SSLV113 - Estimator of error on a cylinder hollow bi--materials

Summary:

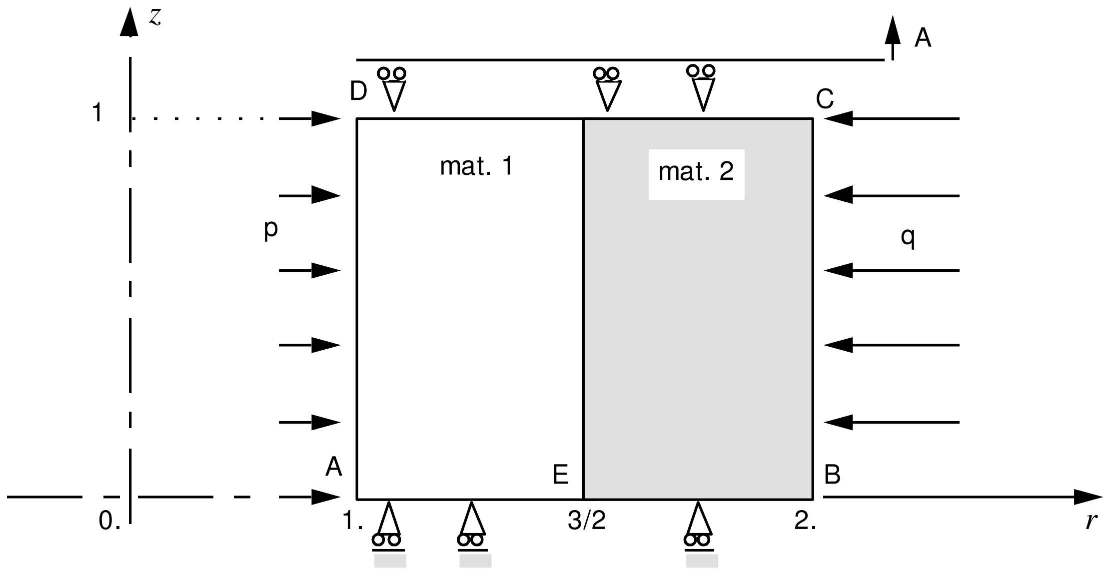
This test validates the estimator of error in pure residue, applied to linear elasticity 2D, in statics. One considers a hollow roll made up of two materials and subjected to internal and external pressures.

2 modelings are axisymmetric, on quadrangles with 8 nodes.

The interest of the test lies in the comparison between the exact and calculated constraints, on the one hand, the estimated error and the exact error, on the other hand. This test also makes it possible to show the validity of the estimator in residue on a structure bimatériau, contrary to the estimator of Zhu-Zienkiewicz which is not applicable on structures presenting of discontinuities in the stress field (here with the interface material).

1 Problem of reference

1.1 Geometry



1.2 Material properties

material 1: $E=2.$ $\nu=0.3$
material 2: $E=1.$ $\nu=0.3$

1.3 Boundary conditions and loadings

On AB , $U_Z=0.$

on DC , $U_Z=0.91333=A$.

Pressure interns on AD , $p=1$.

External pressure on BC , $q=2$.

2 Reference solution

2.1 Method of calculating used for the reference solution

$$\mu_i = \frac{E_i}{2(1+\nu)}$$
$$\lambda_i = \frac{\nu E_i}{(1-2\nu)(1+\nu)}$$

$$\left.\begin{matrix} a_1 = -0.98097 & b_1 & -1.11741 \\ a_2 = -1.34405 & b_2 & -0.30048 \end{matrix}\right\}$$ Numerical data calculated starting from the equations of Navier

