
SSLV130 - Hollow roll into incompressible

Summary:

This test makes it possible to validate the quasi-incompressible elements in statics for a three-dimensional, axisymmetric or two-dimensional problem (plane deformations). One considers a hollow roll subjected to an internal pressure. The material has a Poisson's ratio equal to 0.4999 and one uses the quasi-incompressible elements (modeling INCO_UPG and INCO_UP). In all the cases of modeling, one carries out the test by imposing or not the condition of perfect incompressibility if necessary (DDL_IMPO and GONF=0)

Five modelings are carried out for this problem.

Modeling INCO_UPG is tested in the following configurations:

- modeling *A* : modeling 3D_INCO_UPG with HEXA20;
- modeling *B* : modeling 3D_INCO_UPG with TETRA10;
- modeling *E* : modeling 3D_INCO_UPG with PENTA15;
- modeling *C* : modeling D_PLAN_INCO_UPG with a mixed grid TRIA6/QUAD8;
- modeling *D* : modeling AXIS_INCO_UPG with a mixed grid TRIA6/QUAD8.

Modeling INCO_UP is tested in the following configurations:

- modeling *A* : modeling 3D_INCO_UP with HEXA20;
- modeling *B* : modeling 3D_INCO_UP with TETRA10 and with TETRA4;
- modeling *C* : modeling D_PLAN_INCO_UP with a mixed grid TRIA6/QUAD8 and with TRIA3;
- modeling *D* : modeling AXIS_INCO_UP with a mixed grid TRIA6/QUAD8 and with TRIA3.

Modeling INCO_UPO is tested in the following configurations:

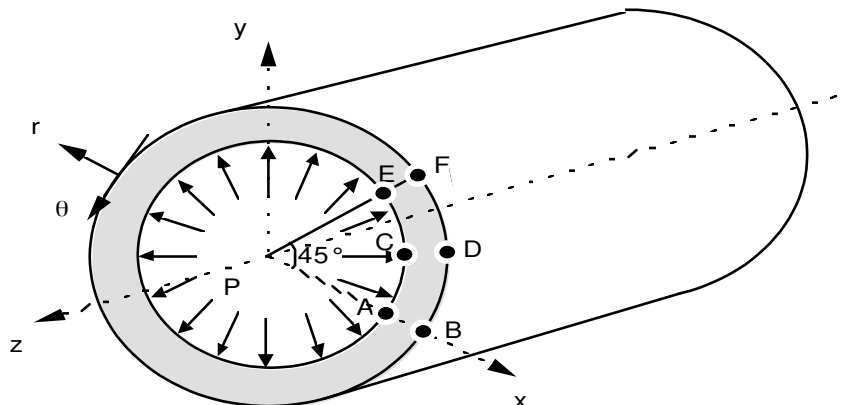
- modeling *A* : modeling 3D_INCO_UPO with HEXA8;
- modeling *B* : modeling 3D_INCO_UPO with TETRA4;
- modeling *E* : modeling 3D_INCO_UPO with PENTA6;
- modeling *C* : modeling D_PLAN_INCO_UPO with a mixed grid TRIA6/QUAD8;
- modeling *D* : modeling AXIS_INCO_UPO with a mixed grid TRIA6/QUAD8.

This test is similar to test SSLV100.

The digital results are satisfactory for all modelings. The fact of imposing the condition of incompressibility explicitly influences only very little the results. The linear elements for modeling with 2 fields are logically a little less precise than the quadratic elements.

1 Problem of reference

1.1 Geometry



Rayon interne $a = 0.1 \text{ m}$
Rayon externe $b = 0.2 \text{ m}$

Internal ray: $a = 0,1 \text{ mm}$
Ray exerne: $b = 0,2 \text{ mm}$

Coordinates of the points (mm)

	A	B	E	F	C	D
x	0.1	0.2	$0.1 \times \cos(45)$	$0.2 \times \cos(45)$	$0.1 \times \cos(22.5)$	$0.2 \times \cos(22.5)$
y	0	0	$0.1 \times \sin(45)$	$0.1 \times \sin(45)$	$0.1 \times \sin(22.5)$	$0.1 \times \sin(22.5)$
z	0	0	0	0	0	0

1.2 Properties of material

The elastic properties of material considered are the following ones:

Young modulus: $E = 2.10^5 \text{ MPa}$

Poisson's ratio: $\nu = 0.4999$

1.3 Boundary conditions and loadings

The imposed internal pressure is equal to $P = 60 \text{ MPa}$.

2 Reference solution

2.1 Method of calculating

The general solution in displacement is the following one:

$$\begin{cases} u_r = \frac{Pa^2}{E(b^2 - a^2)}(1 + \nu) \left[(1 - 2\nu)r + \frac{b^2}{r} \right] \\ u_\theta = u_z = 0 \end{cases}$$

In deformations:

$$\begin{cases} \varepsilon_{rr} = \frac{Pa^2}{E(b^2 - a^2)}(1 + \nu) \left[(1 - 2\nu) - \frac{b^2}{r^2} \right] \\ \varepsilon_{\theta\theta} = \frac{Pa^2}{E(b^2 - a^2)}(1 + \nu) \left[(1 - 2\nu) + \frac{b^2}{r^2} \right] \\ \varepsilon_{r\theta} = \varepsilon_{zz} = 0 \end{cases}$$

