SSLV139 - Buckling of a circular plate subjected to a compressive force uniformly distributed on its contour

Summary:

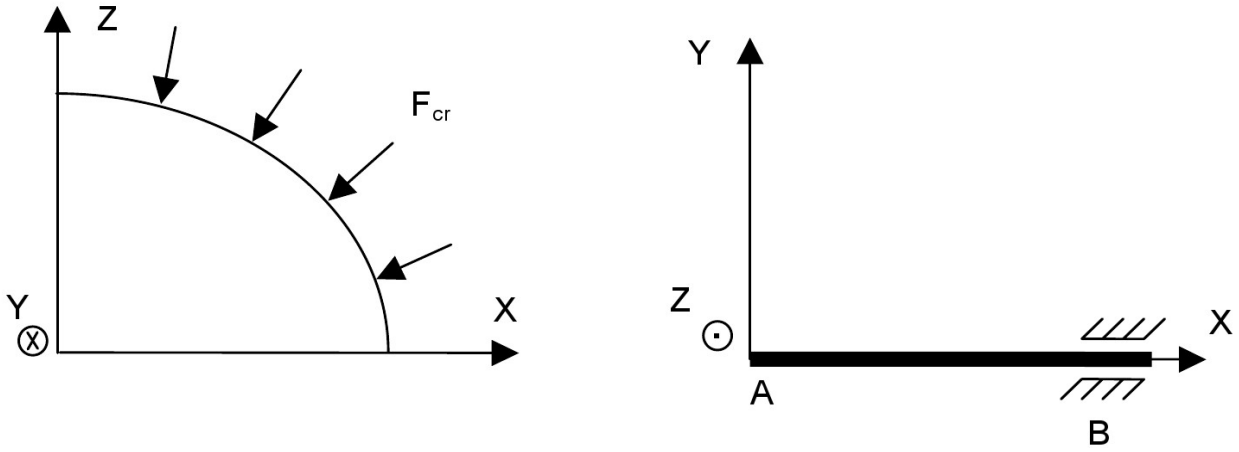
This test represents a calculation of stability of a circular plate subjected to a compressive force uniformly distributed on its contour. We determine the critical load leading to the elastic buckling of Euler as well as the associated modal deformation.

This test validates modeling AXIS_FOURIER for linear buckling with meshs QUAD8 and TRIA6 (circumferential mode equal to zero), and modeling AXIS with meshs QUAD8.

The critical load obtained is compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



- Thickness of the plate: $h=0.0005\,m$
- Ray of the plate : $R=0.115\,m$

1.2 Properties of material

The properties of material constituting the plate are:

$E=2.1\,10^{11}\,Pa$	Young modulus
$\nu=0.3$	Poisson's ratio

1.3 Boundary conditions and loadings

Boundary conditions:

- on the contour of the plate (not B) : following displacement $Y=0$ and rotation around $Z=0$

Loading: one applies a compressive force uniformly distributed F on the external contour of the plate.

1.4 Initial conditions

Without object.

2 Reference solution

2.1 Method of calculating

The value of the critical load is given in [bib1] by the following expression:

$$F_{cr}=\frac{14.68\,D}{R^2}$$

with: D the bending stiffness of the plate (in $N.m$) defined by the following expression:

$$D=\frac{E\,h^3}{12\left(1-\nu^2\right)}$$

This critical load is associated with a circumferential mode equal to 0.

2.2 Sizes and results of reference

For the characteristics given, the critical load is worth:

$F_{cr}=2668.315\,N/m$

2.3 Uncertainties on the solution

Analytical solution

2.4 Bibliographical references

1) S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, second edition, DUNOD (1966)

3 Modeling A

3.1 Characteristics of modeling

Modeling AXIS_FOURIER (QUAD8): 3 degrees of freedom per node (DX, DY, DZ)

Limiting conditions:
- group of meshs AD : $DX=0, DZ=0$
- group of meshs BC : $DY=0, DZ=0$

Characteristics of the discretization
- Sides AB and CD : 460 elements
- Sides AD and BC : 4 elements

3.2 Characteristics of the grid

Many nodes: 6449
Many meshs: 1840 (QUAD8)

3.3 Sizes tested and results

Identification	Reference
Pressure criticizes ($n=0$)	$8.4935\,10^5\,Pa$
Displacement DY with the node D	1

3.4 Remarks

- Critical pressure P_{cr} of reference, used in the command file, was obtained starting from the critical load referred in the paragraph [§2.2]:

$$P_{cr}=\frac{F_{cr}}{2\pi h}=849350.94\,N/m^2$$

- The standardisation of the clean mode for largest of the components of translation implies a value of reference equal to 1 for displacement DY with the node D.

4 Modeling B

4.1 Characteristics of modeling

Modeling AXIS (QUAD8): 2 degrees of freedom per node (DX, DY)

Limiting conditions:
- group of meshs AD : $DX=0$
- group of meshs BC : $DY=0$

Characteristics of the discretization
- Sides AB and CD : 460 elements
- Sides AD and BC : 4 elements

4.2 Characteristics of the grid

Many nodes: 6449
Many meshs: 1840 (QUAD8)

4.3 Sizes tested and results

Identification	Reference
Pressure criticizes ($n=0$)	$8.4935\,10^5\,Pa$
Displacement DY with the node D	1

4.4 Remarks

- Critical pressure P_{cr} of reference, used in the command file, was obtained starting from the critical load referred in the paragraph [§2.2]:

$$P_{cr}=\frac{F_{cr}}{2\pi h}=849350.94\,N/m^2$$

- The standardisation of the clean mode for largest of the components of translation implies a value of reference equal to 1 for displacement DY with the node D.

5 Modeling C

5.1 Characteristics of modeling

Modeling AXIS_FOURIER (TRIA6): 3 degrees of freedom per node (DX, DY, DZ)

Limiting conditions:

- group of meshs AD : $DX=0$, $DZ=0.$
- group of meshs BC : $DY=0$, $DZ=0.$

Characteristic of the discretization

- Sides AB and CD : 690 elements
- Sides AD and BC : 6 elements

5.2 Characteristics of the grid

Many nodes: 17,964
Many meshs: 8,280 (TRIA6)

5.3 Sizes tested and results

Identification	Reference
Pressure criticizes ($n=0$)	$8.4935\,10^5 Pa$
Displacement DY with the node D	1

5.4 Remarks

- Critical pressure P_{cr} of reference, used in the command file, was obtained starting from the critical load referred in the paragraph [§2.2]:

$$P_{cr}=\frac{F_{cr}}{2\pi h}=849350.94\,N/m^2$$

- The standardisation of the clean mode for largest of the components of translation implies a value of reference equal to 1 for displacement DY with the node D .

6 Summary of the results

The got results are very satisfactory for the meshs QUAD8 independently of modeling used (AXIS or AXIS_FOURIER) : uncertainties on the critical pressure do not exceed 0.104%.

However, it will be noted that modeling AXIS_FOURIER is definitely less precise with meshs TRIA6 that with meshs QUAD8.

This test with licence to test and compare modelings AXIS and AXIS_FOURIER in linear buckling of Euler of a circular mean structure subjected to an external force of compression uniformly distributed on its contour.