies in:

- the comparison between solution calculated and analytical solution on the various finite elements,
- the comparison between the results and Code PERMAS on elements TRIA6 (modeling A).

1 Problem of reference

1.1 Geometry



The modelled field is $ACDB$ (plan $\theta=0$).

1.2 Material properties

$$E = 72 \text{ N/m}^2$$

$$\nu = 0.3$$

1.3 Boundary conditions and loadings

$$u_r(A) = u_z(A) = u_\theta(A) = 0$$

$$u_z(AB) = 0$$

$$p = \bar{p} \frac{r}{R} \cos \theta$$

with $\bar{p} = 1$. and $R = 1$ applied in $z = 12$.

1.4 Initial conditions

Without object for the static analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

$$u_r(r, z, \theta) = u(r, z) \cos \theta \quad \text{with } u(r, z) = \frac{M}{2EI} z^2 + \frac{v \bar{p}}{2ER} r^2$$

$$u_z(r, z, \theta) = v(r, z) \cos \theta \quad \text{with } v(r, z) = -\frac{\bar{p}}{2EI} r z$$

$$u_\theta(r, z, \theta) = w(r, z) (-\sin \theta) \quad \text{with } w(r, z) = \frac{M}{2EI} z^2 - \frac{v \bar{p}}{2ER} r^2$$

All the constraints are worthless except $\sigma_{zz}(r, z) = -\frac{\bar{p}}{R} r$.

The data were selected in such way that $u(x) = u(0, l) = 1$.

Displacements are thus written here:

$$u(r, z) = \frac{z^2}{144} + \frac{r^2}{480} ; v(r, z) = -\frac{r z}{72} ; w(r, z) = \frac{z^2}{144} - \frac{r^2}{480}$$

and:

$$\sigma_{zz}(r, z) = -r$$

2.2 Results of reference

$$u, v, w, \sigma_{zz} \quad \text{in } \begin{array}{l} r = 0., 0.5, 1. \\ z = 0., 6., 12. \end{array}$$

$$u_r, u_z, u_\theta \quad \begin{array}{l} r = 0. \\ \text{in } z = 6. \\ \theta = 45^\circ \end{array}$$

2.3 Uncertainty on the solution

Analytical solution.

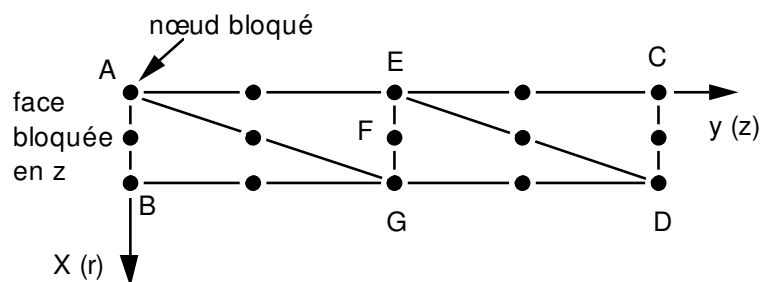
2.4 Bibliographical references

- 1) PERMAS-HS. Axisymmetric Continued with arbitrary loads. Stuttgart 1985. INTES publication n°224 pp 42 - 49.

