SSLV321 - Validation of prepacking XFEM for a shaving interface

Summary:

This test validates the pre-conditioner dedicated to quadratic elements XFEM. The problem corresponds to that of a block 3D cut by an interface XFEM. One applies a multiaxial compression to all the surface of the block, including to interface XFEM. The expected analytical solution, is a displacement P_1 inside the field. This solution is collected naturally by the elements P_1 and P_2 of Code_Aster.

When the interface passes close to the nodes of the grid, the conditioning of the matrix of rigidity increases [1.2]. Usual strategies, used in Code_Aster (readjustment of the interface with Nœuds and elimination of ddls Heaviside), being based on “estimated” criteria, are not always adapted to control the conditioning and the precision of the results [2]. These disadvantages are all the more penalizing, with quadratic elements.

The new strategy (construction of a diagonal pre-conditioner per blocks) allows stage all these disadvantages [3].

This test comprises 5 modelings:

- A : a modeling HEXA8 3D with an interface XFEM of equation $x+y+z+0.01=0$
- B : a modeling HEXA20 3D with an interface XFEM of equation $x+y+z+0.1=0$
- C : a modeling HEXA20 3D with an interface XFEM of equation $x+y+z+0.01=0$
- D : a modeling HEXA8 3D with an interface XFEM of equation $x+y+z+0.011=0$
- E : a modeling HEXA8 3D with an interface XFEM of equation $x+y+z+0.011=0$

1 Problem of reference in 3D

1.1 Geometry

The structure is a cuboidal, separate region into two, by an oblique interface XFEM.

The length of each edge of cube is: $L=4$. The cube is centered in $(0,0)$.

The position of the interface is: $x+y+z+cte=0$

The constant is adjusted so that the interface is shaving with a line of nodes of the grid,
- for modelings A and C , the constant is worth: $cte=0.01$,
- for modeling B , the constant is worth: $cte=0.1$,
- for modelings D and E , the constant is worth: $cte=0.011$

1.2 Material properties

Poisson's ratio: $\nu=0.3$
Young modulus: $E=10^9 N/m^2$

1.3 Boundary conditions and loadings

On each under-field (on both sides of interface XFEM), one imposes a loading in compression. Indeed, uniform pressure $p=-10 MPa$ is applied to each face of the cube and interface XFEM.

Then limiting conditions of Dirichlet are imposed, to fix the analytical solution in displacement and to block the 12 movements of rigid bodies.

***Recall*:**
When interface XFEM cuts all the structure and that the two blocks are not in contact, all occurs as if there were two uncoupled mechanical problems, in the presence of two solids. What thus leads to a total of 12 rigid modes in equivalent problem XFEM.

Some N are thus chosenœuds on both sides of the interface, to fix the solution in displacement:
- 3 nodes are blocked with the top of the interface $\{\mathrm{NS}_1, \mathrm{NS}_2, \mathrm{NS}_4\}$
  $\mathrm{NS}_1=(+2,+2,+2)$
  $\mathrm{NS}_2=(-2,+2,+2)$
  $\mathrm{NS}_4=(-2,-2,+2)$
- 3 Nœuds is blocked below the interface $\{\mathrm{NI}_1, \mathrm{NI}_2, \mathrm{NI}_4\}$
  $\mathrm{NI}_1=(-2,-2,-2)$
  $\mathrm{NI}_2=(+2,-2,-2)$
  $\mathrm{NI}_4=(+2,+2,-2)$

Knowing that, the analytical solution in displacement, is worth in each point of the field:
- $U_x(x,y,z)=\begin{cases} k\times x+2 & \text{si } x+y+z+cte>0 \\ k\times x-2 & \text{si } x+y+z+cte\leq 0 \end{cases}$
- $U_y(x,y,z)=\begin{cases} k\times y+2 & \text{si } x+y+z+cte>0 \\ k\times y-2 & \text{si } x+y+z+cte\leq 0 \end{cases}$

- $U_z(x,y,z)=\begin{cases} k\times z+2 & \text{si} \quad x+y+z+cte>0 \\ k\times z-2 & \text{si} \quad x+y+z+cte\le 0\end{cases}$

