SSLV110 - Elliptic crack in an infinite medium

Summary:

It is about a test in statics for a three-dimensional problem. This test makes it possible to calculate the rate of refund of energy total and local on the bottom of crack by the method theta (order CALC_G).

The rays of the crowns of integration are variable along the crack, and the rate of refund of energy room is calculated according to 3 different methods (LEGENDRE, and LAGRANGE).

The interest of the test is the validation of the method theta in $3\mathrm{D}$ and of the following points:

- comparison between the results and an analytical solution,
- stability of the results according to the crowns of integration,
- comparison between 3 methods different for calculation from G room,
- 2 cases of equivalent loadings (pressure distributed and voluminal loading).

This test contains 5 different modelings.

Modeling E is a data-processing validation of the taking into account of various loadings applied to the lips of the crack in the calculation of G.
Modeling F tests the calculation of $K1$ for a crack nonwith a grid (method X-FEM). It also makes it possible to compare the mistakes made on the calculation of $K1$ with the operator POST_K1_K2_K3 or the operator CALC_G.
Modeling G makes it possible to validate the calculation of factor of intensity of the constraint equivalents in the presence of cohesive zones (see documentation [R7.02.18]), by the operator CALC_G.

1 Problem of reference

1.1 Geometry

It is about an elliptic crack plunged in a presumedly infinite medium. Only one eighth of a parallelepiped is modelled:

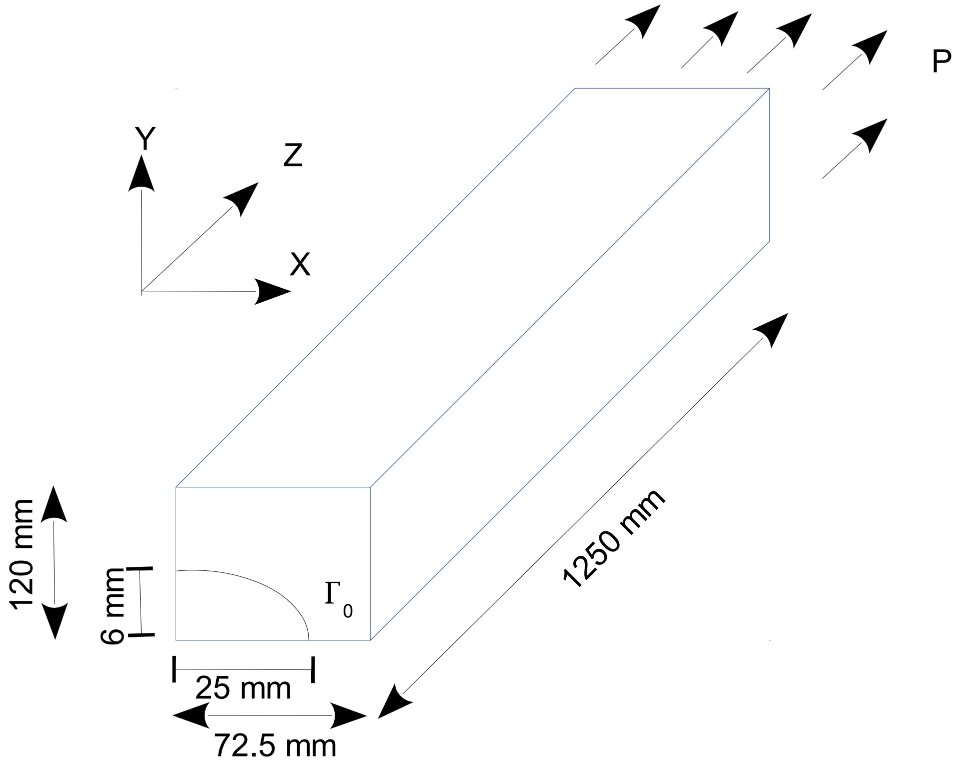


Figure 1.1-1: geometry and elliptic bottom of crack

1.2 Properties materials

$E=210\,000.\text{MPa}$

$\nu=0.3$

1.3 Boundary conditions and loadings

Symmetry compared to the 3 principal plans:

$U_x=0.$ in the plan $X=0$.