For material i , one a:

$$u_r = a_i r + \frac{b_i}{r}$$
$$u_z = A$$

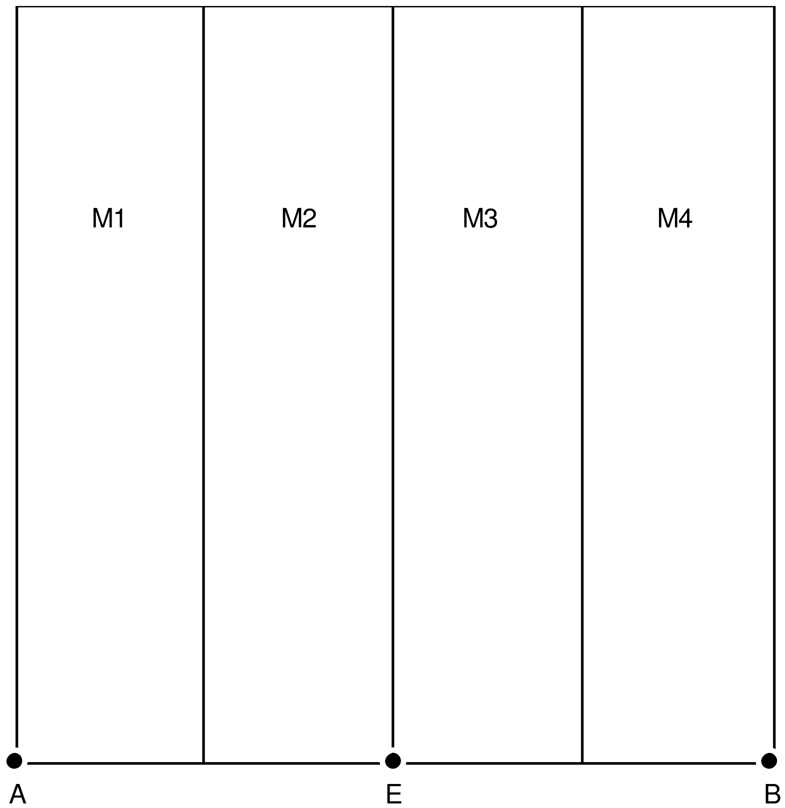
$$\begin{cases} \sigma_{rr} = \lambda_i(2a_i+A)+2\mu_i\left(a_i-\frac{b_i}{r^2}\right) \\ \sigma_{rr} = \lambda_i(2a_i+A)+2\mu_i\left(a_i+\frac{b_i}{r^2}\right) \\ \sigma_{rr} = 2\lambda_i a_i+(\lambda_i+2\mu_i)A \end{cases}$$

2.2 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling



3.2 Characteristics of the grid

Many nodes: 23.

Many meshs and types: 4 `QUAD8`.

3.3 Results and sizes tested

Identification	Reference	*Aster*	% difference	tolerance
A σ_{rr}	– 1.00003	– 1.06833	6.83	7.0
$\sigma_{\theta\theta}$	– 4.43821	– 4.46731	0.66	2.0
σ_{zz}	0.19518	0.16596	14.9	15.0
e_{rel}		2.37%		5.0
E chechmate. 1 σ_{rr}	– 1.95508	– 1.97893	1.22	2.0
$\sigma_{\theta\theta}$	– 3.48316	– 3.49330	0.29	2.0
σ_{zz}	0.19518	0.18498	5.22	6.0
e_{rel}		1.05%		5.0
E chechmate. 2 σ_{rr}	– 1.95508	– 1.98398	1.48	2.0
$\sigma_{\theta\theta}$	– 2.16049	– 2.13394	1.23	2.0

		σ_{zz}	– 0.32135	– 0.32204	0.22	2.0
		e_{rel}		0,152%		5.0
B	σ_{rr}		– 1.99999	– 2.00095	0,048	2.0
	$\sigma_{\theta\theta}$		– 2.11555	– 2.11595	0,012	2.0
	σ_{zz}		– 0.32135	– 0.32174	0.12	2.0
	e_{rel}			0,057%		5.0

