SSLV303 - Cylinder embedded under actual weight and pressure

Summary:

The goal of the test is to validate a load of gravity as well as a pressure, starting from an analysis 2D with decomposition in Fourier series of the load.

Two modelings are adopted for this analysis; they differ from the keyword used to define gravity:

1) modeling a: to validate the keyword GRAVITY,
2) modeling b: to validate the keyword FORCE_INTERNE.

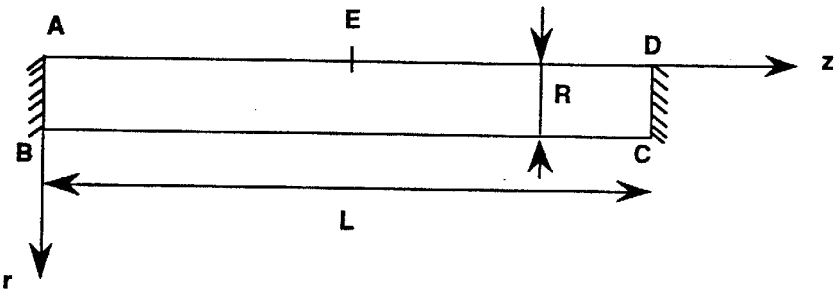
Gravity is calculated in mode 1, and the pressure applied is given in mode 1.

The two loading cases are combined and compared with a digital calculation in modeling 3D (model C).

The fourth modeling (D) is built with an aim of validating the keyword FORCE_INTERNE defined starting from a function.

1 Problem of reference

1.1 Geometry



Length : $L=0.240\,m$
Ray : $R=0.006\,m$

1.2 Material properties

$E=2.1\times10^{11}\,N/m^2$
$\nu=0.3$
$\rho=7800\,kg/m^3$

1.3 Boundary conditions and loadings

Sections AB , CD embedded
Gravity according to r : $g=9.81\,m/s^2$
Pressure given by : $p=p_o\cos\theta$, $p_o=10\,000\,N/m^2$

1.4 Initial conditions

Without object for the static analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

In the case of load of gravity alone:
The value of the field of radial displacement, according to z , is given by:

$$ur = \frac{q}{12EI}\left(\frac{Z^4}{2} - Lz^3 + \frac{L^2}{2}z^2\right)$$

Maximum displacement, in the median section, is worth:

$$ur_{(E)} = \frac{PL^3}{384EI},\ P$$: poids propre du cylindre

In the case of load of pressure, one carries out a comparison with the results of modeling C.

2.2 Results of reference

1) Displacement in the median section, $ur(E) = 0.3566 \times 10^{-6} m$
2) Constraints of embedding at point b: $\sigma_{zz} = -0.2496 \times 10^{6} Pa$

2.3 Uncertainty on the solution

Analytical solution for gravity.