$U_Y=0.$ in the plan $Y=0$.

$U_Z=0.$ in the plan $Z=0.$ out of the crack

The conditions of loadings are is:

$P=1\,MPa$ in the plan $Z=1250\,mm$ (modelings A and B)

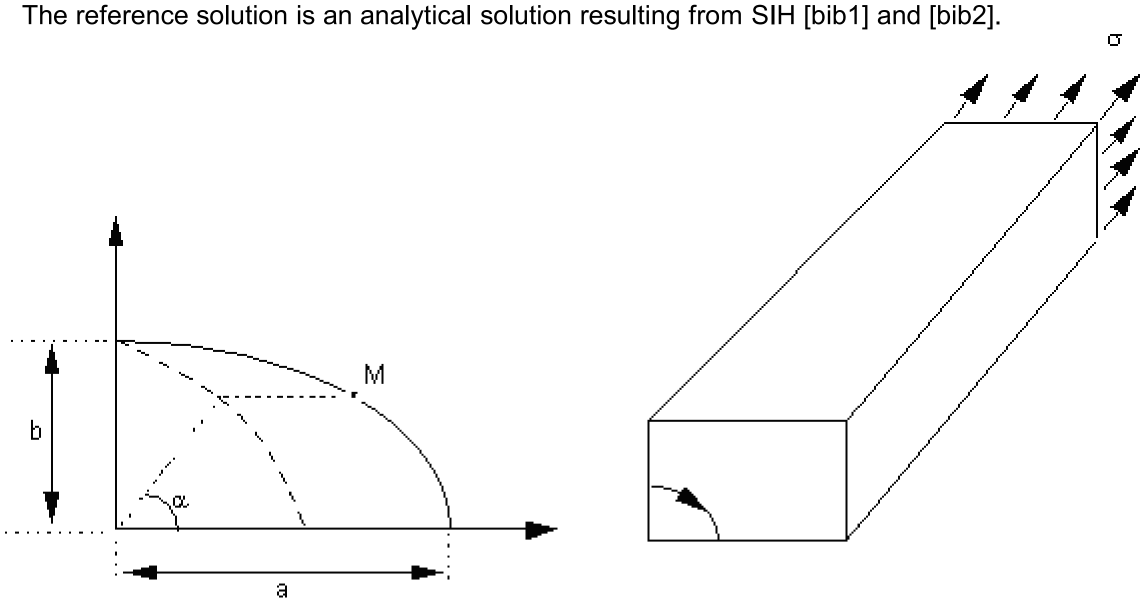
that is to say:

$FZ=8.10^{-4} N/mm^3$ on all the elements of volume (loading are equivalent to the precedent) (modelings C, D, F and G).

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is an analytical solution resulting from SIH [bib1] and [bib2].



It is noted that the angle α indicate here the parametric angle of the point M (angle compared to the axis Ox project of M on the circle of radius b) and not the polar coordinate of this point.

Here: $a=25\,mm$ and $b=6\,mm$, therefore $k=0{,}9707728$

Values of the elliptic integral $E(k)$ are tabulées in [bib3], according to $asin(k)$ who is worth here $76{,}11°$. One finds then: $E(k)=1{,}0672$.

From where the factor of intensity of the constraints in $MPa.\sqrt{mm}$:

$$K_I(\alpha)=4.0680\left[\sin^2\alpha+\frac{b^2}{a^2}\cos^2\alpha\right]^{1/4}$$

Then, starting from the formula of Irwin (plane deformation):

The total rate of refund of energy G_{ref} is calculated by integration of $G(\alpha)$:

$G_{ref}=5{,}76.10^{-3}\,J/mm$.

2.2 Bibliography

1) G.C. SIH: Mathematical Theories of Brittle Fractures - FRACTURE, flight II - Academic Close - 1968

2) M.K. KASSIN and G.C. SIH: Three-dimensional stress distribution around year elliptical ace under arbitrary loadings J. Appl. Mech., 88,601-611, 1966.