3.4 Remarks

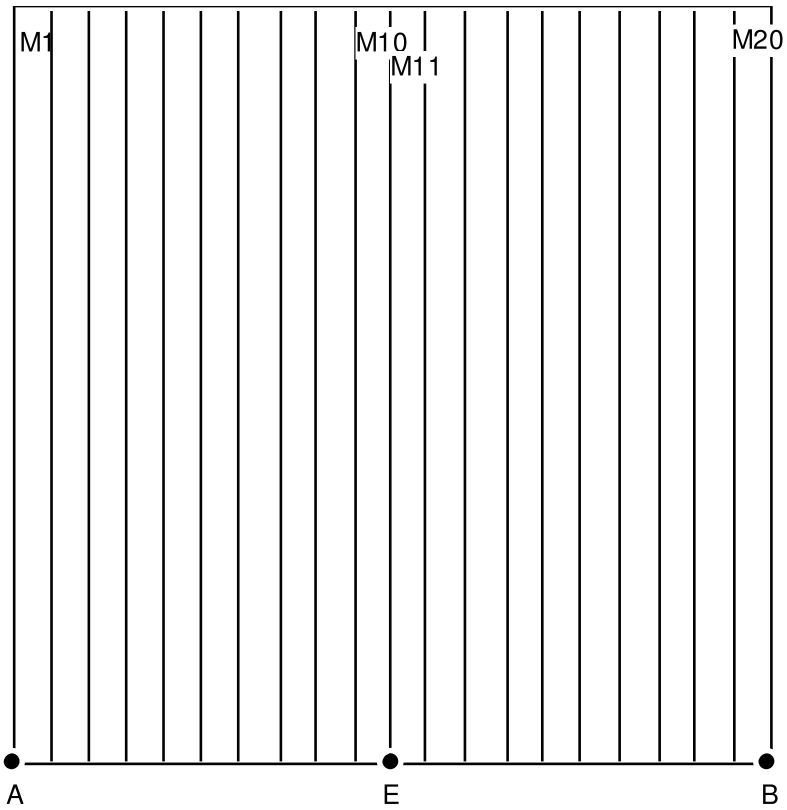
Grid being coarse (4 elements according to Or), certain constraints close to the axis of axisymetry are badly approximated. The jump of $\sigma_{\theta\theta}$ and σ_{zz} with the interface of 2 materials is on the other hand well detected.

3.5 Remarks

Relative error considered total = 1.40%.

4 Modeling B

4.1 Characteristics of modeling



4.2 Characteristics of the grid

Many nodes:

Many meshs and types: 20 QUAD8.

4.3 Results and sizes tested

Identification	Reference	*Aster*	% difference	tolerance
A σ_{rr}	– 1.00003	– 1.00351	0.35	0.5
$\sigma_{\theta\theta}$	– 4.43821	– 4.43970	0,034	0.05
σ_{zz}	0.19518	0.19369	0.76	0.8
e_{rel}		0.57%		0.6
E chechmate. 1 σ_{rr}	– 1.95508	– 1.95583	0,039	0.05
$\sigma_{\theta\theta}$	– 3.48316	– 3.48347	0,009	0.01
σ_{zz}	0.19518	0.19486	0.16	0.2
e_{rel}		0.14%		0.2
E chechmate. 2 σ_{rr}	– 1.95508	– 1.96166	0.34	0.5
$\sigma_{\theta\theta}$	– 2.16049	– 2.15403	0,299	0.5

	σ_{zz}	– 0.32135	– 0.32138	0,009	0.01
	e_{rel}		0,027%		0.03
B	σ_{rr}	– 1.99999	– 2.00003	0,002	0.01
	$\sigma_{\theta\theta}$	– 2.11555	– 2.11558	0,001	0.01
	σ_{zz}	– 0.32135	– 0.32135	0,002	0.01
	e_{rel}		0.0084%		0.01

4.4 Notice

Relative error considered total = 0.24%.

5 Summary of the results

The estimator of error in residue ERRE_ELEM_SIGM give good performances on the bi--material problems.

Note:

The estimator of error of Zhu-Zienkiewicz does not give correct results. Indeed, with the interface it detects a strong error because it carries out a continuous smoothing of the constraints whereas there exists a jump for σ_{zz} *and* $\sigma_{\theta\theta}$ *.*