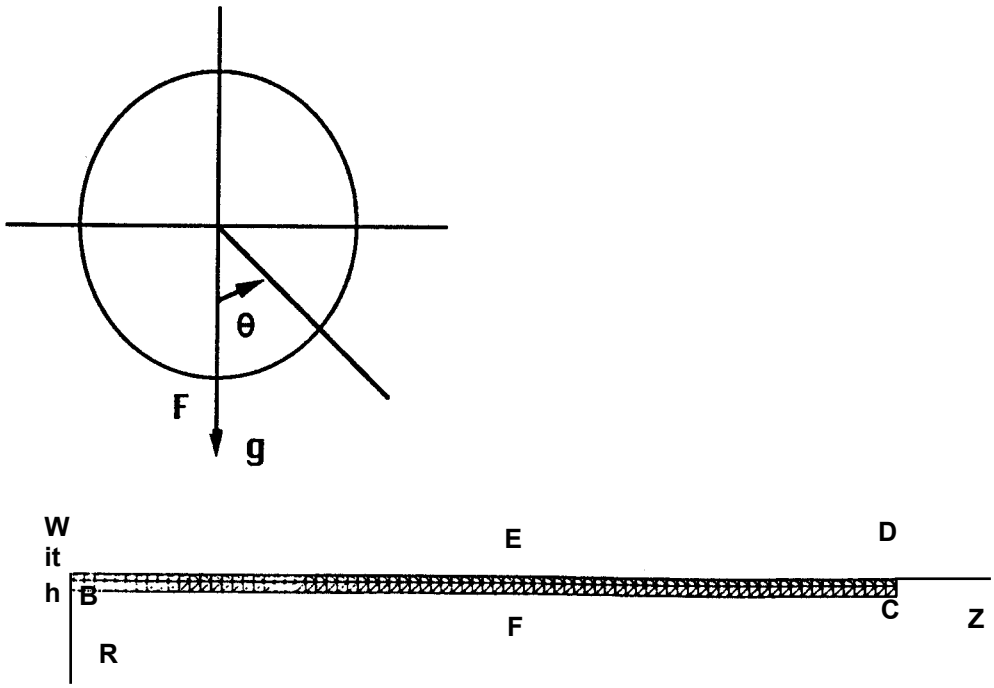
2.4 Bibliographical reference

1) S. TIMOSHENKO: Resistance of materials, 1st part. Polytechnic bookstore CH. Béranger, Paris, 1947

3 Modeling A

3.1 Characteristics of modeling A

AXIS_FOURIER, T6 nets



Cutting: 80 elements according to the length
2 elements according to the ray

Loadings:

$C1$: vertical gravity (field Ug) ($g\sqrt{2}$)
$C2$: pressure (field Up) $p_o = 10\,000\,N/m^2$

Components of displacements: u_r (radial), u_z (axial), u_θ (circumferential)

Names of the nodes:

$A = N1$ $B = N2$ $C = N3$ $D = N4$ $E = N249$ $F = N87$

3.2 Characteristics of the grid

Many nodes: 805
Many meshs and types: 320 TRIA6

3.3 Values tested

Values provided for $\theta = 0$.

Localization	Type of value	Reference	*Aster*	% difference
Field Ug (for $\theta=0$)				
Not E, F	$u_r(m)$	3,566 X 10^{-7}	3,541 X 10^{-7}	– 0,701
	$u_\theta(m)$	0.	3.94 10^{-14}	
Not B	$\sigma_{zz}(Pa)$	– 2,496 X 10^5	– 2,598 X 10^5	+ 4.09
Field Up (for $\theta=0$)				
Not E	$u_r(m)$	– 7.82 X 10^{-6}	– 7.71 X 10^{-6}	– 1.4
Not F		– 7.82 X 10^{-6}	– 7.70 X 10^{-6}	– 1.5
Not B	$\sigma_{rr}(Pa)$	1.63 X 10^6	1.41 X 10^6	– 13.4
	$\sigma_{zz}(Pa)$	5.51 X 10^6	5.65 X 10^6	2.7
	$\sigma_{\theta\theta}(Pa)$	1.65 X 10^6	1.89 X 10^6	14.7
Field $Up+Ug$ (for $\theta=0$)				
Not E	$u_r(m)$	– 7.46 X 10^{-6}	– 7,358 X 10^{-6}	– 1.3
Not F	$u_r(m)$	– 7.44 X 10^{-6}	– 7,348 X 10^{-6}	– 1.2
Not B	$\sigma_{rr}(Pa)$	1.56 X 10^6	1.34 X 10^6	– 13.7
	$\sigma_{zz}(Pa)$	5.25 X 10^6	5,398 X 10^6	2.8
	$\sigma_{\theta\theta}(Pa)$	1.57 X 10^6	1.80 X 10^6	15.0

3.4 Remarks

1) The values of reference for the pressure (Up field) are obtained in modeling C, starting from a grid 3D.
2) For gravity, it should be stressed that the order of the components in GRAVITY is: r, θ, z (whereas in FORCE_INTERNE the order is r, z, θ).