3) H. TADA, P.PARIS, G. IRWIN: The Stress Analysis of Cracks Handbook - Third Edition - International ASM - 2000

3 Modeling A

3.1 Characteristics of modeling

$A=N01099$ ($s=0.$)
$B=N01259$ ($s=26.68$)
$C=N01179$ ($s=17.8$; $\alpha=\pi/4$)

Loading: Unit pressure distributed on the face of the block opposed to the plan of the lip:

$P=1.\mathrm{MPa}$ in the plan $Z=1250.\mathrm{mm}$.

3.2 Characteristics of the grid

Many nodes: 1716
Many meshs and types: 304 `PENTA15` and 123 `HEXA20`

3.3 Sizes tested and results

The values tested are:

- the total rate of refund of energy G ,
- the rate of refund of energy room g in all the nodes of the bottom of crack.

  The grid understands only one of the lips of the crack, it is thus necessary to use the keyword `'SYME'` automatically to multiply by 2 in calculation Aster the rate of refund of energy calculated by virtual extension of the single lip.

  In the same way, it G total calculated here corresponds to the quarter of G of reference defined previously, only a eighth of parallelepiped being represented.

Identification	Reference	$\%$ tolerance
G Crown C_1	1.44 10^{-3}	-2.1
G Crown C_2	1.44 10^{-3}	0.8
G Crown C_3	1.44 10^{-3}	-1.1
$g(A)$ crown C_1 **('LEGENDRE')**	7,171 10^{-5}	-4.8
$g(A)$ crown C_2 **('LEGENDRE')**	7,171 10^{-5}	0.95
$g(A)$ crown C_3 **('LEGENDRE')**	7,171 10^{-5}	-4.3
$g(B)$ crown C_1 **('LEGENDRE')**	1,721 10^{-5}	-13.8
$g(B)$ crown C_2 **('LEGENDRE')**	1,721 10^{-5}	-8.7
$g(B)$ crown C_3 **('LEGENDRE')**	1,721 10^{-5}	-6.9
$g(C)$ crown C_1 **('LEGENDRE')**	5,215 10^{-5}	-4.3
$g(C)$ crown C_2 **('LEGENDRE')**	5,215 10^{-5}	-1.7
$g(C)$ crown C_3 **('LEGENDRE')**	5,215 10^{-5}	-3.9

3.4 Notice

The results are rather stable between the crowns except at the point B where variation of $g(s)$ is larger and the results far away from the reference solution. One can explain this variation by the poor grid of quality.

4 Modeling B

4.1 Characteristics of modeling

$A=N01099$ ($s=0.$)
$B=N01259$ ($s=26.68$)
$C=N01179$ ($s=17.8$)

Loading: Unit pressure distributed on the face of the block opposed to the plan of the lip:

$P=1\,MPa$ in the plan $Z=1250\,mm$.

4.2 Characteristics of the grid

Many nodes: 1716
Many meshs and types: 304 PENTA15 and 123 HEXA20

4.3 Sizes tested and results

The values tested are:

- the total rate of refund of energy G ,
- the rate of refund of energy room g in all the nodes of the bottom of crack.

  The grid understands only one of the lips of the crack, it is thus necessary to use the keyword 'SYME' automatically to multiply by 2 in calculation Aster the rate of refund of energy calculated by virtual extension of the single lip.

  In the same way, it G total calculated here corresponds to the quarter of G of reference defined previously, only a eighth of parallelepiped being represented.

Identification	Reference	% tolerance
G Crown C_1	1.44 10^{-3}	-2.1
G Crown C_2	1.44 10^{-3}	0.8
G Crown C_3	1.44 10^{-3}	-1.1
$g(A)$ crown C_1	7,171 10^{-5}	-0.7
$g(A)$ crown C_2	7,171 10^{-5}	3.9
$g(A)$ C_ crown$_3$	7,171 10^{-5}	3.6
$g(B)$ crown C_1	1,721 10^{-5}	-6.6
$g(B)$ crown C_2	1,721 10^{-5}	-3.4
$g(B)$ crown C_3	1,721 10^{-5}	-0.9
$g(C)$ crown C_1	5,215 10^{-5}	-4.5
$g(C)$ crown C_2	5,215 10^{-5}	-2.3
$g(C)$ crown C_3	5,215 10^{-5}	-3.9

Notice

The results are better than in modeling A at the point B, but the disparity between the crowns remains strong.