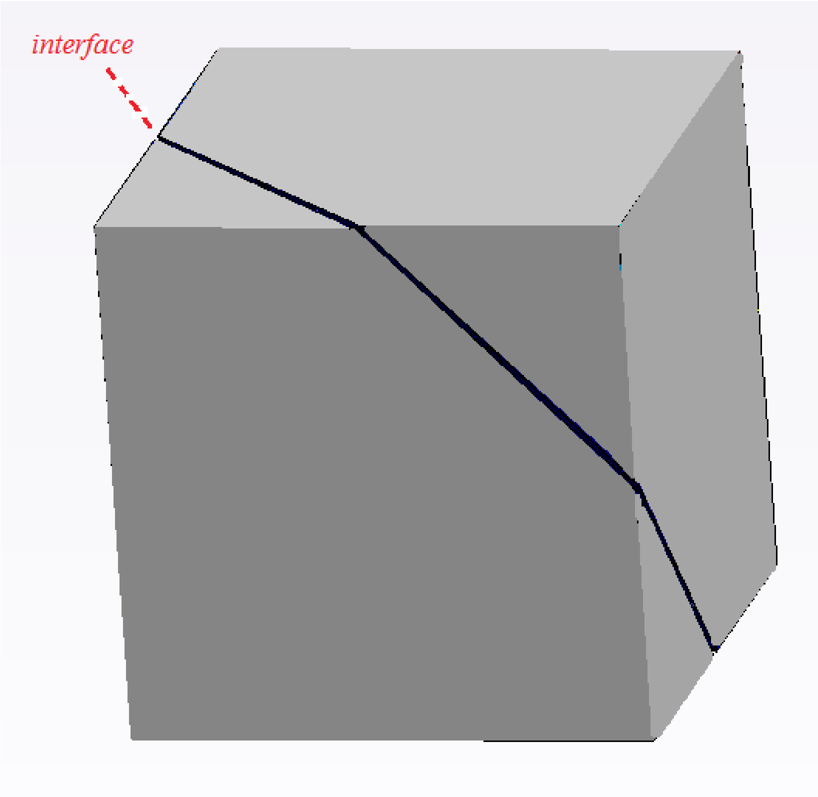

Figure 1.3-1: Geometry of the field and position of the interface

where, $k=-\frac{p(1-2\nu)}{E}=4.10^{-3}$.

1.4 Bibliographical references

[1] E. Bechet, H. Minnebo, NR. Moes, B. Burgardt, Improved implementation and robustness study of the x-fem method for stress analysis around aces, International Newspaper for Numerical Methods in Engineering, 64: 1033-1056, 2005.

[2] Mr. Siavelis, M.L.E. Guiton, P. Massin, NR. Moës, Broad sliding contact along branched discontinuities with X-FEM, Computational Mechanics, 52-1: 201-219, 2013.

[3] Extended Finite Method Element, Code_Aster documentation, R7.02.12

2 Reference solution

2.1 Method of calculating used for the reference solution

The analytical solution in displacement, in the canonical base $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$, is worth:

$$U(x,y,z)=k\begin{Bmatrix}x\\y\\z\end{Bmatrix} \pm 2 \text{ where, } k=-\frac{p(1-2\nu)}{E}=4.10^{-3}.$$

Thereafter, it is checked that this expression of the field displacement, is the single solution of the problem of balance on both sides of interface XFEM, i.e., that the suggested solution §2.1, observes the limiting conditions and the assumptions of the problem HP.

By construction, the solution in displacement checks the conditions of Dirichlet of the §1.3.

The tensor of the deformations is worth: $\underline{\underline{\epsilon}}=\frac{1}{2}(\nabla U+\nabla U^T)=\begin{bmatrix}k&0&0\\0&k&0\\0&0&k\end{bmatrix}=kI$.

While taking $k=0{,}4\,\%$, the assumption of the small disturbances is respected.