In constraints:

$$\begin{cases} \sigma_{rr} = P \frac{a^2}{b^2 - a^2} \left[1 - \frac{b^2}{r^2} \right] \\ \sigma_{\theta\theta} = P \frac{a^2}{b^2 - a^2} \left[1 + \frac{b^2}{r^2} \right] \\ \sigma_{zz} = 2\nu P \frac{a^2}{b^2 - a^2} \\ \sigma_{r\theta} = 0 \end{cases}$$

One obtains for a perfectly incompressible cylinder ($\nu = 0.5$) :

	$r=0,1$	$r=0,2$
u_r	$6.10^{-5} mm$	$3.10^{-5} mm$
ε_{rr}	-6.10^{-4}	$-1,5 10^{-4}$
$\varepsilon_{\theta\theta}$	-6.10^{-4}	$1,5 10^{-4}$
σ_{rr}	$-60 MPa$	$0 MPa$
$\sigma_{\theta\theta}$	$100 MPa$	$40 MPa$
σ_{zz}	$20 MPa$	$20 MPa$

The passage in the Cartesian system is done using the following relations:

$$\begin{aligned} \sigma_{xx} &= \sigma_{rr} \cos^2(\theta) + \sigma_{\theta\theta} \sin^2(\theta) - 2\sigma_{r\theta} \sin(\theta)\cos(\theta) \\ \sigma_{yy} &= \sigma_{rr} \sin^2(\theta) + \sigma_{\theta\theta} \cos^2(\theta) + 2\sigma_{r\theta} \sin(\theta)\cos(\theta) \\ \sigma_{xy} &= \sigma_{rr} \sin(\theta)\cos(\theta) - \sigma_{\theta\theta} \cos(\theta)\sin(\theta) - 2\sigma_{r\theta}(\cos^2(\theta) - \sin^2(\theta)) \end{aligned}$$

2.2 Sizes and results of reference

One compared to values of reference:

- displacements (u, v) at the points A and F ,
- deformations $(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy})$ and constraints $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy})$ at the points A and F ,
- equivalent strains and stresses equivalent to the point A .

Lastly, to test the passage of the sizes of the points of Gauss to the nodes for the nodes mediums, one also tests the nonworthless strains and stresses in a node medium of the structure.

2.3 Bibliographical references

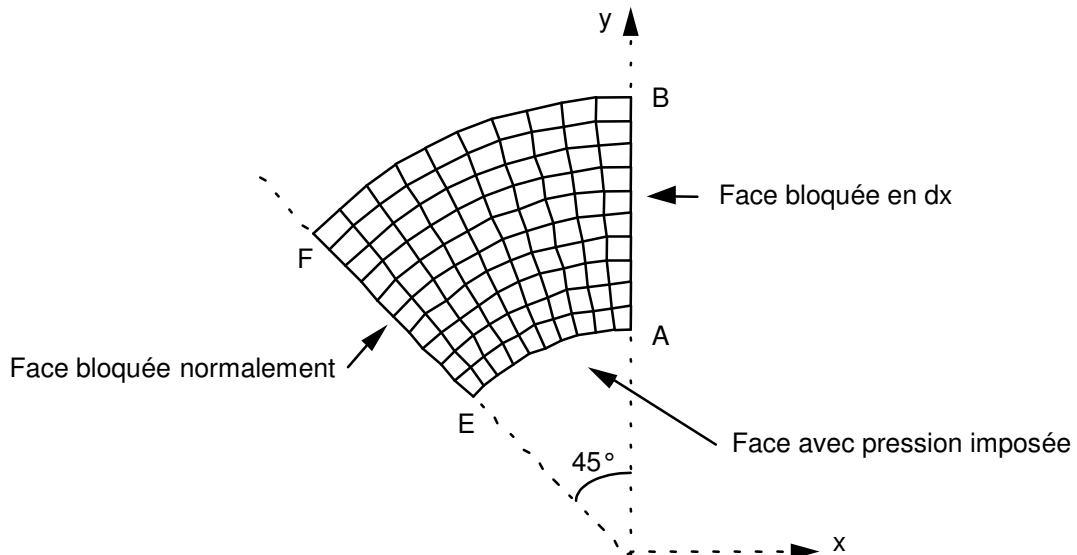
- 1) Y.C. FUNG: Foundations of solid mechanics. Prentice-hall, Inc. Englewood Cliffs. NJ. 1965, p. 243-245
- 2) [V3.04.100] Hollow roll in plane deformations

3 Modeling A

3.1 Characteristics of modeling

Grids:

1. of type HEXA20 for modelings 3D_INCO_UPG and 3D_INCO_UP
2. of type HEXA8 for modeling 3D_INCO_UPO



Along the axis z :

- total thickness $e=0.01\text{ mm}$
- 2 layers of elements

For the needs for examination in a node medium, one defines the node $NOEUMI = A + (0.0 \cdot e / 4)$ where the strains and the stresses are the same ones as in A .

