

## SSLV316 – Cracking with propagation imposed with X-FEM

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### Summary:

The goal of this test is to check that the methods simplex, upwind and geometrical of the operator `PROPA_FISS` correctly calculate the position of the bottom of a crack 3D which propagates in mixed mode.

One simulates several propagations of a crack by imposing a projection and a given direction of propagation. The position of the crack after each propagation is thus known and one can check if the position of the bottom calculated by the operator `PROPA_FISS` is correct.

## 1 Problem of reference

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### 1.1 Geometry

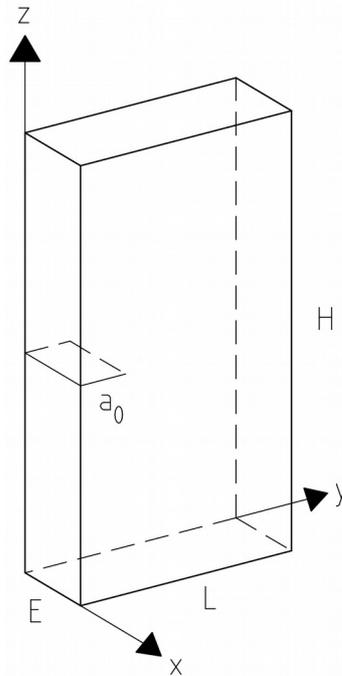


Figure 1.1-a: geometry of the fissured plate

Geometrical dimensions of the fissured plate:

width	$L = 8\text{m}$
thickness	$E = 1\text{m}$
height	$H = 18\text{m}$

Initial length of the plane crack:  $a_0 = 2\text{m}$

The crack is positioned in the middle of the height of the plate ( $H/2$ ).

### 1.2 Properties of material

Young modulus  $E = 205\,000\text{ MPa}$

Poisson's ratio  $\nu = 0.3$

### 1.3 Boundary conditions and loadings

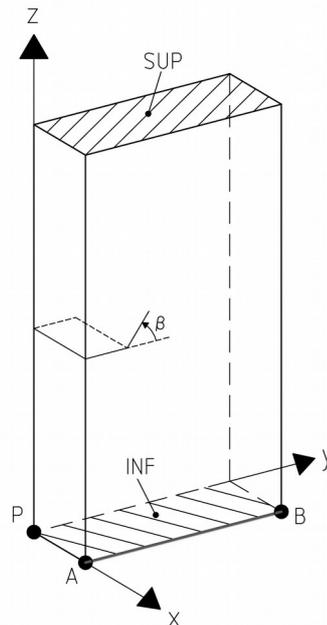


Figure 1.3-a: boundary conditions and loadings

Boundary conditions:

Not  $P$  :  $\Delta X = \Delta Y = \Delta Z = 0$

Points on the segment  $AB$  :  $\Delta X = \Delta Z = 0$

Points on surface  $INF$  :  $\Delta Z = 0$

Loading:

Pressure on surface  $SUP$  :  $P = -1 \text{ MPa}$

The loading is constant during the propagation. Three calls to the operator `PROPA_FISS` are made to simulate a propagation of the initial crack already present in the structure. With each call advance and direction of propagation of **each point** bottom of crack are imposed:

Advance of the point of the bottom:  $\Delta a = 0.4 \text{ m}$

Angle of propagation:  $\beta = 30^\circ$

Positive direction of the angle  $\beta$  is visible on Figure 1.3-a.

The bottom of the crack remains **always right** during all the propagation. The motivation of this choice will be explained in the paragraph 4.

## 2 Reference solution

### 2.1 Method of calculating

One wants to check that the position of the bottom after the propagation, calculated by the operator `PROPA_FISS`, is correct. One must thus calculate the theoretical position given by the advance and the imposed direction of propagation.

As already noticed, the bottom of crack remains right during all the propagation. The bottom is always perpendicular to two surfaces of the plate parallel with the plan  $YZ$ , like magazine Figure 1.3-a. The position of the bottom can thus be indicated by using only the coordinates  $Y$  and  $Z$ .