The law of Hooke, for an isotropic material, being worth:

$$\underline{\underline{\sigma}}=\frac{E}{1+\nu}\left(\underline{\underline{\epsilon}}+\frac{\nu}{1-2\nu}Tr(\underline{\underline{\epsilon}})I\right)=k\frac{E}{1-2\nu}I$$

however, by construction, one a: $k=-\frac{p(1-2\nu)}{E}$

with, $\underline{\underline{\sigma}}=-pI$.

Lastly, limiting conditions of Neumann to be imposed on the edges of the field $-\underline{\underline{\sigma}}.\vec{n}=p$, correspond well to the loading of the §1.3.

Like, the tensor of the constraints is uniform, it respects the static equilibrium equation, $\operatorname{div}\underline{\underline{\sigma}}=0$.

2.2 Results of reference

The error is tested maximum, on displacements in some points located on the interface $x+y+z+cte=0$. The value of displacement in these points corresponds to the interpolation of the field of displacement calculated by Aster. The displacement calculated by Aster, is then compared to the analytical value of displacement given to the §2.1:

$$U(x,y,z)=4.10^{-3}\begin{Bmatrix}x\\y\\z\end{Bmatrix}+sign(x+y+z+cte)\times 2$$

3 Modeling A

3.1 Characteristics of modeling

It is a modeling $3D$ XFEM with linear elements.

This modeling shows the good performance of the “estimated” criteria with linear elements. The interface is positioned such as the threshold of readjustment of the interface with 1% is reached: the interface then is modified and readjusted with the closest nodes. This modification makes it possible to control conditioning. On the other hand, the readjustment of the interface appreciably degrades the quality of the results for the linear elements (for the quadratic elements, one removes this readjustment, to replace it by an algebraic prepacking).

The equation of interface XFEM is: $x+y+z+0.01=0$.

3.2 Characteristics of the grid

Many nodes: 125
Grid: 64 meshs of the type HEXA8

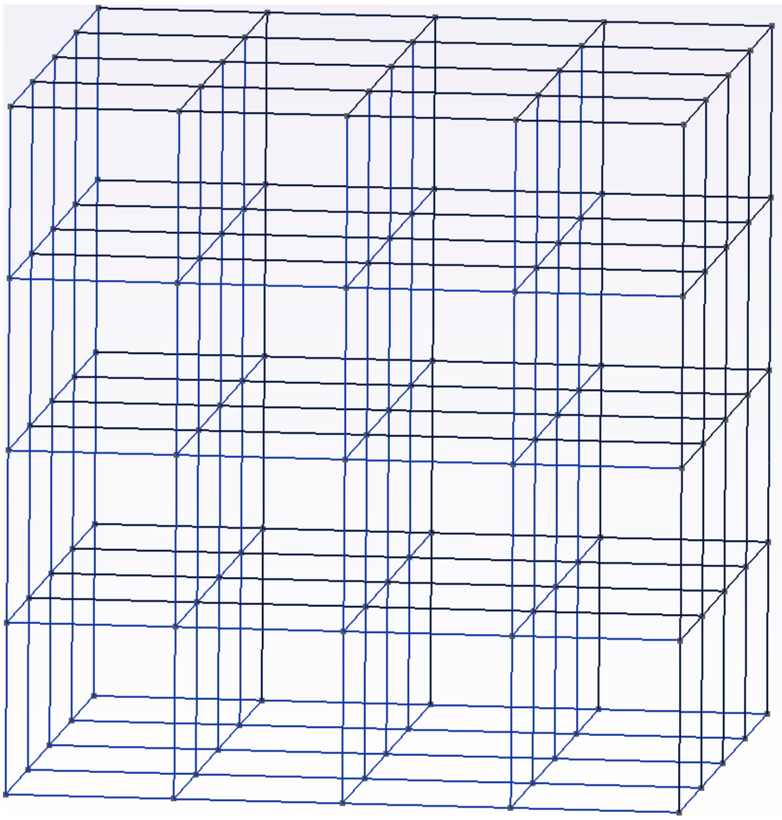

Figure 3.2-1: linear grid

3.3 Sizes tested and results

One tests the maximum error on displacement, in absolute value, along interface XFEM. One extracts a list from points located on the interface, as well as the field of displacement interpolated in these points.