Conditions limits :

```
DDL_IMPO= GROUP_NO = ' FACSUP', DZ = 0
GROUP_NO = ' FACINF', DZ = 0      faces AEFD ( z=0 and z=0.01 )
GROUP_NO = ' FACEAB', DX = 0      face AB
FACE_IMPO =GROUP_MA = ' FACEEF', DNOR = 0      face EF
PRES_REP =GROUP_MA = ' FACEAE', CLOSE = 60 face AE
```

3.2 Characteristics of the grids

Grid 1 HEXA20:
Many nodes: 1501 nodes
Many meshes: 240 HEXA20

Grid 2 HEXA8:
Many nodes: 429 nodes
Many meshes: 240 HEXA 8

3.3 Sizes tested and results

Results at the point A :

- first column 3D_INCO_UPG without imposing $GONF = 0$
- second column 3D_INCO_UPG while imposing $GONF = 0$
- third column 3D_INCO_UP
- fourth column 3D_INCO_UPO

Identification	Type of reference	Value of reference	Tolerance			
			1	2	3	4
ν	'ANALYTICAL'	6.10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
σ_{xx}	'ANALYTICAL'	100.	0,10%	0,10%	0,10%	0.15%
σ_{yy}	'ANALYTICAL'	- 60.	0,50%	0,50%	0,50%	9.0%
σ_{zz}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	5.0%
σ_{xy}	'ANALYTICAL'	0.	0,50%	0,50%	0,50%	50,00%
ϵ_{xx}	'ANALYTICAL'	6.10^{-4}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ϵ_{yy}	'ANALYTICAL'	- 6.10^{-4}	0,50%	0,50%	0,50%	7.0%
ϵ_{zz}	'ANALYTICAL'	0.	0,50%	0,50%	0,50%	0,50%
ϵ_{xy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	$3,10^{-5}$
$\epsilon_{eq} - I NVA_2$	'ANALYTICAL'	$6.92 10^{-4}$	10^{-5}	10^{-5}	10^{-5}	$5,10^{-2}$
$\epsilon_{eq} - PRIN_1$	'ANALYTICAL'	- 6.10^{-4}	0,50%	0,50%	0,50%	7.0%
$\epsilon_{eq} - PRIN_2$	'ANALYTICAL'	0.	0,50%	0,50%	0,50%	10^{-5}
$\epsilon_{eq} - PRIN_3$	'ANALYTICAL'	6.10^{-4}	10^{-5}	10^{-5}	10^{-5}	$5,10^{-2}$
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	0,50%	0,50%	0,50%	5,00%
$\sigma_{eq} - TRESCA$	'ANALYTICAL'	160.	0,50%	0,50%	0,50%	5,00%
$\sigma_{eq} - PRIN_1$	'ANALYTICAL'	-60.	0,50%	0,50%	0,50%	10.0%
$\sigma_{eq} - PRIN_2$	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	5,00%
$\sigma_{eq} - PRIN_3$	'ANALYTICAL'	100.	0,50%	0,50%	0,50%	1.50%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	0,50%	0,50%	0,50%	5.0%

Results at the point F :

- first column 3D_INCO_UPG without imposing $GONF = 0$
- second column 3D_INCO_UPG while imposing $GONF = 0$
- third column 3D_INCO_UP
- fourth column 3D_INCO_UPO

Identification	Type of reference	Value of reference	Tolerance			
			1	2	3	4
u	'ANALYTICAL'	$-2.12 \cdot 10^{-5}$	0,10%	0,10%	0,10%	0,10%
v	'ANALYTICAL'	$2.12 \cdot 10^{-5}$	0,10%	0,10%	0,10%	0,10%
σ_{xx}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	3.0%
σ_{yy}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	6.0%
σ_{zz}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	1.0%
σ_{xy}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	2.0%
ε_{xx}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{yy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{xy}	'ANALYTICAL'	$1.5 \cdot 10^{-4}$	0,50%	0,50%	0,50%	2.0%

Checking of the passage to the nodes for the nodes mediums (only for the result got without imposing $GONF = 0$) - value with the node *NOEUMI* :

Identification	Type of reference	Value of reference	Tolerance (%)
σ_{xx}	'ANALYTICAL'	100.	0,50%
σ_{yy}	'ANALYTICAL'	- 60.	0,50%
σ_{zz}	'ANALYTICAL'	20.	0,50%
ε_{xx}	'ANALYTICAL'	$6 \cdot 10^{-4}$	0,50%
ε_{yy}	'ANALYTICAL'	$-6 \cdot 10^{-4}$	0,50%

3.4 Remarks

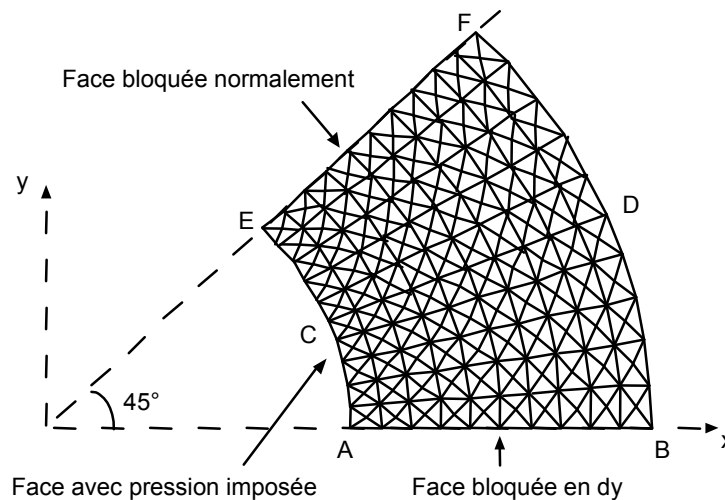
Except for 3D_INCO_UPO, one obtains very good performances which is the formulation selected since for all the examined sizes, the difference between the solution obtained with the code and the analytical solution is lower than 0.2%. It is seen that the variation enters the solutions obtained by imposing or not the condition $tr(\varepsilon) = 0$ is unimportant.