The initial position is the following one:

$$y_0 = a_0 = 2.0$$

$$z_0 = 9.0$$

After the propagation  $i$ , the new position of the bottom can be calculated like this:

$$y_i = y_{i-1} + \Delta a \cdot \cos(i \cdot \beta)$$

$$z_i = z_{i-1} + \Delta a \cdot \sin(i \cdot \beta)$$

## 2.2 Sizes and results of reference

For the three propagations calculated in the tests, the position of the bottom is the following one:

Propagation	Coordinate $y_i$	Coordinate $z_i$
1	2.34641	9.19999
2	2.54642	9.54640
3	2.54644	9.94640

**Table 2.1**

In the current version of Code Aster, the coordinates of the points of the bottom of crack are available only in the file .mess and thus one cannot check them directly in the command file.

However, for this case test, one knows the theoretical position and the form (a segment) of the bottom of the crack. In fact the bottom is always coincide with the edge which connects the two points  $(0, y_i, z_i)$  and  $(1, y_i, z_i)$ . While using `MACR_LIGN_COUPE`, one can to raise values of the level sets at the points of intersection between this edge and faces of the elements of the grid. If the position of the bottom after the propagation is calculated correctly by `PROPA_FISS`, the value of both level sets must be equal to zero for all the found points of intersection because, by definition, the bottom of crack is formed by all the points where the level set tangent and normal are equal to zero.

That explains why one decided to give the same advance and direction of propagation to all the points of the bottom of crack. Even if the bottom of crack is located by two only coordinates, the propagation is 3D and the algorithms implemented in `PROPA_FISS` calculate a propagation in 3D.

## 3 Modeling A

### 3.1 Characteristics of modeling

Method **upwind** is used by `PROPA_FISS` to solve the equations of propagation of the crack.

**No auxiliary grid** is not used. That is possible because the grid of the structure is very regular. The field of calculation is **located** around the bottom of the crack.

### 3.2 Characteristics of the grid

The structure is modelled by a grid made up of 6720 elements HEXA8 (see Figure 3.2-a).

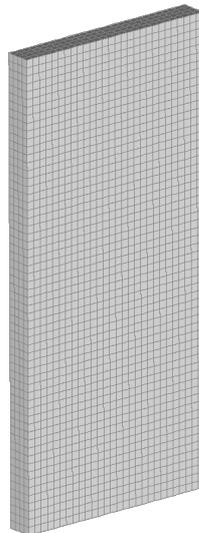


Figure 3.2-a: grid of the structure

The grid is not very refined to reduce the computing time. The size of the elements is uniform and equal to  $0.29 \times 0.33 \times 0.25 \text{ m}$ .

### 3.3 Sizes tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the faces of the elements of the grid while using `MACR_LIGN_COUPE` (see Table 2.1). For each one of these points, one calculates the value of the level set normal ( $LSN$ ) and tangent ( $LST$ ) by using the operator `POST_RELEVE_T` and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	-2.64E-16	-8.40E-16	9,71E-16	-4,16E-17
2	-3.30E-04	-3.30E-04	-2.23E-06	-2.25E-06
3	-3.44E-03	-3.53E-03	-2.20E-04	-2.22E-04

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost worthless. By considering that the grid is coarse, one affect a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

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Tolerance used =  $0.15 \times 0.33 = 0.05 \text{ m}$

## 3.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the method upwind is correct and that the localization of the field goes well for the method upwind without auxiliary grid.

It should be noticed that after the three simulated propagations, the crack deviated of  $90^\circ$ . On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method upwind calculated well the position of the crack and one thus checked his robustness.

## 4 Modeling B

### 4.1 Characteristics of modeling

Method **upwind** is used by `PROPA_FISS` to solve the equations of propagation of the crack. An **auxiliary grid** is used.

The same model that described for modeling A is used. The field of calculation is **located** around the bottom of the crack.

### 4.2 Characteristics of the grid

One uses the same grid as that of modeling A.