As the field of displacement is discontinuous, there exists in each point of the interface 2 analytical values of the field of displacement $U^{+}(x,y,z)=\{U_1^+, U_2^+, U_3^+\}$ and $U^{-}(x,y,z)=\{U_1^-, U_2^-, U_3^-\}$. One compares these analytical values, with the field of displacement interpolated in each point.

In practice, in Code_Aster to take account of discontinuity during the interpolation, each point on the interface is transformed into nodes doubled bloom (NP* and NM*) to which are associated values of displacements $DX_i(NP)$ and $DX_i(NM)$.

- For the nodes “MORE” (noted NP* by default in the code_Aster), one calculates the table of following difference then $DIFF(NP)_i=\left|U_i^+(x_{NP}, y_{NP}, z_{NP})-DX_i(NP)\right|$;
- for the nodes “LESS” (noted NM* by default in the code_Aster), one calculates the table of following difference then $DIFF(NM)_i=\left|U_i^-(x_{NM}, y_{NM}, z_{NM})-DX_i(NM)\right|$.

Identification	Reference	Type	% tolerance
$\mathrm{DIFF(NP)}_X$ (MAX)	0.0	Analytical	0.2
$\mathrm{DIFF(NP)}_Y$ (MAX)	0.0	Analytical	0.2
$\mathrm{DIFF(NP)}_Z$ (MAX)	0.0	Analytical	0.2
$\mathrm{DIFF(NM)}_X$ (MAX)	0.0	Analytical	0.2
$\mathrm{DIFF(NM)}_Y$ (MAX)	0.0	Analytical	0.2
$\mathrm{DIFF(NM)}_Z$ (MAX)	0.0	Analytical	0.2

Table 3.3-1: synthetic results

3.4 Notice

The analytical tolerances are much higher than for quadratic modelings B and C .

4 Modeling B

4.1 Characteristics of modeling

It is a modeling $3D$ XFEM with quadratic elements.

This modeling shows the need for the pre-conditioner for calculation with quadratic elements. The interface is positioned to 10 % (the length of the edge) beyond the threshold of readjustment of the interface with 1%, but the conditioning of the matrix of rigidity is already high: without the pre-conditioner, the test cannot turn in a robust way. With the pre-conditioner, the direct solvor calculates a robust solution of manner.

The equation of interface XFEM is: $x+y+z+0.1=0$.