Results got with 3D_INCO_UPO are less precise. That is explained by the fact why the elements used are linear. To find a more precise result, the grid would have to be refined.

4 Modeling B

4.1 Characteristics of modeling

Grid with elements 3D incompressible of type TETRA10 only



AB is on the axis OX (contrary to modeling A).

For the needs for examination, one defines the node $NOEUMI = A + (0.0.e/4)$ where the strains and the stresses are the same ones as in A .

Conditions limits :

```
DDL_IMPO= GROUP_NO = ' FACSUP', DZ = 0
GROUP_NO = ' FACINF', D Z = 0 faces AEFD ( z=0 and z=0.01 )
GROUP_NO = ' FACEAB', DY = 0 face AB
FACE_IMPO =GROUP_MA = ' FACEEF', DNOR = 0 face EF
PRES_REP =GROUP_MA = ' FACEAE', CLOSE = 60 face AE
```

4.2 Characteristics of the grid

Many nodes: 13907

Many meshes: 8519 TETRA10

4.3 Sizes tested and results

Result at the point A :

- first column 3D_INCO_UPG without imposing $GONF = 0$
- second column 3D_INCO_UPG while imposing $GONF = 0$
- third column 3D_INCO_UP with quadratic elements
- fourth column 3D_INCO_UP with linear elements
- fifth column 3D_INCO_UPO with linear elements

Identification	Type of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	'ANALYTICAL'	$6. 10^{-5}$	0,50%	0,50%	0,50%	0,50%	0,50%
v	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
σ_{xx}	'ANALYTICAL'	- 60.	1,00%	1,00%	1,00%	4.00%	3.00%
σ_{yy}	'ANALYTICAL'	100.	1,00%	1,00%	1,00%	1.50%	1,00%
σ_{zz}	'ANALYTICAL'	20.	2,50%	2,50%	2,50%	10.00%	3,00%
σ_{xy}	'ANALYTICAL'	0.	2,5	2,5	2,5	2,5	2,5
ϵ_{xx}	'ANALYTICAL'	$-6. 10^{-4}$	0,50%	0,50%	0,50%	1,50%	2.0%
ϵ_{yy}	'ANALYTICAL'	$6. 10^{-4}$	0,50%	0,50%	0,50%	1,00%	1,00%
ϵ_{xy}	'ANALYTICAL'	0.	$3. 10^{-5}$	$3. 10^{-5}$	$3. 10^{-5}$	$3. 10^{-5}$	$3. 10^{-5}$
$\epsilon_{eq} - I NVA_2$	'ANALYTICAL'	$6.92 10^{-4}$	0,50%	0,50%	0,50%	1,00%	1.50%
$\epsilon_{eq} - PRIN_1$	'ANALYTICAL'	$- 6. 10^{-4}$	0,50%	0,50%	0,50%	1,50%	2.0%
$\epsilon_{eq} - PRIN_2$	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
$\epsilon_{eq} - PRIN_3$	'ANALYTICAL'	$6. 10^{-4}$	0,50%	0,50%	0,50%	1,00%	1.0%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	1,00%	1,00%	1,00%	1,00%	1.5%
$\sigma_{eq} - TRESCA$	'ANALYTICAL'	160.	1,00%	1,00%	1,00%	1,00%	1.5%
$\sigma_{eq} - PRIN_1$	'ANALYTICAL'	-60.	3,00%	3,00%	3,00%	4.00%	3,00%
$\sigma_{eq} - PRIN_2$	'ANALYTICAL'	20.	3,00%	3,00%	3,00%	10.00%	3,00%
$\sigma_{eq} - PRIN_3$	'ANALYTICAL'	100.	1,00%	1,00%	1,00%	1.50%	1,00%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	1,00%	1,00%	1,00%	1,00%	1.5%

Result at the point F :

- first column 3D_INCO_UPG without imposing $GONF = 0$
- second column 3D_INCO_UPG while imposing $GONF = 0$
- third column 3D_INCO_UP with quadratic elements
- fourth column 3D_INCO_UP with linear elements
- fifth column 3D_INCO_UPO with linear elements

Identification	Type of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	'ANALYTICAL'	$2.12 \cdot 10^{-5}$	0,50%	0,50%	0,50%	0,50%	0,50%
v	'ANALYTICAL'	$2.12 \cdot 10^{-5}$	0,50%	0,50%	0,50%	0,50%	0,50%
σ_{xx}	'ANALYTICAL'	20.	1,00%	1,00%	1,00%	1,50%	1,50%
σ_{yy}	'ANALYTICAL'	20.	1,00%	1,00%	1,00%	1,00%	1,00%
σ_{zz}	'ANALYTICAL'	20.	1,00%	1,00%	1,00%	1,50%	1,00%
σ_{xy}	'ANALYTICAL'	-20.	1,00%	1,00%	1,00%	1,00%	1,00%
ϵ_{xx}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ϵ_{yy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ϵ_{xy}	'ANALYTICAL'	$-1.5 \cdot 10^{-4}$	0,50%	0,50%	0,50%	1,00%	1,00%

Checking of the passage to the nodes for the nodes mediums (only for the result got without imposing $GONF = 0$) - value with the node *NOEUMI* :