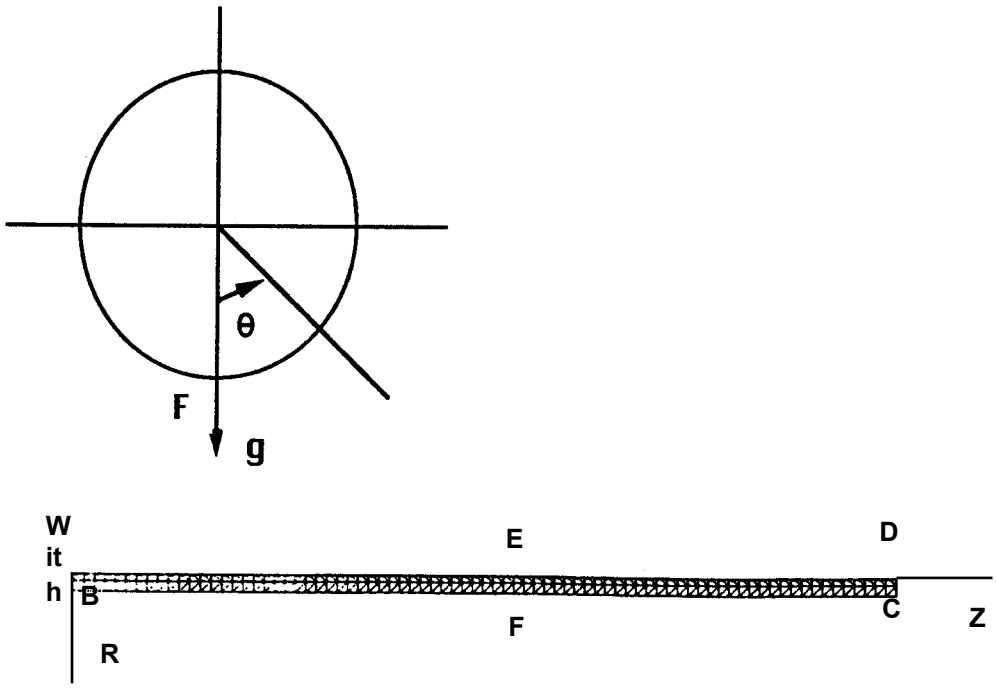
3.5 Contents of the file results

Displacements, constraints.

4 Modeling B

4.1 Characteristics of modeling B

AXIS_FOURIER, T6 meshs



Cutting: 80 elements according to the length
2 elements according to the ray

Loadings:

$C1$: vertical gravity (field Ug) in the form of voluminal density of forces $\rho g = 76518 Pa$
$C2$: pressure (field Up)

Components of displacements: u_r (radial), u_z (axial), u_θ (circumferential)

Names of the nodes:

$A = N1$ $B = N2$ $C = N3$ $D = N4$ $E = N249$ $F = N87$

4.2 Characteristics of the grid

Many nodes: 805
Many meshs and types: 320 TRIA6

4.3 Values tested

Localization	Type of value	Reference	*Aster*	% difference

Field Ug (for $\theta=0.$)				
Not E, F	$u_r(m)$	3,566 X 10^{-7}	3,541 X 10^{-7}	– 0.70
	$u_\theta(m)$	0.	0.	
Not B	$\sigma_{zz}(Pa)$	– 2,496 X 10^5	– 2.60 X 10^5	+ 4.1
Field Up (for $\theta=0.$)				
Not E	$u_r(m)$	– 7.82 X 10^{-6}	– 7.71 X 10^{-6}	– 1.4
Not F		– 7.82 X 10^{-6}	– 7.70 X 10^{-6}	– 1.5
Not B	$\sigma_{rr}(Pa)$	1.63 X 10^6	1.41 X 10^6	– 13.4
	$\sigma_{zz}(Pa)$	5.51 X 10^6	5.65 X 10^6	2.7
	$\sigma_{\theta\theta}(Pa)$	1.65 X 10^6	1.89 X 10^6	14.7
Field $Up+Ug$ (for $\theta=0.$)				
Not E	$u_r(m)$	– 7.46 X 10^{-6}	– 7,358 X 10^{-6}	– 1.3
Not F	$u_r(m)$	– 7.46 X 10^{-6}	– 7,348 X 10^{-6}	– 1.5
Not B	$\sigma_{rr}(Pa)$	1.56 X 10^6	1.34 X 10^6	– 13.7
	$\sigma_{zz}(Pa)$	5.25 X 10^6	5,398 X 10^6	2.8
	$\sigma_{\theta\theta}(Pa)$	1.57 X 10^6	1.80 X 10^6	15.0

4.4 Remarks

- Values of reference for the pressure (field Up) are obtained in modeling C, starting from a grid 3D.
- The got results are rigorously identical to those of modeling A with GRAVITY.