4.2 Characteristics of the grid

Many nodes: 425
Grid: 64 meshs of the type `HEXA20`

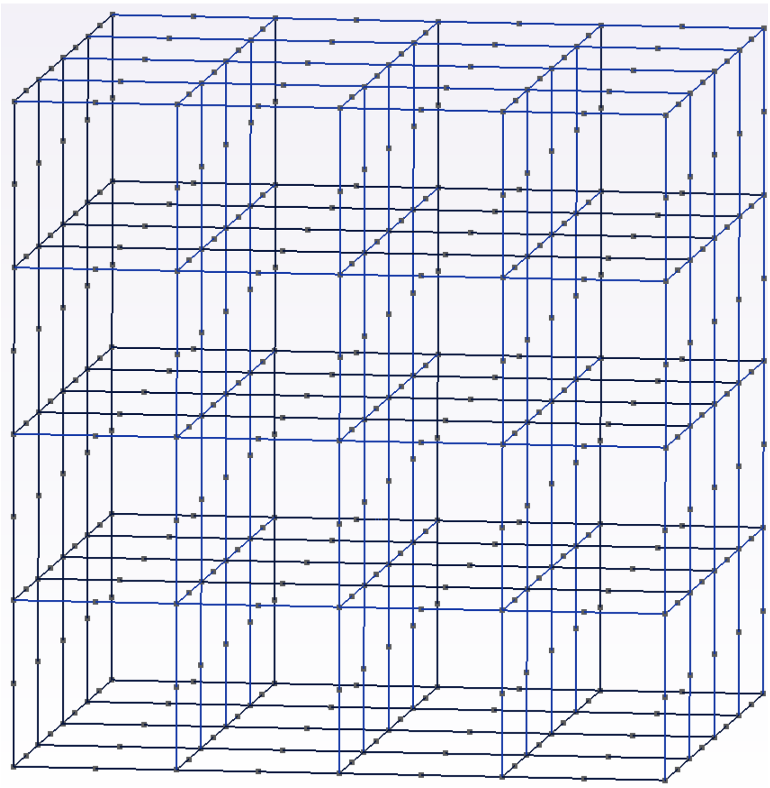

Figure 4.2-1: quadratic grid

4.3 Sizes tested and results

One tests the maximum error on displacement, in absolute value, along interface XFEM. One extracts a list from points located on the interface, as well as the field of displacement interpolated in these points.

As the field of displacement is discontinuous, there exists in each point of the interface 2 analytical values of the field of displacement $U^{+}(x,y,z)=\{U_1^{+},U_2^{+},U_3^{+}\}$ and $U^{-}(x,y,z)=\{U_1^{-},U_2^{-},U_3^{-}\}$. One compares these analytical values, with the field of displacement interpolated in each point by ASTER.

In practice, in Code_Aster to take account of discontinuity during the interpolation, each point on the interface is transformed into nodes doubled bloom (NP* and NM*) to which are associated values of displacements $DX_i(NP)$ and $DX_i(NM)$.

- For the nodes "MORE" (noted NP* by default in the code_Aster), one calculates the table of following difference then $DIFF(NP)_i=\left|U_i^{+}(x_{NP},y_{NP},z_{NP})-DX_i(NP)\right|$;
- for the nodes "LESS" (noted NM* by default in the code_Aster), one calculates the table of following difference then $DIFF(NM)_i=\left|U_i^{-}(x_{NM},y_{NM},z_{NM})-DX_i(NM)\right|$.

Identification	Reference	Type	% tolerance
$\mathrm{DIFF(NP)}_X$ (MAX)	0.0	Analytical	1.E-09
$\mathrm{DIFF(NP)}_Y$ (MAX)	0.0	Analytical	1.E-09
$\mathrm{DIFF(NP)}_Z$ (MAX)	0.0	Analytical	1.E-09
$\mathrm{DIFF(NM)}_X$ (MAX)	0.0	Analytical	1.E-04
$\mathrm{DIFF(NM)}_Y$ (MAX)	0.0	Analytical	1.E-04
$\mathrm{DIFF(NM)}_Z$ (MAX)	0.0	Analytical	1.E-04

Table 4.3-1 : synthetic results

4.4 Notice

The analytical tolerances are much weaker than for modeling A . This difference is explained by the readjustment of the interface to the nodes for the linear elements of modeling A . If the interface is positioned far from the threshold of readjustment to 1%, one notes an improvement of the results for the linear elements.

5 Modeling C

5.1 Characteristics of modeling

It is a modeling $3D$ XFEM with quadratic elements.

The interface is positioned exactly as in modeling A (to 1% of the edge). Like, the estimated criteria are not adapted to the quadratic elements, only the pre-conditioner controls the conditioning of the matrix of rigidity. The test shows that the pre-conditioner returns on the one hand, possible a positioning interface XFEM, completely independent of the proximity of the nodes of the grid and in addition, increases considerably the precision of the results compared to modeling A .

The equation of interface XFEM is: $x+y+z+0.01=0$, in the same configuration as modeling A .

5.2 Characteristics of the grid

Even grid that modeling B .

5.3 Sizes tested and results

One tests the maximum error on displacement, in absolute value, along interface XFEM. One extracts a list from points located on the interface, as well as the field of displacement interpolated in these points.

As the field of displacement is discontinuous, there exists in each point of the interface 2 analytical values of the field of displacement $U^{+}(x,y,z)=\{U_1^{+},U_2^{+},U_3^{+}\}$ and $U^{-}(x,y,z)=\{U_1^{-},U_2^{-},U_3^{-}\}$. One compares these analytical values, with the field of displacement interpolated in each point by ASTER.