The auxiliary grid used consists of 1296 regular elements HEXA8 of dimension  $0.25 \times 0.25 \times 0.25 \text{ m}$  (see Figure 4.2-a).

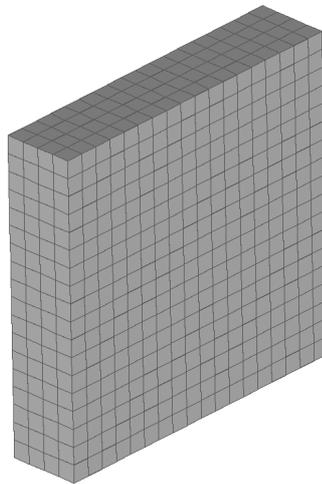


Figure 4.2-a: grid used to define the auxiliary grid

The grid is extended to the only zone of the structure interested by the propagation of the crack.

### 4.3 Sizes tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the faces of the elements of the grid by using the operator `MACR_LIGN_COUPE` (see Table 2.1). For each one of these points, one calculates the value of the level set normal ( $LSN$ ) and tangent ( $LST$ ) by using the operator `POST_RELEVE_T` and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	9.02E-17	-2.77E-16	-3.88E-16	-5.7E-16
2	-1.94E-03	-2.08E-03	-2.09E-06	-2.11E-06
3	9.19E-04	-4.52E-05	-1.23E-03	-1.17E-03

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level

sets calculated are almost worthless. By considering that the grid is coarse, one affect a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

Tolerance used =  $0.15 \times 0.33 = 0.05 \text{ m}$

## 4.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the auxiliary method upwind+grille is correct and that the localization of the field goes well for the auxiliary method upwind+grille.

It should finally be noticed that after the three simulated propagations, the crack deviated of  $90^\circ$ . On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the auxiliary method upwind+grille calculated well the position of the crack and one thus checked his robustness.

## 5 Modeling C

### 5.1 Characteristics of modeling

Method **simplex** is used by `PROPA_FISS` to solve the equations of propagation of the crack. **No auxiliary grid** is not used. The field of calculation is **located** around the bottom of the crack. The same model that described for modeling A is used.

### 5.2 Characteristics of the grid

One uses the same grid as that of modeling A.

### 5.3 Sizes tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the faces of the elements of the grid while using `MACR_LIGN_COUPE` (see Table 2.1). For each one of these points, one calculates the value of the level set normal (  $LSN$  ) and tangent (  $LST$  ) by using the operator `POST_RELEVE_T` and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	-4,43E-15	-4,65E-15	3,01E-15	2,76E-15
2	-0.0007	-0.0007	2.0122E-16	-1.311E-15
3	0.00036	0.00036	-0.00041	-0.00041

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed, the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost worthless. By considering that the grid is coarse, one affects a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

Tolerance used =  $0.15 \times 0.33 = 0.05 \text{ m}$

### 5.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the method simplex is correct and that the localization of the field goes well for the method simplex.

It should finally be noticed that after the three simulated propagations, the crack deviated of  $90^\circ$ . On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method simplex calculated well the position of the crack and one thus checked his robustness.

## 6 Modeling D

### 6.1 Characteristics of modeling

This modeling is identical to modeling A except for the field of calculation which **is not localised** around the bottom of the crack. The update of the level sets is thus made under all the model.

### 6.2 Characteristics of the grid

One uses the same grid as that of modeling A.

### 6.3 Sizes tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the faces of the elements of the grid while using `MACR_LIGN_COUPE` (see Table 2.1). For each one of these points, one calculates the value of the level set normal (  $LSN$  ) and tangent (  $LST$  ) by using the operator `POST_RELEVE_T` and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	-2.63E-16	-8.39E-16	-4.163E-17	-4.996E-16
2	-0.00033	-0.00033	-2.2346E-06	-2.249E-06
3	-0.00347	-0.00356	-0.00022	-0.00022

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost worthless. By considering that the grid is coarse, one affects a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

Tolerance used =  $0.15 \times 0.33 = 0.05 \text{ m}$

### 6.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the method upwind is correct.

It should be noticed that after the three simulated propagations, the crack deviated of  $90^\circ$ . On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method upwind calculated well the position of the crack and one thus checked his robustness.