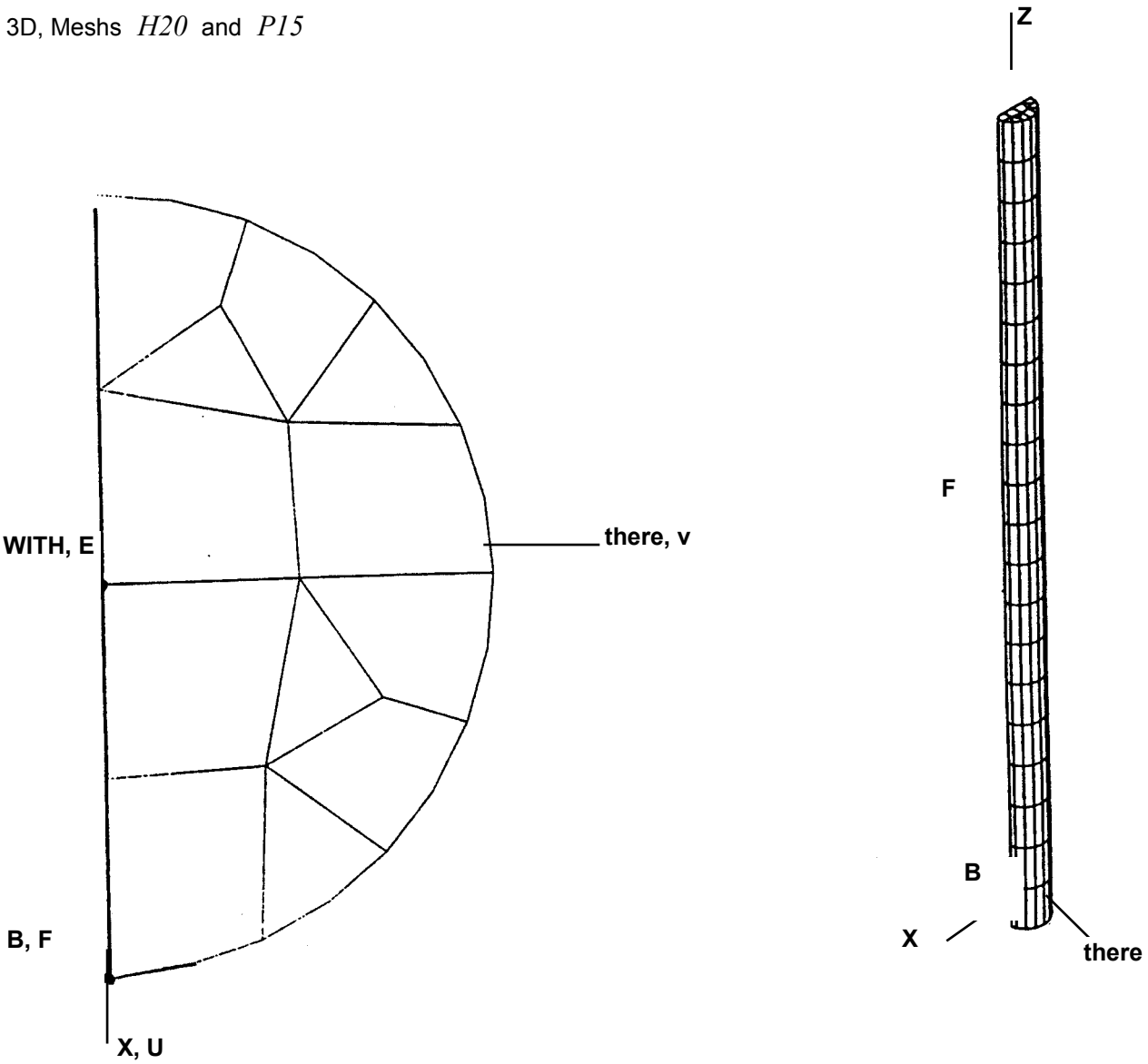
4.5 Contents of the file results

Displacements, constraints.

5 Modeling C

5.1 Characteristics of modeling C

3D, Meshs $H20$ and $P15$



Position of the points: A, B in the section $z=0$

E, F in the median section $z=L/2$

Cutting: 20 elements according to the length
2 elements according to the ray, 8 elements according to the circumference.