In practice, in Code_Aster to take account of discontinuity during the interpolation, each point on the interface is transformed into nodes doubled bloom (NP* and NM*) to which are associated values of displacements $DX_i(NP)$ and $DX_i(NM)$.

- For the nodes "MORE" (noted NP* by default in the code_Aster), one calculates the table of following difference then $DIFF(NP)_i=\left|U_i^{+}(x_{NP},y_{NP},z_{NP})-DX_i(NP)\right|$;
- for the nodes "LESS" (noted NM* by default in the code_Aster), one calculates the table of following difference then $DIFF(NM)_i=\left|U_i^{-}(x_{NM},y_{NM},z_{NM})-DX_i(NM)\right|$.

Identification	Reference	Type	% tolerance
$\mathrm{DIFF(NP)}_X$ (MAX)	0.0	Analytical	1.E-09
$\mathrm{DIFF(NP)}_Y$ (MAX)	0.0	Analytical	1.E-09
$\mathrm{DIFF(NP)}_Z$ (MAX)	0.0	Analytical	1.E-09
$\mathrm{DIFF(NM)}_X$ (MAX)	0.0	Analytical	1.E-03
$\mathrm{DIFF(NM)}_Y$ (MAX)	0.0	Analytical	1.E-03
$\mathrm{DIFF(NM)}_Z$ (MAX)	0.0	Analytical	1.E-03

Table 5.3-1: synthetic results

6 Modeling D

6.1 Characteristics of modeling

It is a modeling $3D$ XFEM with linear elements.

This modeling makes it possible to carry out a direct comparison between 2 methods of prepacking: the elimination of the ddls Heaviside with the criterion of rigidity (modeling D) and pre-conditioner XFEM (cf modeling E). One places oneself very right before the threshold of readjustment of the interface, to 1.1 % length of the edge. In the order MODI_MODELE_XFEM, one leaves option PRETRAITEMENTS=' AUTO' [U4.41.11] , which disables pre-conditioner XFEM for the linear elements.

The equation of interface XFEM is: $x+y+z+0.011=0$. The pre-conditioner is not activated in PRETRAITEMENTS=' AUTO' [U4.41.11] for this modeling. The criterion of elimination of ddls Heaviside is thus activated.

6.2 Characteristics of the grid

Even grid that modeling A .

6.3 Sizes tested and results

One tests the maximum error on displacement, in absolute value, along interface XFEM. One extracts a list from points located on the interface, as well as the field of displacement interpolated in these points.

As the field of displacement is discontinuous, there exists in each point of the interface 2 analytical values of the field of displacement $U^+(x,y,z)=\{U_1^+,U_2^+,U_3^+\}$ and $U^-(x,y,z)=\{U_1^-,U_2^-,U_3^-\}$. One compares these analytical values, with the field of displacement interpolated in each point.

In practice, in Code_Aster to take account of discontinuity during the interpolation, each point on the interface is transformed into nodes doubled bloom (NP* and NM*) to which are associated values of displacements $DX_i(NP)$ and $DX_i(NM)$.

- For the nodes “MORE” (noted NP* by default in the code_Aster), one calculates the table of following difference then $DIFF(NP)_i=\left|U_i^+(x_{NP},y_{NP},z_{NP})-DX_i(NP)\right|$;
- for the nodes “LESS” (noted NM* by default in the code_Aster), one calculates the table of following difference then $DIFF(NM)_i=\left|U_i^-(x_{NM},y_{NM},z_{NM})-DX_i(NM)\right|$.