## 7 Modeling E

### 7.1 Characteristics of modeling

This modeling is identical to modeling B except for the field of calculation which **is not localised** around the bottom of the crack. The update of the level sets is thus made under all the model.

### 7.2 Characteristics of the grid

One uses the same grid as that of modeling A and the same auxiliary grid as that of modeling B.

### 7.3 Sizes tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the faces of the elements of the grid while using `MACR_LIGN_COUPE` (see Table 2.1). For each one of these points, one calculates the value of the level set normal (  $LSN$  ) and tangent (  $LST$  ) by using the operator `POST_RELEVE_T` and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	9.0205E-17	-2.7755E-16	-3.885E-16	-5.689E-16
2	-0.0019	-0,002	-2.064E-06	-2.1013E-06
3	0.00091	-4.822E-05	-0.0011	-0.0012

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost worthless. By considering that the grid is coarse, one affects a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

Tolerance used =  $0.15 \times 0.33 = 0.05 \text{ m}$

### 7.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the auxiliary method upwind+grille is correct.

It should finally be noticed that after the three simulated propagations, the crack deviated of  $90^\circ$ . On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the auxiliary method upwind+grille calculated well the position of the crack and one thus checked his robustness.

## 8 Modeling F

### 8.1 Characteristics of modeling

This modeling is identical to modeling C except for the field of calculation which **is not localised** around the bottom of the crack. The update of the level sets is thus made under all the model.

### 8.2 Characteristics of the grid

One uses the same grid as that of modeling C.

### 8.3 Sizes tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the faces of the elements of the grid while using `MACR_LIGN_COUPE` (see Table 2.1). For each one of these points, one calculates the value of the level set normal (  $LSN$  ) and tangent (  $LST$  ) by using the operator `POST_RELEVE_T` and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	-4.430E-15	-4.654E-15	3.010E-15	2.7616E-15
2	-0.00067	-0.00067	2.0122E-16	-1.3114E-15
3	0.00046	0.00046	-0.00041	-0.00041

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed, the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost worthless. By considering that the grid is coarse, one affects a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

$$\text{Tolerance used} = 0.15 \times 0.33 = 0.05 \text{ m}$$

### 8.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the method simplex is correct.

It should finally be noticed that after the three simulated propagations, the crack deviated of  $90^\circ$ . On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method simplex calculated well the position of the crack and one thus checked his robustness.

## 9 Modeling G

### 9.1 Characteristics of modeling

Method **geometrical** is used by PROPA\_FISS for the calculation of the new position of the crack. The field of calculation is **located** around the bottom of the crack. **No auxiliary grid** is not used.

### 9.2 Characteristics of the grid

One uses the same grid as that of modeling A.

### 9.3 Sizes tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the faces of the elements of the grid while using MACR\_LIGN\_COUPE (see Table 2.1). For each one of these points, one calculates the value of the level set normal (  $LSN$  ) and tangent (  $LST$  ) by using the operator POST\_RELEVE\_T and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	5.55E-16	2.22E-16	-1.94E-16	-3.747E-16
2	-0.0123	-0.0123	-1.97E-4	-1.97E-4
3	-0,027	-0,027	-6.72E-3	-6.72E-3

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed, the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost worthless. By considering that the grid is coarse, one affects a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

$$\text{Tolerance used} = 0.15 \times 0.33 = 0.05 \text{ m}$$

### 9.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the geometrical method is correct.

It should finally be noticed that after the three simulated propagations, the crack deviated of  $90^\circ$ . On the other hand the total projection is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the geometrical method calculated well the position of the crack and one thus checked his robustness.

## 10 Modeling L

### 10.1 Characteristics of modeling

This modeling is intended to validate the propagation of the surface of potential cracking for a calculation of *propagation with cohesive elements*.