The loading being symmetrical, the half only of the cylinder is modelled.

Boundary conditions:

1) section embedded ends ($u=v=w=0$)
2) conditions of symmetry in the plan xz : $v=0$

1) Pressure on the circumference (field Up)
The surface of the cylinder is divided into 8 lines of elements according to the circumference (1 line of elements represents a sector of $\pi/8$ radians.
Pressure being in $\cos\theta$, it is supposed to be uniform on each line. For any point of the surface of angle θ , (including enters θ_1 and θ_2 , $\theta_1=(n-1)\frac{\pi}{8}$, $\theta_2=n\frac{\pi}{8}$, $1\leq n\leq 8$, the value of the pressure assigned to the line of elements containing this point is taken equalizes with:
$\frac{p0}{2}(\cos\theta_1+\cos\theta_2)$.

2) Vertical gravity according to x (field Ug)

Names of the nodes:

$A=N845$ $B=N965$ $E=N865$ $F=N995$

5.2 Characteristics of the grid

Many nodes: 1285
Many meshs and types: 160 HEXA20, 80 PENTA15

5.3 Values tested

Localization	Type of value	Reference	*Aster*	% difference
Field Up				
Not E	$u(m)$		– 7.82 X 10^{-6}	
	$v(m)$	0.	10^{-21}	
Not F	$u(m)$		– 7,816 X 10^{-6}	
	$v(m)$	0.	10^{-21}	
Not B	$\sigma_{xx}(Pa)$		1.63 X 10^6	
	$\sigma_{yy}(Pa)$		1.65 X 10^6	
	$\sigma_{zz}(Pa)$		5.51 X 10^6	
Field $Up+Ug$				
Not E	$u(m)$		– 7.46 X 10^{-6}	
	$v(m)$	0.	10^{-21}	
Not F	$u(m)$		– 7.44 X 10^{-6}	
	$v(m)$	0.	10^{-21}	
Not B	$\sigma_{xx}(Pa)$		1.56 X 10^6	
	$\sigma_{yy}(Pa)$		1.57 X 10^6	
	$\sigma_{zz}(Pa)$		5.25 X 10^6	

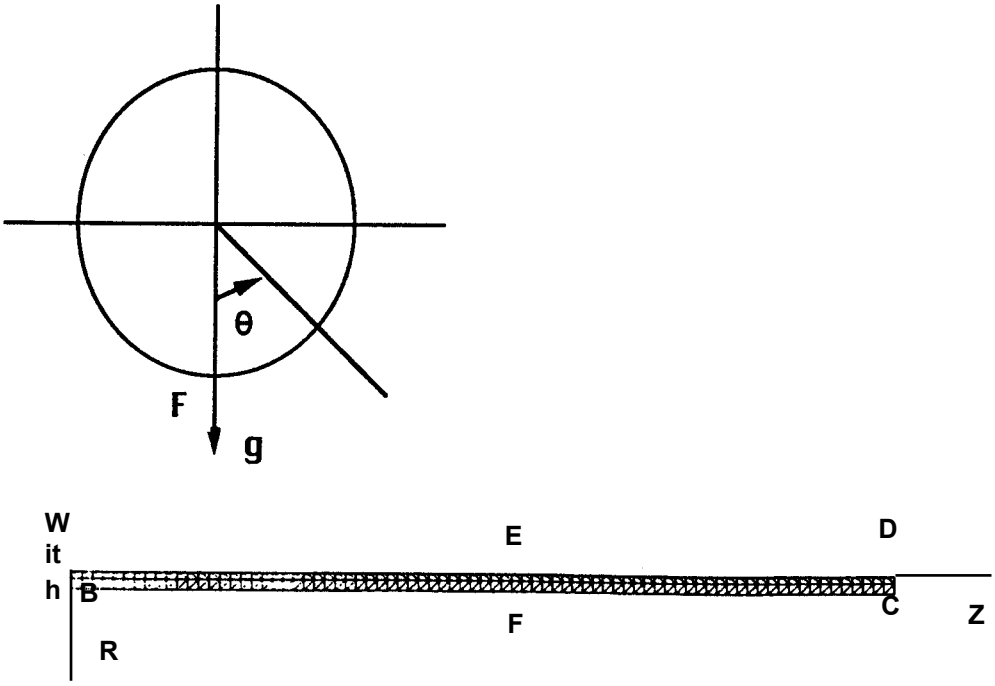
5.4 Remarks

- There are no values of reference for this modeling. The results are to be compared with those of modelings AXIS_FOURIER (A, B, D).
- At the point B (located in the symmetry plane), one a: $\sigma_{rr} = \sigma_{xx}$, $\sigma_{\theta\theta} = \sigma_{yy}$