5 Modeling D

5.1 Characteristics of modeling

$A=N01099$ ($s=0.$)
$B=N01259$ ($s=26.68$)
$C=N01179$ ($s=17.8$)

Loading: Voluminal force Fz equivalent to a unit pressure distributed on the face of the block opposed to the plan of the lip:

FORCE_INTERNE: FZ = $8.10^{-4} N/mm^3$ on all the elements of volume.

5.2 Characteristics of the grid

Many nodes: 1716
Many meshs and types: 304 PENTA15 and 123 HEXA20

5.3 Sizes tested and results

The values tested are:

- the total rate of refund of energy G,
- the rate of refund of energy room g in all the nodes of the bottom of crack.

The grid understands only one of the lips of the crack, it is thus necessary to use the keyword 'SYME' automatically to multiply by 2 in calculation Aster the rate of refund of energy calculated by virtual extension of the single lip.

In the same way, it G total calculated here corresponds to the quarter of G of reference defined previously, only a eighth of parallelepiped being represented.

Identification	Reference	% tolerance
G Crown C_1	1.44 10^{-3}	-0.2
G Crown C_2	1.44 10^{-3}	2.7
G Crown C_3	1.44 10^{-3}	0.7
$g(A)$ crown C_1	7,171 10^{-5}	1.2
$g(A)$ crown C_2	7,171 10^{-5}	5.9
$g(A)$ crown C_3	7,171 10^{-5}	5.7
$g(B)$ crown C_1	1,721 10^{-5}	– 4.9
$g(B)$ crown C_2	1,721 10^{-5}	– 1.7
$g(B)$ crown C_3	1,721 10^{-5}	0.7
$g(C)$ crown C_1	5,215 10^{-5}	– 2.7
$g(C)$ crown C_2	5,215 10^{-5}	0.4
$g(C)$ crown C_3	5,215 10^{-5}	– 2.1

Notice

The results are better than in modeling C at the point B.

6 Modeling E

The grid is the same one as that of modeling D.

The goal of this modeling is only data-processing: to test that the order CALC_G function well for loads of pressure on the lips of the crack. The pressure is modelled in 3 different ways:

- a pressure function (AFFE_CHAR_MECA_F/PRES_REP),
- a force distributed constant (AFFE_CHAR_MECA/FORCE_FACE)
- and a force distributed function (AFFE_CHAR_MECA_F/FORCE_FACE).

It should be noted that one only mechanical resolution is carried out with a load of constant pressure, and that the 3 various loads detailed above are transmitted to 3 CALC_G different *via* the keyword EXCIT.

6.1 Sizes tested and results

The values tested are:

- the total rate of refund of energy G,
- the rate of refund of energy room g with node A of the bottom of crack (in $s=0$).

The choice of the crowns for the method theta is that of the crown n°2 of modeling D (crown C_2).

G total calculated here corresponds to the quarter of G of reference defined previously because the loading is symmetrical.

Identification	Type of reference	Value of reference	Tolerance
G pressure function	'ANALYTICAL'	$1.44\ 10^{-3}$	1,00%
G pressure function	'NON_REGRESSION'	$1.449052\ 10^{-3}$	10^{-4}%
G constant force	'AUTRE_ASTER'	$1.449052\ 10^{-3}$	10^{-4} %
G force function	'AUTRE_ASTER'	$1.449052\ 10^{-3}$	10^{-4} %
$g(A)$ pressure function	'ANALYTICAL'	$7.16\ 10^{-5}$	3,00%
$g(A)$ pressure function	'NON_REGRESSION'	$6.98287\ 10^{-5}$	10^{-4}%
$g(A)$ constant force	'AUTRE_ASTER'	$6.98287\ 10^{-5}$	10^{-4} %
$g(A)$ force function	'AUTRE_ASTER'	$6.98287\ 10^{-5}$	10^{-4} %

7 Modeling F

7.1 Characteristics of modeling

In this modeling, the crack is not with a grid. Method X-FEM is used.