Identification	Reference	Type	% tolerance
$\mathrm{DIFF(NP)}_X$ (MAX)	0.0	Analytical	1.E-07
$\mathrm{DIFF(NP)}_Y$ (MAX)	0.0	Analytical	1.E-07
$\mathrm{DIFF(NP)}_Z$ (MAX)	0.0	Analytical	1.E-07
$\mathrm{DIFF(NM)}_X$ (MAX)	0.0	Analytical	1.E-01
$\mathrm{DIFF(NM)}_Y$ (MAX)	0.0	Analytical	1.E-01
$\mathrm{DIFF(NM)}_Z$ (MAX)	0.0	Analytical	1.E-01

Table 6.3-1: synthetic results

6.4 Notice

One gains several orders of magnitude on the precision of the results, compared to modeling A , knowing that, the interface was shifted of 0.1% .
The readjustment of the interface (with a threshold with 1%) thus has a harmful influence on the quality of the results.
Nevertheless, to control conditioning in modeling D , one notes the activation of the elimination of ddls Heavisides, thanks to a criterion of estimate of rigity [R7.02.12] .

7 Modeling E

7.1 Characteristics of modeling

It is a modeling 3D XFEM with linear elements.

This modeling makes it possible to carry out a direct comparison between 2 methods of prepacking: the elimination of the ddls Heaviside with the criterion of rigidity (modeling D) and pre-conditioner XFEM (modeling E). As in modeling D , one places oneself very right before the threshold of readjustment of the interface, to 1.1 % length of the edge. In the order `MODI_MODELE_XFEM` , the option is passed `PRETRAITEMENTS=' FORCE'` [U4.41.11] , who activates pre-conditioner XFEM. The use of pre-conditioner XFEM makes it possible to gain 3 orders of magnitude on the precision of the results.

The equation of interface XFEM is: $x+y+z+0.011=0$. The pre-conditioner is activated by `PRETRAITEMENTS=' FORCE'` [U4.41.11] for this modeling. The criterion of elimination of ddls Heaviside is thus not activated.

7.2 Characteristics of the grid

Even grid that modeling A .

7.3 Sizes tested and results

One tests the maximum error on displacement, in absolute value, along interface XFEM. One extracts a list from points located on the interface, as well as the field of displacement interpolated in these points.

As the field of displacement is discontinuous, there exists in each point of the interface 2 analytical values of the field of displacement $U^+(x,y,z)=\{U_1^+,U_2^+,U_3^+\}$ and $U^-(x,y,z)=\{U_1^-,U_2^-,U_3^-\}$. One compares these analytical values, with the field of displacement interpolated in each point.

In practice, in Code_Aster to take account of discontinuity during the interpolation, each point on the interface is transformed into nodes doubled bloom (NP* and NM*) to which are associated values of displacements $DX_i(NP)$ and $DX_i(NM)$.

- For the nodes “MORE” (noted NP* by default in the code_Aster), one calculates the table of following difference then $DIFF(NP)_i=|U_i^+(x_{NP},y_{NP},z_{NP})-DX_i(NP)|$;
- for the nodes “LESS” (noted NM* by default in the code_Aster), one calculates the table of following difference then $DIFF(NM)_i=|U_i^-(x_{NM},y_{NM},z_{NM})-DX_i(NM)|$.

Identification	**Reference**	**Type**	**% tolerance**
$\text{DIFF}(\text{NP})_X$ (MAX)	0.0	Analytical	1.E-09
$\text{DIFF}(\text{NP})_Y$ (MAX)	0.0	Analytical	1.E-09
$\text{DIFF}(\text{NP})_Z$ (MAX)	0.0	Analytical	1.E-09
$\text{DIFF}(\text{NM})_X$ (MAX)	0.0	Analytical	1.E-03
$\text{DIFF}(\text{NM})_Y$ (MAX)	0.0	Analytical	1.E-03
$\text{DIFF}(\text{NM})_Z$ (MAX)	0.0	Analytical	1.E-03

Table 7.3-1: synthetic results

7.4 Notice

One gains several orders of magnitude on the precision of the results, compared to modeling D , for the same position of the interface. In modeling E , conditioning is controlled by the pre-conditioner and of the same order of magnitude as for the modelation D (around 10^3).

8 Summary of the results

This test makes it possible to validate the operation of the prepacking of the ddls Heavisides for quadratic elements XFEM.

It is noted that this strategy of prepacking gives better results, that the readjustment of the level-set with linear elements, as well as the elimination of the ddls by the criterion of rigidity.

For the quadratic elements, the pre-conditioner is essential to obtain a robust solution.