3 Modeling A

3.1 Characteristics of modeling

Number of the nodes: $A=N1$ $B=N3$ $C=N13$
 $D=N15$ $E=N7$ $F=N8$ $G=N9$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With          DX = 0.  DY = 0.  DZ = 0.)
face AB        ( GROUP_NO : AB        DY = 0.)
```

Pressure on the face CD : PRES_REP (GROUP_MA: End NEAR: p)

p being defined by AFFE_CHAR_MECA_F by $p(X) = -X$

3.2 Characteristics of the grid

Many nodes: 15

Many meshes and types: 4 TRIA6, 1SEG3 on segment CD

4 Results of modeling A

4.1 Values tested

Node	Size	Reference
<i>B</i>	<i>u</i>	$2.0833 \cdot 10^{-3}$
	<i>v</i>	0.
	<i>w</i>	$-2.0833 \cdot 10^{-3}$
	σ_{zz}	-1.
<i>E</i>	<i>u</i>	0.25
	<i>v</i>	0.
	<i>w</i>	0.25
	σ_{zz}	0.
<i>F</i>	<i>u</i>	0.250521
	<i>v</i>	-0.04166
	<i>w</i>	0.0249479
	σ_{zz}	-0.5
<i>G</i>	<i>u</i>	0.252083
	<i>v</i>	-0.083333
	<i>w</i>	0.247917
	σ_{zz}	-1.
<i>C</i>	<i>u</i>	1.
	<i>v</i>	0.
	<i>w</i>	1.
	σ_{zz}	0.
<i>D</i>	<i>u</i>	1.00208
	<i>v</i>	-0.16666
	<i>w</i>	0.99791
	σ_{zz}	-1.

4.2 Remarks

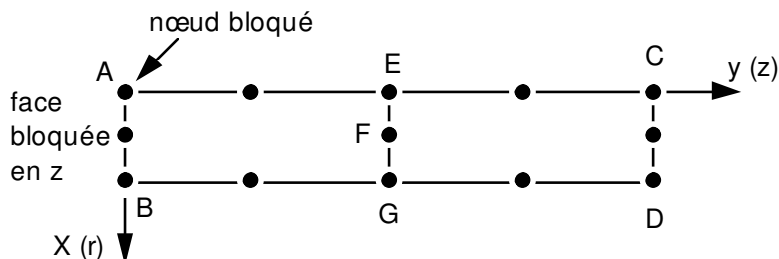
The analytical solution is found with a precision < 0.02 for displacements and < 0.1 for the constraints.

With a digital formula of integration at 6 points of GAUSS (instead of 3) to calculate the stiffness, one would find the relation with 10^{-10} close (like PERMAS).

5 Modeling B

5.1 Characteristics of modeling

Number of the nodes: $A = N1$ $B = N3$ $C = N13$
 $D = N15$ $E = N7$ $F = N8$ $G = N9$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With          DX = 0.  DY = 0.  DZ = 0.)
face AB        ( GROUP_NO : AB          DY = 0.)
```

Pressure on the face CD : PRES_REP (GROUP_MA: End NEAR: p)

p being defined by AFFE_CHAR_MECA_F by $p(X) = -X$

5.2 Characteristics of the grid

Many nodes: 15

Many meshes and types: 2 QUAD8, 1 SEG3 on segment CD

6 Results of modeling B

6.1 Values tested

Node	Size	Reference
<i>B</i>	<i>u</i>	$2.0833 \cdot 10^{-3}$
	<i>v</i>	0.
	<i>w</i>	$-2.0833 \cdot 10^{-3}$
	σ_{zz}	-1.
<i>E</i>	<i>u</i>	0.25
	<i>v</i>	0.
	<i>w</i>	0.25
	σ_{zz}	0.
<i>F</i>	<i>u</i>	0.250521
	<i>v</i>	-0.04166
	<i>w</i>	0.0249479
	σ_{zz}	-0.5
<i>G</i>	<i>u</i>	0.252083
	<i>v</i>	-0.08333
	<i>w</i>	0.247917
	σ_{zz}	-1.
<i>C</i>	<i>u</i>	1.
	<i>v</i>	0.
	<i>w</i>	1.
	σ_{zz}	0.
<i>D</i>	<i>u</i>	1.00208
	<i>v</i>	-0.16666
	<i>w</i>	0.99791
	σ_{zz}	-1.

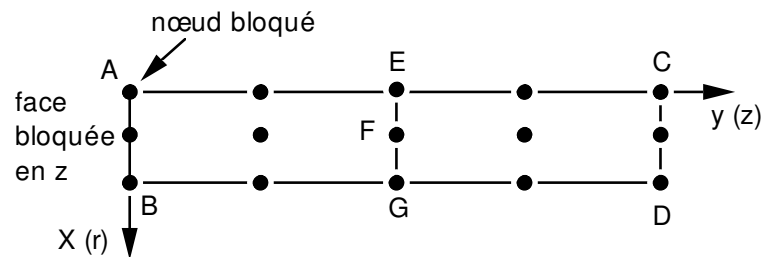
6.2 Remarks

The analytical solution is found with 10 or 11 significant figures.