Taking into account symmetries of the problem, it is possible to represent only one eighth of the structure (as that is done in modeling A). However, with method X-FEM, it is not possible to represent a crack which is located in a symmetry plane (on the edge of the modelled field). One thus models in this modeling a quarter of the structure, i.e. a portion of $90°$ ellipse.

The grid is composed of meshs HEXA8, uniformly distributed along the axes X and Y and divided into geometric progression along the axis Z so that in the plan $Z=0$, the meshs are approximately cubes of with dimensions $10\,mm$.

Conditions of symmetry are applied to the faces in $X=0$ and $Y=0$. Rigid mode following the axis Z is blocked by blocking following displacement Z point located in $(0,0,-1250\,mm)$.

Loading: Unit pressure distributed on the two normal faces of the block:

$P=1\,MPa$ in the plan $Z=\pm 1250\,mm$.

7.2 Characteristics of the grid

Many nodes: 21000
Many meshs and types: 13000 PENTA6 and 12500 HEXA8 (linear grid)

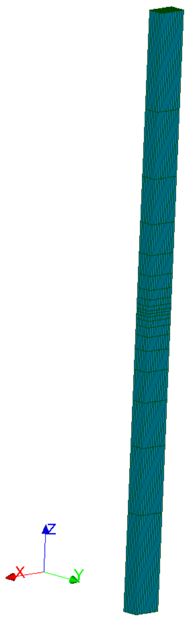


Figure 7.2-1 : initial grid, overall picture

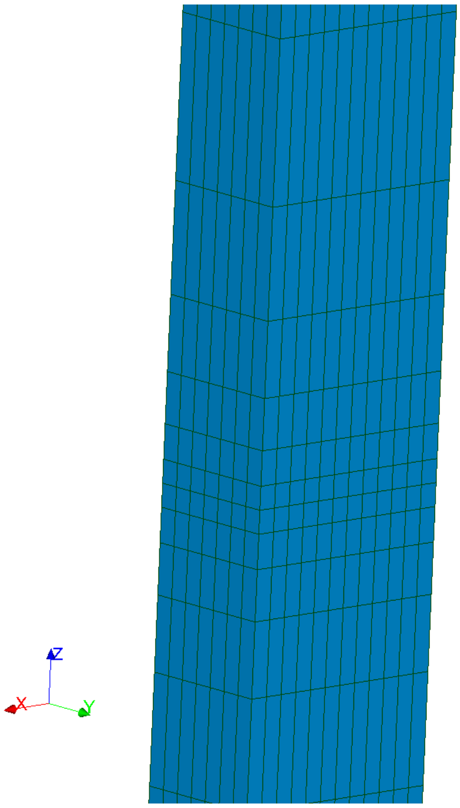


Figure 7.2-2: initial grid, zoom in the medium plan

As this initial grid is well too coarse for a precise calculation of the stress intensity factors along the bottom of the crack, an automatic procedure of refinement of the meshs close to the bottom of crack is used, as recommended in documentation [U2.05.02].

The target size of the meshs wished is $b/9$. That will induce 5 successive refinements. The size of the meshs of the grid thus refined is then $h=0{,}39\,mm$.

