# SSLV304 – Cylinder under variable external pressure

**Summary:**

The objective of this CAS-test is to validate the application of a pressure on an axisymmetric structure, starting from a decomposition in Fourier series of the load (modeling AXIS_FOURIER).

The pressure applied is function of the three coordinates of space $(r, \theta, z)$.

# 1 Problem of reference

## 1.1 Geometry

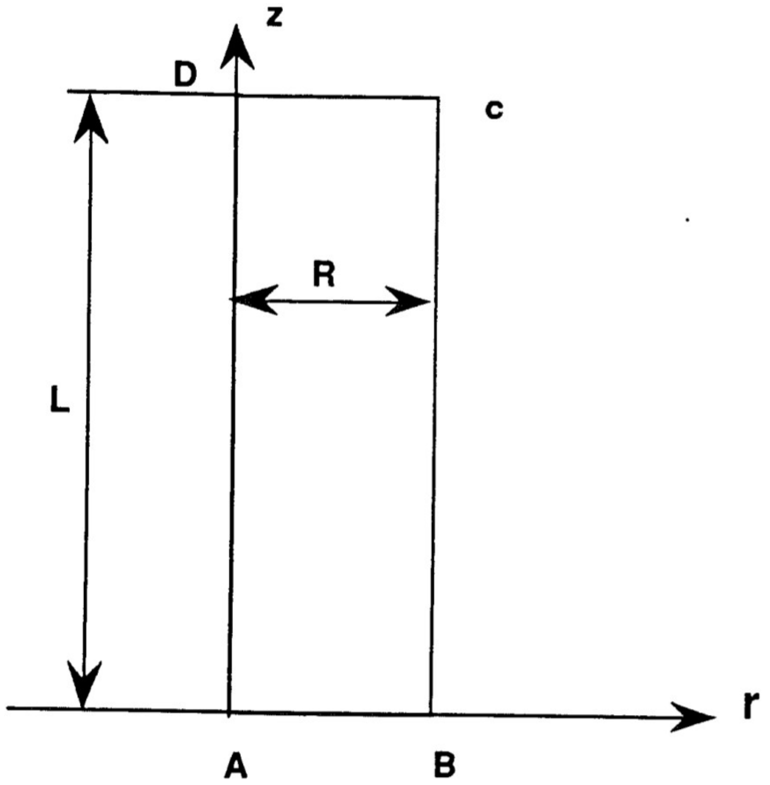


**Figure 1.1 Geometry of the problem and system of loading**

Length: $L=0.24\,m$
Ray: $R=0.006\,m$

## 1.2 Properties of material

| | |
|---|---|
| Young modulus | $E=2.1\times10^{11}\,Pa$ |
| Poisson's ratio | $\nu=0.3$ |

## 1.3 Boundary conditions and loadings

Imposed displacement:

| | |
|---|---|
| Embedding on the side $AB$ | $DX=0$ , $DY=0$ , $DZ=0$ |

Imposed loading: $p_0=10\,000\,Pa$

| | |
|---|---|
| External pressure on the side $BC$ | Radial component: $-p_0\dfrac{R}{L}\sin(\theta)$ (normal with the circumference) |

| | |
|---|---|
| | Axial component: $p_0 \frac{z}{L} \sin(\theta)$ (according to $z$ ) |
| External pressure on the side $DC$ | Radial component: $p_0 \sin(\theta)$ (according to $r$ )<br>Circumferential component: $p_0 \cos(\theta)$ (according to $\theta$ )<br>Axial component: $\frac{1-\nu}{\nu} \frac{p_0 r}{L} \sin(\theta)$ (normal with the section) |

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

The deformation due to direct compression is given by:

$$u_r=\frac{p_0}{\nu E L}\times\frac{1}{1-2\nu}\times\frac{z^2}{4}\sin(\theta)\ ;\ u_z=\frac{-p_0}{2 nu E L} rz\sin(\theta)\ ;\ u_\theta=\frac{p_0}{\nu E L}\times\frac{1}{1-2\nu}\times\frac{z^2}{4}\cos(\theta)\ .$$

The stress field is worth:

$$\sigma_{rr}=-\frac{p_0 r\sin(\theta)}{L}\ ;\ \sigma_{zz}=-\frac{1-\nu}{\nu L}p_0 r\sin(\theta)\ ;\ \sigma_{\theta\theta}=-\frac{p_0 r\sin(\theta)}{L}\ ;$$

$$\sigma_{rz}=\frac{p_0 z\sin(\theta)}{L}\ ;\ \sigma_{r\theta}=0\ ;\ \sigma_{\theta z}=\frac{p_0 z\cos(\theta)}{L}\ .$$

## 2.2 Results of reference

Radial displacements ( $DX$ ), vertical ( $DY$ ) and ortho-radial ( $DZ$ ) at the points $C$ and $D$ for an angle $\theta=45°$ .
Constraints at the points $A$ and $B$ .

## 2.3 Uncertainty on the solution

Analytical solution.

# 3 Modeling A

## 3.1 Characteristics of modeling A

Modeling AXIS_FOURIER in mode 1(the decomposition of the load is carried out according to mode 1).

## 3.2 Characteristics of the grid

Many nodes: 455
Many meshs and types: 400 TRIA3, 160 QUAD4

## 3.3 Sizes tested and results

For $\theta=45$ ,

| Size | Component | Localization | Type of reference | Value of reference | Tolerance |
|---|---|---|---|---|---|
| DEPL | DX | Not D ( $N461$ ) | 'ANALYTICAL' | $1.683587\times10^{-8}$ m | 0.25 % |
| DEPL | DY | Not D ( $N461$ ) | 'ANALYTICAL' | $0.0$ | 1. E14 Pa |
| DEPL | DZ | Not D ( $N461$ ) | 'ANALYTICAL' | $1.683587\times10^{-8}$ | 0.25 % |
| DEPL | DX | Not C ( $N460$ ) | 'ANALYTICAL' | $1.683587\times10^{-8}$ | 0.25 % |
| DEPL | DY | Not C ( $N460$ ) | 'ANALYTICAL' | $-3.36717\times10^{-10}$ | 5.2 % |
| DEPL | DZ | Not C ( $N460$ ) | 'ANALYTICAL' | $1.683587\times10^{-8}$ | 0.25 % |
| SIGM_ELNO | SIXX | Not B ( $N5$ ) | 'ANALYTICAL' | $-176.77$ | 1.3 % |
| SIGM_ELNO | SIYY | Not B ( $N5$ ) | 'ANALYTICAL' | $-412.48$ | 3.7 % |
| SIGM_ELNO | SIZZ | Not B ( $N5$ ) | 'ANALYTICAL' | $-176.77$ | 2.4 % |
| SIGM_ELNO | SIXY | Not B ( $N5$ ) | 'ANALYTICAL' | $0.0$ | 5.0 Pa |
| SIGM_ELNO | SIXX | Not A ( $N1$ ) | 'ANALYTICAL' | $0.0$ | 45.0 Pa |
| SIGM_ELNO | SIYY | Not A ( $N1$ ) | 'ANALYTICAL' | $0.0$ | 105.0 Pa |
| SIGM_ELNO | SIZZ | Not A ( $N1$ ) | 'ANALYTICAL' | $0.0$ | 45.0 Pa |
| SIGM_ELNO | SIXY | Not A ( $N1$ ) | 'ANALYTICAL' | $0.0$ | 30.0 Pa |

### 3.4 Remarks

The test reveals the need, in modelings with linear elements, to use very fine grids to arrive to satisfactory results.

# 4 Summary of the results

The results are in concord with the analytical solution.