7 Modeling C

7.1 Characteristics of modeling

Number of the nodes: $A = N1$ $B = N3$ $C = N13$
 $D = N15$ $E = N7$ $F = N8$ $G = N9$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With          DX = 0.  DY = 0.  DZ = 0.)
face AB        ( GROUP_NO : AB        DY = 0.)
```

Pressure on the face CD : PRES_REP (GROUP_MA = End, CLOSE = p)

p being defined by AFFE_CHAR_MECA_F by $p(x) = -x$

7.2 Characteristics of the grid

Many nodes: 15

Many meshes and types: 2 QUAD9, 1 SEG3 on segment CD

8 Results of modeling C

8.1 Values tested

Node	Size	Reference
<i>B</i>	<i>u</i>	$2.0833 \cdot 10^{-3}$
	<i>v</i>	0.
	<i>w</i>	$-2.0833 \cdot 10^{-3}$
	σ_{zz}	-1.
<i>E</i>	<i>u</i>	0.25
	<i>v</i>	0.
	<i>w</i>	0.25
	σ_{zz}	0.
<i>F</i>	<i>u</i>	0.250521
	<i>v</i>	-0.04166
	<i>w</i>	0.0249479
	σ_{zz}	-0.5
<i>G</i>	<i>u</i>	0.252083
	<i>v</i>	-0.08333
	<i>w</i>	0.247917
	σ_{zz}	-1.
<i>C</i>	<i>u</i>	1.
	<i>v</i>	0.
	<i>w</i>	1.
	σ_{zz}	0.
<i>D</i>	<i>u</i>	1.00208
	<i>v</i>	-0.16666
	<i>w</i>	0.99791
	σ_{zz}	-1.

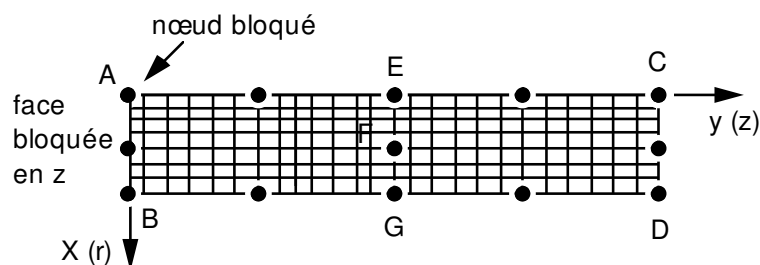
8.2 Remarks

The analytical solution is found with 10 or 11 significant figures.

9 Modeling D

9.1 Characteristics of modeling

Number of the nodes: $A=N1$ $B=N1129$ $C=N1369$
 $D=N2169$ $E=N141$ $F=N705$ $G=N1269$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With          DX = 0.  DY = 0.  DZ = 0.)
face AB        ( GROUP_NO : AB        DY = 0.)
```

Pressure on the face CD : PRES_REP (GROUP_MA: End NEAR: p)

p being defined by AFFE_CHAR_MECA_F by $p(x) = -x$

9.2 Characteristics of the grid

Many nodes: 2169

Many meshes and types: 1920 QUAD4, 8 SEG2 on segment CD

10 Results of modeling D

10.1 Values tested

Node	Size	Reference
<i>B</i>	<i>u</i>	$2.0833 \cdot 10^{-3}$
	<i>v</i>	0.
	<i>w</i>	$-2.0833 \cdot 10^{-3}$
	σ_{zz}	-1.
<i>E</i>	<i>u</i>	0.25
	<i>v</i>	0.
	<i>w</i>	0.25
	σ_{zz}	0.
<i>F</i>	<i>u</i>	0.250521
	<i>v</i>	-0.04166
	<i>w</i>	0.0249479
	σ_{zz}	-0.5
<i>G</i>	<i>u</i>	0.252083
	<i>v</i>	-0.083333
	<i>w</i>	0.247917
	σ_{zz}	-1.
<i>C</i>	<i>u</i>	1.
	<i>v</i>	0.
	<i>w</i>	1.
	σ_{zz}	0.
<i>D</i>	<i>u</i>	1.00208
	<i>v</i>	-0.16666
	<i>w</i>	0.99791
	σ_{zz}	-1.