Identification	Type of reference	Value of reference	Tolerance (%)
σ_{xx}	'ANALYTICAL'	-60.	1,70%
σ_{yy}	'ANALYTICAL'	100.	0,60%
σ_{zz}	'ANALYTICAL'	20.	3,50%
ϵ_{xx}	'ANALYTICAL'	$-6. \cdot 10^{-4}$	0,50%
ϵ_{yy}	'ANALYTICAL'	$6. \cdot 10^{-4}$	0,50%

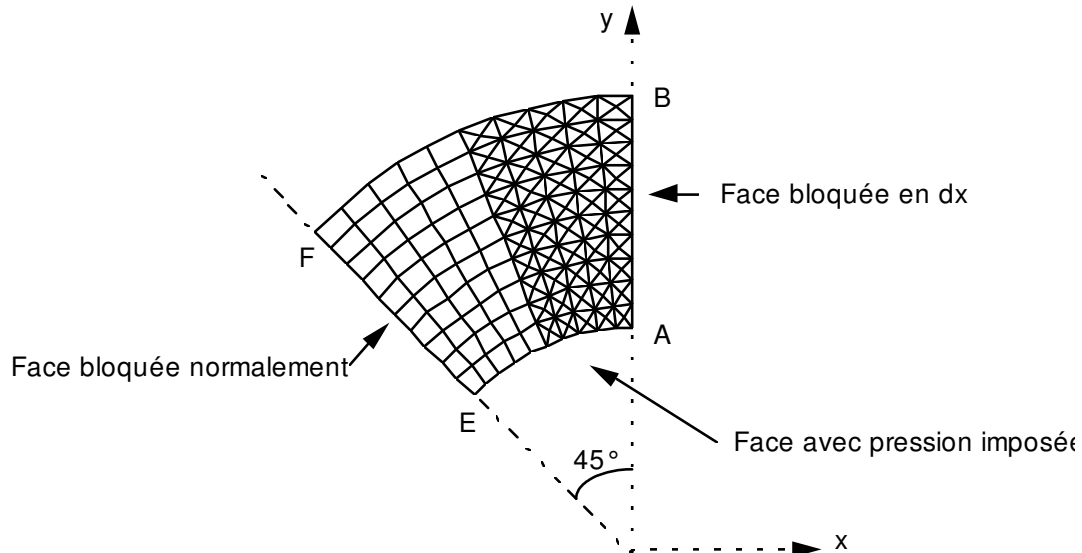
4.4 Remarks

The results got here are a little less good than in the case of modeling A , but the discretization is coarser since there are approximately 2 times less nodes in this CAS-test. The results are satisfactory all the same since the variations are lower than 0.2 % for displacements, lower than 0.5 % for the deformations and lower than 1 % for the constraints. It is noted again that there is no significant improvement of the result when one imposes explicitly $tr \epsilon = 0$.

5 Modeling C

5.1 Characteristics of modeling

Grid with elements 2D incompressible of type QUAD8 and TRIA6.



Limiting conditions:

```
DDL_IMPO= GROUP_NO = ' GRNM11', DX = 0           side AB
FACE_IMPO =GROUP_MA = ' GRMA12', DNOR = 0        dimensioned EF
PRES_REP  =GROUP_MA = ' GRMA13', CLOSE = 60     face AE
```

Name of the nodes:

$A=N2$, $B=N361$, $C=N121$, $D=N584$, $E=N155$, $F=N503$

5.2 Characteristics of the grid

Many nodes: 591

Many meshes: 200 TRIA6, 50 QUAD8.

5.3 Sizes tested and results

Results at the point A :

- first column D_PLAN_INCO_UPG without imposing $GONF = 0$
- second column D_PLAN_INCO_UPG while imposing $GONF = 0$
- third column D_PLAN_INCO_UP with quadratic grid
- fourth column D_PLAN_INCO_UP with linear grid
- fifth column D_PLAN_INCO_UPO with linear elements

Identification	Type of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
v	'ANALYTICAL'	$6. 10^{-5}$	0,50%	0,50%	0,50%	0,50%	0,50%
σ_{xx}	'ANALYTICAL'	100.	0,50%	0,50%	0,50%	0.60%	0,50%
σ_{yy}	'ANALYTICAL'	- 60.	0,50%	0,50%	0,50%	2.50%	2.10%
σ_{zz}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	5.00%	1.60%
σ_{xy}	'ANALYTICAL'	0.	0,3	0,3	0,3	1.5	0,9
ϵ_{xx}	'ANALYTICAL'	$6. 10^{-4}$	0,50%	0,50%	0,50%	0,50%	0,50%
ϵ_{yy}	'ANALYTICAL'	- $6. 10^{-4}$	0,50%	0,50%	0,50%	1,50%	1,50%
ϵ_{xy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
$\epsilon_{eq} - I NVA_2$	'ANALYTICAL'	$6.92 10^{-4}$	0,50%	0,50%	0,50%	1,00%	1,00%
$\epsilon_{eq} - PRIN_1$	'ANALYTICAL'	- $6. 10^{-4}$	0,50%	0,50%	0,50%	1,50%	1,50%
$\epsilon_{eq} - PRIN_2$	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
$\epsilon_{eq} - PRIN_3$	'ANALYTICAL'	$6. 10^{-4}$	0,50%	0,50%	0,50%	0,50%	0,50%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	1,00%	1,00%	1,00%	1,00%	1,00%
$\sigma_{eq} - TRESCA$	'ANALYTICAL'	160.	1,00%	1,00%	1,00%	1,00%	1,00%
$\sigma_{eq} - PRIN_1$	'ANALYTICAL'	-60.	1,00%	1,00%	1,00%	2.50%	2.10%
$\sigma_{eq} - PRIN_2$	'ANALYTICAL'	20.	1,00%	1,00%	1,00%	5.00%	1.60%
$\sigma_{eq} - PRIN_3$	'ANALYTICAL'	100.	1,00%	1,00%	1,00%	1,00%	1,00%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	1,00%	1,00%	1,00%	1,00%	1,00%