6 Modeling D

6.1 Characteristics of modeling D

AXIS_FOURIER, T6 meshs



Cutting: 80 elements according to the length
2 elements according to the ray

Loadings:

$C1$: vertical gravity (field Ug)
$C2$: pressure (field Up)

Components of displacements: u_r (radial), u_z (axial), u_θ (circumferential)

Names of the nodes:

$A = N1$ $B = N2$ $C = N3$ $D = N4$ $E = N249$ $F = N87$

6.2 Characteristics of the grid

Many nodes: 805
Many meshs and types: 320 TRIA6

6.3 Values tested

Localization	Type of value	Reference	*Aster*	% difference
Field Ug				

(for $\theta=0.$)				
Not E, F	$u_r(m)$	3,566 X 10^{-7}	3,535 X 10^{-7}	– 0.84
	$u_\theta(m)$	0.	0.	
Not B	$\sigma_{zz}(Pa)$	– 2,496 X 10^5	– 2.60 X 10^5	+ 4.1
Field Up				
(for $\theta=0.$)				
Not E	$u_r(m)$	– 7.82 X 10^{-6}	– 7.71 X 10^{-6}	1.4
Not F		– 7.82 X 10^{-6}	– 7.70 X 10^{-6}	1.5
Not B	$\sigma_{rr}(Pa)$	1.63 X 10^6	1.41 X 10^6	– 13.4
	$\sigma_{zz}(Pa)$	5.51 X 10^6	5.65 X 10^6	2.7
	$\sigma_{\theta\theta}(Pa)$	1.65 X 10^6	1.89 X 10^6	14.7

6.4 Remarks

- Values of reference for the pressure (field Up) are obtained in modeling C, starting from a model 3D.
- The got results are identical to those of modelings A and B.

6.5 Contents of the file results

Displacements, constraints.

7 Summary of the results

Maximum differences (in %) between modelings AXIS_FOURIER and modeling 3D, observed at the points E, F, B (in the plan $\theta=0°$), on the combined loading cases.

	Localization	Variation AXIS_FOURIER/3D In (%)
Displacements U : = u in 3D = u_r in AXI	NOT F	1.5
Constraints σ_{zz}	NOT B	2.8
Constraints $\sigma_{xx}(3D)$ = $\sigma_{rr}(AXI)$	NOT B	– 14.1
Constraints $\sigma_{yy}(3D)$ = $\sigma_{\theta\theta}(AXI)$	NOT B	14.6

- Results between modelings 3D on the one hand and AXIS_FOURIER in addition, are concordant with regard to displacements (variation of 1.5%) and the bending stress σ_{zz} (variation of 2.8%).
- With embedding, the relation $\sigma_{xx}=\sigma_{yy}=0$ involve:

$$\sigma_{xx}=\sigma_{yy}=\frac{\nu}{1-\nu}\sigma_{zz}$$

The relation of embedding is well checked at the point B, in modeling 3D.

- In addition, at the point B, one has moreover:

$$\sigma_{xx}=\sigma_{rr}$$
$$\sigma_{yy}=\sigma_{\theta\theta}$$

In modeling AXIS_FOURIER, the difference between the two constraints is approximately 25%.

- The second calculation on the model AXIS_FOURIER was realized with a finer grid: 4 elements in the thickness instead of 2, denser grid in the vicinity of embedding AB (total 800 TRIA6).

The variation observed on the constraints σ_{rr} and $\sigma_{\theta\theta}$ at the point AB remain: $\sigma_{rr}=1.51\times10^{6}$, $\sigma_{\theta\theta}=2.08\times106$ (loading case combined).

The relation of embedding $\sigma_{xx}=\sigma_{yy}$ thus is checked much better on the model 3D, with a grid in the thickness however coarse.