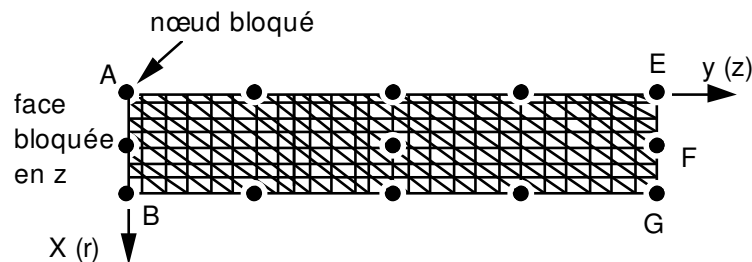
10.2 Remarks

To obtain one precision of about 1% on the constraints, it is necessary to model the structure very finely (8 elements radially and 240 axially).

11 Modeling E

11.1 Characteristics of modeling

Number of the nodes: $A = N1$ $B = N2421$
 $E = N121$ $F = N1331$ $G = N2541$



Limiting conditions:

DDL_IMPO: (NODE : With DX = 0. DY = 0. DZ = 0.)
face AB (GROUP_NO : AB DY = 0.)

Pressure on the face EG : PRES_REP (GROUP_MA: End NEAR: p)

p being defined by AFFE_CHAR_MECA_F by $p(x) = -x$

11.2 Characteristics of the grid

Many nodes: 2541

Many meshes and types: 4800 TRIA3, 20 SEG2 on segment EG

11.3 Remarks

To decrease the number of nodes, one modelled the structure for $y \leq 6$.
The precision on the results is nevertheless less than for elements QUAD4.

12 Results of modeling E

12.1 Values tested

Node	Size	Reference
<i>B</i>	<i>u</i>	$2.0833 \cdot 10^{-3}$
	<i>v</i>	0.
	<i>w</i>	$-2.0833 \cdot 10^{-3}$
	σ_{zz}	-1.
<i>E</i>	<i>u</i>	0.25
	<i>v</i>	0.
	<i>w</i>	0.25
	σ_{zz}	0.
<i>F</i>	<i>u</i>	0.250521
	<i>v</i>	-0.04166
	<i>w</i>	0.249479
	σ_{zz}	-0.5
<i>G</i>	<i>u</i>	0.252083
	<i>v</i>	-0.083333
	<i>w</i>	0.247917
	σ_{zz}	-1.

12.2 Remarks

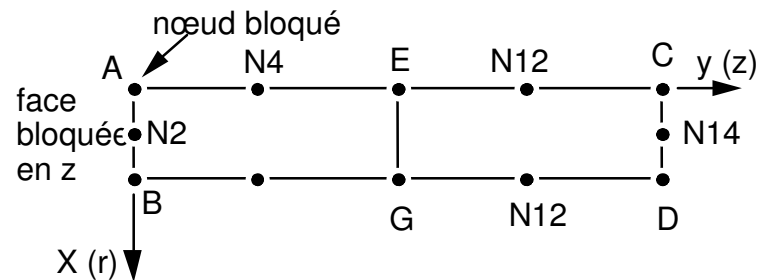
The precision on displacements is lower than 3%, that on the constraints lower than 2%.

On this example, the TRIA3 converge definitely less quickly than the QUAD4 towards the exact solution.

13 Modeling F

13.1 Characteristics of modeling

Number of the nodes: $A = N1$ $B = N3$ $C = N13$
 $D = N15$ $E = N7$ $G = N9$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With          DX = 0.  DY = 0.  DZ = 0.)
face AB        ( GROUP_NO : AB        DY = 0.)
```

Pressure on the face CD : PRES_REP (GROUP_MA: End NEAR: p)

p being defined by AFFE_CHAR_MECA_F by $p(X) = -X$

13.2 Characteristics of the grid

Many nodes: 15

Many meshes and types: 2 QUAD8, 1SEG3 on segment CD

14 Results of modeling F

14.1 Values tested

Node	Size	Reference
N2	u	2.6041666
	w	- 2.6041666
A	σ_{zz}	0.
B	σ_{zz}	- 1.
N4	u	0.0625
	w	0.0625
E	u	0.25
	w	0.25
	σ_{zz}	0.
G	v	- 0.083333
	σ_{zz}	- 1.
N10	u	0.5625
	w	0.5625
N12	v	- 0,125
C	u	1.
	w	1.
	σ_{zz}	0.
N14	v	- 0.083333
D	v	- 0.166666
	σ_{zz}	- 1.

15 Summary of the results

The elements of order 2 give the analytical solution.

The elements of order 1 converge slowly towards the solution and require very fine grids. Times calculations remain however reasonable.