Results at the point F :

- first column D_PLAN_INCO_UPG without imposing $GONF=0$
- second column D_PLAN_INCO_UPG while imposing $GONF=0$
- third column D_PLAN_INCO_UP with quadratic grid
- fourth column D_PLAN_INCO_UP with linear grid
- fifth column D_PLAN_INCO_UPO with linear elements

Identification	Type of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	'ANALYTICAL'	$-2.12 \cdot 10^{-5}$	0,50%	0,50%	0,50%	0,50%	0,50%
v	'ANALYTICAL'	$2.12 \cdot 10^{-5}$	0,50%	0,50%	0,50%	0,50%	0,50%
σ_{xx}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	1,00%	1,00%
σ_{yy}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	2,00%	1,50%
σ_{zz}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	0,50%	0,50%
σ_{xy}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	0,50%	0,50%
ϵ_{xx}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ϵ_{yy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ϵ_{xy}	'ANALYTICAL'	$1.5 \cdot 10^{-4}$	0,50%	0,50%	0,50%	0,50%	0,50%

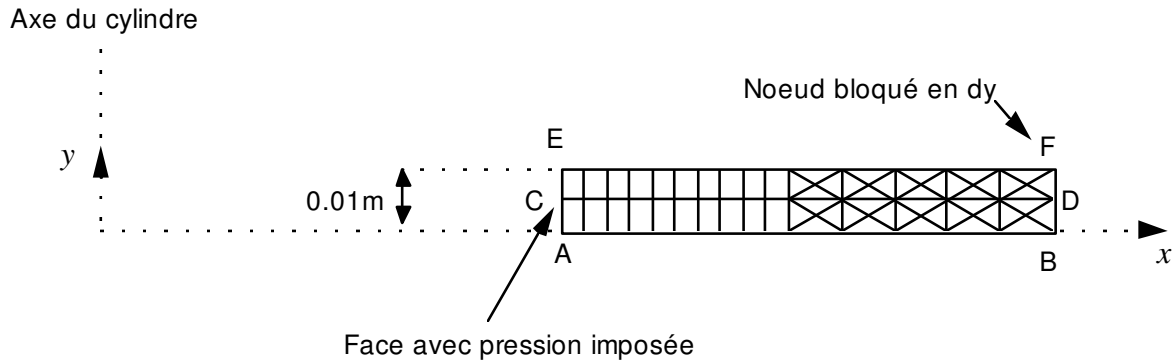
5.4 Remarks

As for modeling 3D, the got results are completely satisfactory.

6 Modeling D

6.1 Characteristics of modeling

Elements AXI incompressible (TRIA6 + QUAD8)



For the needs for examination, one defines the nodes:

- $NOEUMIA = A + (0.01/4)$ where the strains and stresses are the same one as in A
- $NOEUMIB = B + (0.01/4)$ where the strains and stresses are the same one as in B

Limiting conditions:

```
DDL_IMPO= GROUP_NO = ' FACSUP', DY = 0          y=0.1
          GROUP_NO = ' FACINF', DY = 0          y=0
PRES_REP  =GROUP_MA = ' FACEAE', CLOSE = 60face AE
```

6.2 Characteristics of the grid

Many nodes: 175.

Many meshes and types: 20 QUAD8, 40 TRIA6.

6.3 Sizes tested and results

Results at the point A :

- first column AXIS_INCO_UPG without imposing $GONF = 0$
- second column AXIS_INCO_UPG while imposing $GONF = 0$
- third column AXIS_INCO_UP with quadratic grid
- fourth column AXIS_INCO_UP with linear grid
- fifth column AXIS_INCO_UPO with linear elements

Identification	Type of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	'ANALYTICAL'	$6. 10^{-5}$	0,10%	0,10%	0,10%	0,10%	0,10%
v	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
σ_{xx}	'ANALYTICAL'	- 60.	0,50%	0,50%	0,50%	0.70%	1.50%
σ_{yy}	'ANALYTICAL'	20/03/12	0,50%	0,50%	0,50%	1,00%	9.00%
σ_{zz}	'ANALYTICAL'	100.	0,50%	0,50%	0,50%	0,50%	6.00%
σ_{xy}	'ANALYTICAL'	0.	10^{-5}	0,01	10^{-5}	0.08	0.2
ϵ_{xx}	'ANALYTICAL'	$-6. 10^{-4}$	0,50%	0,50%	0,50%	0,60%	0,60%
ϵ_{yy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ϵ_{xy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
$\epsilon_{eq} - I NVA_2$	'ANALYTICAL'	$6.92 10^{-4}$	0,50%	0,50%	0,50%	0,50%	3.0%
$\epsilon_{eq} - PRIN_1$	'ANALYTICAL'	$- 6. 10^{-4}$	0,50%	0,50%	0,50%	1,00%	6.0%
$\epsilon_{eq} - PRIN_2$	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
$\epsilon_{eq} - PRIN_3$	'ANALYTICAL'	$6. 10^{-4}$	0,50%	0,50%	0,50%	0,50%	0.60%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	1,00%	1,00%	1,00%	1,00%	3.00%
$\sigma_{eq} - TRESCA$	'ANALYTICAL'	160.	1,00%	1,00%	1,00%	1,00%	3.00%
$\sigma_{eq} - PRIN_1$	'ANALYTICAL'	-60.	1,00%	1,00%	1,00%	1,00%	2.00%
$\sigma_{eq} - PRIN_2$	'ANALYTICAL'	20.	1,00%	1,00%	1,00%	1,00%	23, %
$\sigma_{eq} - PRIN_3$	'ANALYTICAL'	100.	1,00%	1,00%	1,00%	1,00%	5.5%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	1,00%	1,00%	1,00%	1,00%	3.00%