The refined grid (that on which mechanical calculation is carried out) has as characteristics:

- 26484 nodes
- 7720 TETRA4, 10650 PYRAM5 and 20080 HEXA8

This grid induces 99 points along the bottom of crack and taking into account as of conditions blocking 118404 equations in the system to solve

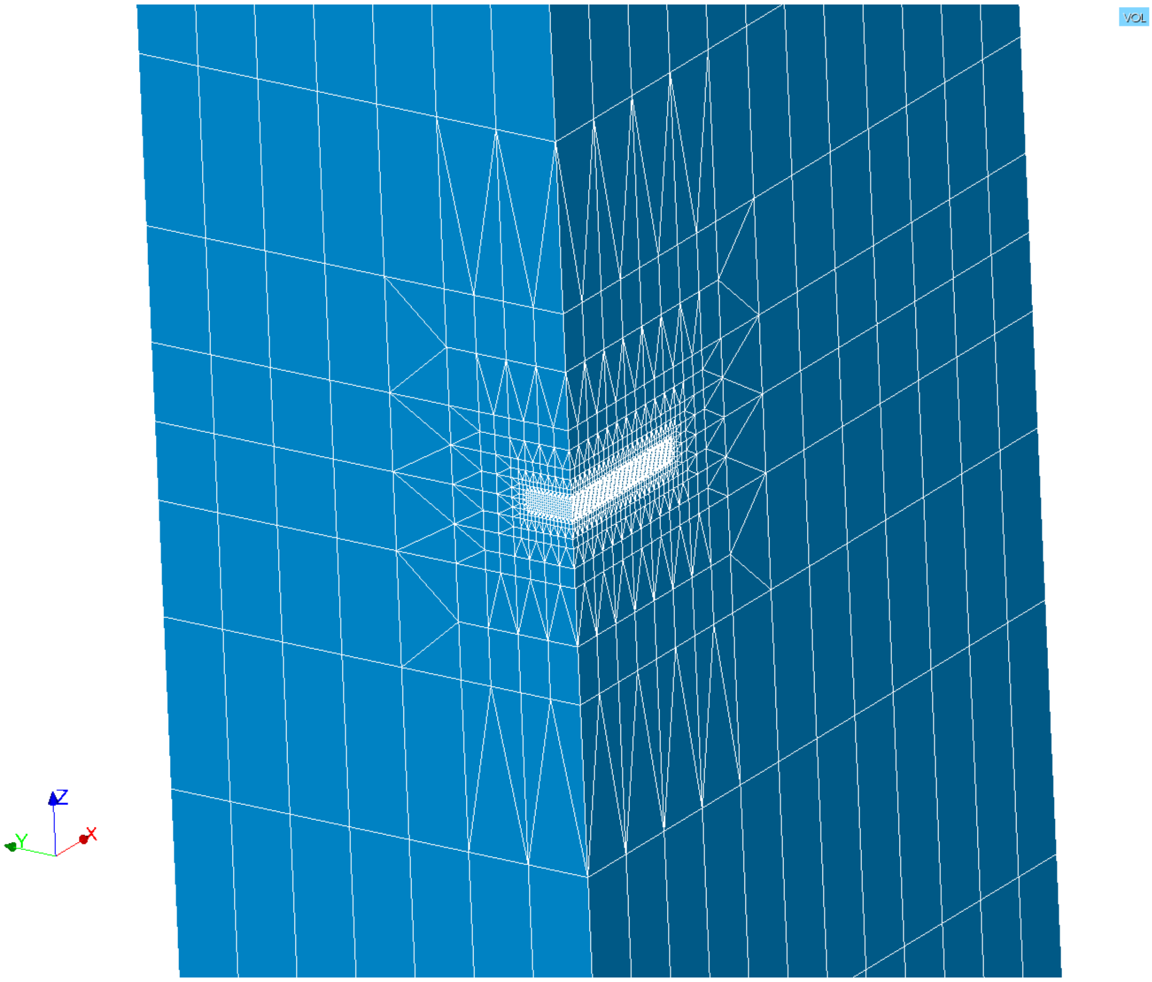


Figure 7.2-3: grid refined, zoom on the zone close to the crack

7.3 Sizes tested and results

The values tested are the factors of intensity of the constraints $K1$ along the bottom of crack, calculated either by CALC_G, that is to say by POST_K1_K2_K3.

For CALC_G, the crown of integration is worth $2h-4h$. Smoothing by default (Legendre) is used.
For POST_K1_K2_K3, the maximum curvilinear X-coordinate is worth $4h$. In order to reduce the computing times of POST_K1_K2_K3, one post-draft that on 20 points distributed uniformly along the bottom of crack.
Let us note that the computing time for Legendre smoothing of CALC_G is insensitive to this number.