Method **geometrical** is used by `PROPA_FISS` for the calculation of the new position of the crack, with the operation `PROPA_COHESIF`. The field of calculation is not **located** and **no auxiliary grid** is not used.

Three calls to the operator `PROPA_FISS` are made to simulate a propagation of crack *by cohesive elements*, starting from the initial crack already present in the structure. With each call advance and direction of propagation of **each point** bottom of crack are imposed:

Advance of the point of the bottom:  $\Delta a = 2 m$

Angle of propagation:  $\beta = 30^\circ$  for the first two calls to `PROPA_FISS`,  $\beta = 70^\circ$  for the last

### 10.2 Characteristics of the grid

A structured grid is used for which the initial crack is with a grid. The structure is modelled by a grid made up of 8120 elements `HEXA8`.

### 10.3 Sizes tested and results

One calculates the points of intersection between the line which gives the theoretical position of the bottom and the faces of the elements of the grid while using `MACR_LIGN_COUPE` (see Table 2.1). For each one of these points, one calculates the value of the level set normal ( *LSN* ) and tangent ( *LST* ) by using the operator `POST_RELEVE_T` and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	-4.87110352E-14	-1.0999534E-13	9.31824062E-14	-6.34770014E-14
2	-4.87110352E-14	-1.0999534E-13	9.31824062E-14	-6.34770014E-14
3	8.07479083E-14	-1.80688797E-14	-2.95111157E-14	-4.96512553E-14

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed, the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost worthless. By considering that the grid is coarse, one affects a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

Tolerance used =  $0.15 \times 0.25 = 0.0375 m$

### 10.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the geometrical method is correct. The remarks are thus the same ones as for modeling I.

## 11 Modeling M

### 11.1 Characteristics of modeling

Method **geometrical** with the rotation of discharge activated is used by `PROPA_FISS` for the calculation of the new position of the crack. The dumping angle  $\gamma$  is forced to zero. The field of calculation is **located** around the bottom of the crack. **No auxiliary grid** is not used.

### 11.2 Characteristics of the grid

One uses the same grid as that of modeling A.

### 11.3 Sizes tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the faces of the elements of the grid while using `MACR_LIGN_COUPE` (see Table 2.1). For each one of these points, one calculates the value of the level set normal (  $LSN$  ) and tangent (  $LST$  ) by using the operator `POST_RELEVE_T` and it is checked that the values maximum and minimal are almost worthless:

Propag. $i$	Max $LSN_i$	Min $LSN_i$	Max $LST_i$	Min $LST_i$
1	5.55E-16	2.22E-16	-1.94E-16	-3.747E-16
2	-0.0123	-0.0123	-1.97E-4	-1.97E-4
3	-0,027	-0,027	-6.72E-3	-6.72E-3

The values obtained are calculated starting from the values with the nodes of the grid by using the functions of form of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the grid. Indeed, the precision of representation of the level set is it even dependent in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost worthless. By considering that the grid is coarse, one affects a tolerance equal to 15% length of the backbone of the grid in the zone of propagation:

$$\text{Tolerance used} = 0.15 \times 0.33 = 0.05 \text{ m}$$

### 11.4 Remarks

All the values tested respect the tolerance used. That means that the position of the bottom of crack calculated by the geometrical method is correct.

It should finally be noticed that after the three simulated propagations, the crack deviated of  $90^\circ$ . On the other hand the total projection is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the geometrical method calculated well the position of the crack and one thus checked his robustness.

## 12 Summary of the results

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All the methods used (upwind, upwind+grille auxiliary, simplex and geometrical) made it possible to calculate the position of a crack well propagating in mixed mode in severe conditions. That also made it possible to validate the implementation of these methods in the operator `PROPA_FISS` and in particular the possibility of locating the zone of update of the level-sets.

If it is considered that the grid used in the cases test is coarse, one can say that the methods calculate in a sufficiently precise way the position of the bottom of crack.