Results at the point F :

- first column AXIS_INCO_UPG without imposing $GONF=0$
- second column AXIS_INCO_UPG while imposing $GONF=0$
- third column AXIS_INCO_UP with quadratic grid
- fourth column AXIS_INCO_UP with linear grid
- fifth column AXIS_INCO_UPO with linear elements

Identification	Type of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	'ANALYTICAL'	$3. 10^{-5}$	0,10%	0,10%	0,10%	0,10%	2.0%
v	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
σ_{xx}	'ANALYTICAL'	0.	0.03	0.03	0.03	0.10	0.70
σ_{yy}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	0,50%	1.1%
σ_{zz}	'ANALYTICAL'	40.	0,50%	0,50%	0,50%	0,50%	0.60%
σ_{xy}	'ANALYTICAL'	0.	$5. 10^{-3}$	$5. 10^{-3}$	$5. 10^{-3}$	0,01	0.2
ϵ_{xx}	'ANALYTICAL'	$-1.5 10^{-4}$	0,50%	0,50%	0,50%	0,50%	1, 0%
ϵ_{yy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ϵ_{xy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}

Checking of the passage to the nodes for the nodes mediums (only for the result got without imposing $GONF=0$)

NOEUMIA

Identification	Type of reference	Value of reference	Tolerance (%)
σ_{xx}	'ANALYTICAL'	-60.	0,50%
σ_{yy}	'ANALYTICAL'	20.	0,50%
σ_{zz}	'ANALYTICAL'	100.	0,50%
ϵ_{xx}	'ANALYTICAL'	$-6. 10^{-4}$	0,50%
ϵ_{yy}	'ANALYTICAL'	$6. 10^{-4}$	0,50%

NOEUMIB

Identification	Type of reference	Value of reference	Tolerance (%)
σ_{xx}	'ANALYTICAL'	0.	0,50%
σ_{yy}	'ANALYTICAL'	20.	0,50%
σ_{zz}	'ANALYTICAL'	40.	0,50%
ϵ_{xx}	'ANALYTICAL'	$-1.5 10^{-4}$	0,50%

ε_{yy}	'ANALYTICAL'	$1.5 \cdot 10^{-4}$	0,50%
--------------------	--------------	---------------------	-------

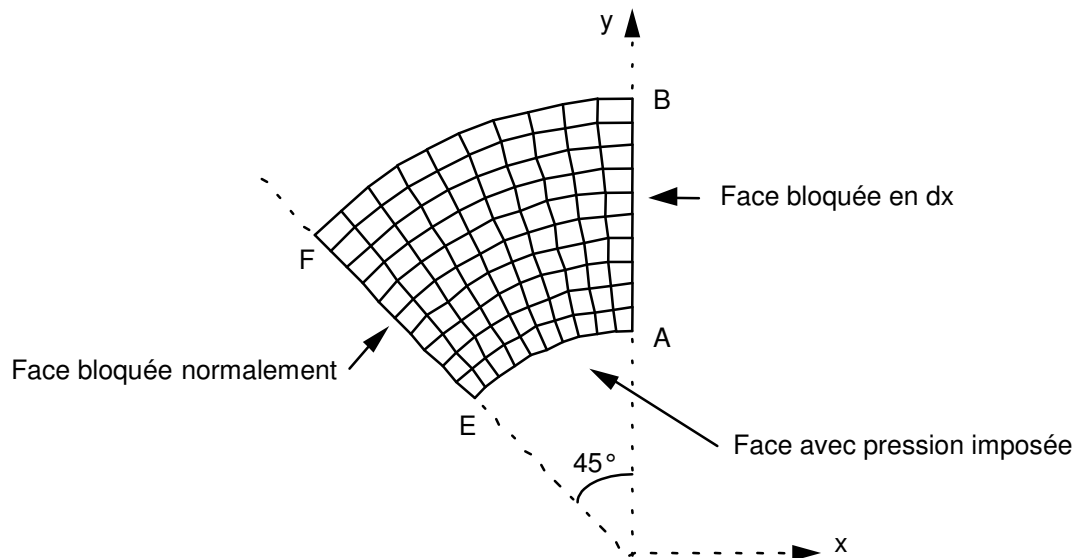
6.4 Remarks

The precision obtained is very good.

7 Modeling E

7.1 Characteristics of modeling

Grid with elements 3D incompressible of type PENTA15 only



Along the axis z :

- total thickness $e=0.01\text{ mm}$
- 2 layers of elements

For the needs for examination in a node medium, one defines the node $NOEUMI = A + (0.0 \cdot e/4)$ where the strains and the stresses are the same ones as in A .