One tests the values at the points A ($s=0$) and B ($s=26,7$).

Identification	Type of reference	Value of reference	Tolerance
CALC_G : $K1(A)$	'ANALYTICAL'	0,000	2,0%
CALC_G : $K1(B)$	'ANALYTICAL'	4,068	2,0%
POST_K1_K2_K3 : $K1(A)$	'ANALYTICAL'	0,000	6,0%
POST_K1_K2_K3 : $K1(B)$	'ANALYTICAL'	4,068	6,0%

For the operator CALC_G, smoothings of the type LAGRANGE do not allow to have easily useable results; a smoothing of the type LEGENDRE is thus to privilege.

8 Modeling G

8.1 Characteristics of modeling

The geometry and the loading are identical to modeling F: a quarter of the structure is modelled, a pressure is applied to the higher face. In this modeling, the initial crack is with a grid. Compared to modeling F, one introduces cohesive zones into the prolongation of the crack. This prolongation is represented by level-sets, so that discontinuity is taken into account by a modeling XFEM, as for modeling F. The cohesive law CZM_LIN_MIX is introduced into this model XFEM by the order DEFI_CONTACT.

Loading: Unit pressure distributed on the face of the block opposed to the plan of the lip:

$P=1\,MPa$ in the plan $Z=1250\,mm$.

The cohesive parameters are selected so that this causes to open some cohesive elements in the vicinity of the bottom of initial crack:

– Not to have a complete rupture, but simply a decoherence close to the point of initial crack, one takes $G_c>G_{max}$, with G_{max} G maximum room the long face, while preserving the same order of magnitude for the two values. In our case, $G_c=2.5\times10^{-4}\,N.mm^{-1}$ against $G_{max}=7.2\times10^{-5}\,N.mm^{-1}$.

– For all the same observing a decoherence in the vicinity of the point, size characteristic of the cohesive zone $l_c=\frac{E\,G_c}{(1-\nu^2)\sigma_c^2}$ is selected of kind to cover some elements while remaining small in front of the size of the structure $h\leq l_c\leq a$. In this case test, to reduce the computing time, one took $l_c=14\,mm$, which leads to $\sigma_c=2\,MPa$, to compare with typical sizes of elements $h=1\,mm$ on the small side of the ellipse, and $h=2\,mm$ on the large side.

8.2 Characteristics of the grid

Many nodes: 4522
Many meshs and types: 22300 TETRA4

8.3 Sizes tested and results

The values tested are the factors of intensity of the constraints equivalents $K1$ along the bottom of crack, calculated by CALC_G. Withend to get a regular result, one post-draft that on 5 points distributed uniformly along the bottom of crack (NB_POINT_FOND=5). One tests the values at the points ends of the face A ($s=0$) and B ($s=26{,}7$).

Smoothing 'LAGRANGE' is used for θ and 'LAGRANGE_NO_NO' for G .

Identification	Type of reference	Value of reference	Tolerance
CALC_G : $K1(A)$	'ANALYTICAL'	0,000	4.0%
CALC_G : $K1(B)$	'ANALYTICAL'	4,068	8.0%

Notice

For this test, there is a variation of a few percent. The precision can be improved by choosing a cohesive zone of lower size and by still refining the grid. This was not done here so that modeling can turn in less than a minute.

9 Summary of the results

Calculation of G or of K room:

- for a crack with a grid, them 2 methods (LEGENDRE and LAGRANGE) the same results give appreciably (less 5% of error compared to the analytical solution) except at the point B (not end of the ellipse on the main roads) where the Lagrange method is most precise;
- loading case: the values obtained with the voluminal loading are slightly higher than those obtained with constraints imposed (including for the values of G). The differences are tiny and due to digital integrations different on the term from volume and the term of edge;
- method $\mathrm{X\text{-}FEM}$ allows to evaluate the factors of intensity of the constraints K on a grid not fissured with an error lower than 10%.