Limiting conditions:

```
DDL_IMPO= GROUP_NO = ' FACSUP', DZ = 0
GROUP_NO = ' FACINF', DZ = 0      faces AEFD ( z=0 and z=0.01 )
GROUP_NO = ' FACEAB', DX = 0      face AB
FACE_IMPO =GROUP_MA = ' FACEEF', DNOR = 0      face EF
PRES_REP  =GROUP_MA = ' FACEAE', CLOSE = 60. face AE
```

7.2 Characteristics of the grid

Many nodes: 1501 nodes
Many meshes: 480 PENTA15

7.3 Sizes tested and results

Results at the point A :

- first column 3D_INCO_UPG without imposing $GONF = 0$
- second column 3D_INCO_UPG while imposing $GONF = 0$
- third column 3D_INCO_UP
- fourth column 3D_INCO_UPO with linear elements

Identification	Type of reference	Value of reference	Tolerance			
			1	2	3	4
u	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
v	'ANALYTICAL'	$6. \cdot 10^{-5}$	0,10%	0,10%	0,10%	0.50%
σ_{xx}	'ANALYTICAL'	100.	0,50%	0,50%	0,50%	6.0%
σ_{yy}	'ANALYTICAL'	- 60.	0,50%	0,50%	0,50%	15.0%
σ_{zz}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	5,00%
σ_{xy}	'ANALYTICAL'	0.	0,6	0,6	0,6	06/03/12
ϵ_{xx}	'ANALYTICAL'	$6. \cdot 10^{-4}$	0,50%	0,50%	0,50%	1.0%
ϵ_{yy}	'ANALYTICAL'	- $6. \cdot 10^{-4}$	0,50%	0,50%	0,50%	1.0%
ϵ_{zz}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ϵ_{xy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	$5,10^{-5}$
$\epsilon_{eq} - I NVA_2$	'ANALYTICAL'	$6.92 \cdot 10^{-4}$	0,50%	0,50%	0,50%	8.0%
$\epsilon_{eq} - PRIN_1$	'ANALYTICAL'	- $6. \cdot 10^{-4}$	0,50%	0,50%	0,50%	8.0%
$\epsilon_{eq} - PRIN_2$	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
$\epsilon_{eq} - PRIN_3$	'ANALYTICAL'	$6. \cdot 10^{-4}$	0,50%	0,50%	0,50%	8.0%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	0,50%	0,50%	0,50%	10, %
$\sigma_{eq} - TRESCA$	'ANALYTICAL'	160.	0,50%	0,50%	0,50%	10, %
$\sigma_{eq} - PRIN_1$	'ANALYTICAL'	-60.	0,50%	0,50%	0,50%	12, %
$\sigma_{eq} - PRIN_2$	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	5,00%
$\sigma_{eq} - PRIN_3$	'ANALYTICAL'	100.	0,50%	0,50%	0,50%	6.0%
$\sigma_{eq} - VMIS$	'ANALYTICAL'	138.56	0,50%	0,50%	0,50%	10, %

Results at the point F :

- first column 3D_INCO_UPG without imposing $GONF=0$
- second column 3D_INCO_UPG while imposing $GONF=0$
- third column 3D_INCO_UP
- fourth column 3D_INCO_UPO with linear elements

Identification	Type of reference	Value of reference	Tolerance			
			1	2	3	4
u	'ANALYTICAL'	$-2.12 \cdot 10^{-5}$	0,10%	0,10%	0,10%	0.30%
v	'ANALYTICAL'	$2.12 \cdot 10^{-5}$	0,10%	0,10%	0,10%	0.30%
σ_{xx}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	50,00%
σ_{yy}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	6.0%
σ_{zz}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	0,50%
σ_{xy}	'ANALYTICAL'	20.	0,50%	0,50%	0,50%	1.50%
ε_{xx}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{yy}	'ANALYTICAL'	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{xy}	'ANALYTICAL'	$1.5 \cdot 10^{-4}$	0,50%	0,50%	0,50%	1.50%

Checking of the passage to the nodes for the nodes mediums (only for the result got without imposing $GONF=0$) - value with the node *NOEUMI* :

Identification	Type of reference	Value of reference	Tolerance (%)
σ_{xx}	'ANALYTICAL'	100.	0,50%
σ_{yy}	'ANALYTICAL'	- 60.	0,50%
σ_{zz}	'ANALYTICAL'	20.	0,50%
ε_{xx}	'ANALYTICAL'	$6 \cdot 10^{-4}$	0,50%
ε_{yy}	'ANALYTICAL'	$- 6 \cdot 10^{-4}$	0,50%

7.4 Remarks

Except for modeling 3D_INCO_UPO, one obtains very good performances some is the formulation selected since for all the examined sizes, the difference between the solution obtained with the code and the analytical solution is lower than 0.5%. It is seen that the variation enters the solutions obtained by imposing or not the condition $tr(\varepsilon)=0$ is unimportant.

Results got with 3D_INCO_UPO are less precise. That is explained by the fact why the elements used are linear. To find a more precise result, the grid would have to be refined.

8 Summary of the results

With a Poisson's ratio ν very near to 0.5, one finds the results of the incompressible analytical solution with a weak difference. It is noticed that it is not necessary to impose the condition of incompressibility explicitly $\text{tr} \varepsilon = 0$ to obtain good performances since the results are quasi-identical that one activates or not, the condition $GONF = 0$ with `DDL_IMPO`.