TPLP305 – Bar at temperatures imposed with adiabatic interface of type X-FEM

Summary:

This test is the equivalent in stationary linear thermics of mechanical test X-FEM SSNV173 [V6.04.173]. The objective of this test is to validate two aspects of elementary calculation within the framework of the resolution of thermal problems with X-FEM [R7.02.12]:

- the integration of a discontinuous size thanks to a under-cutting of the element
- the enrichment of the functions of forms by the Heaviside function

One considers a parallelepipedic bar crossed by an adiabatic interface, the two parts of the bar thus defined are thus thermically insulated one compared to the other. Each one of these parts is then subjected to a temperature imposed on one of its edges.

All thermal elements X-FEM with Heaviside enrichment are tested, the results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry 3D

The structure is a right parallelepiped at square base, its dimensions (see Figure 1.1-1) are the following ones:

$Lx=1\text{m}$, $Ly=1\text{m}$ and $Lz=5\text{m}$

The interface is defined by functions of level (level sets) directly in the command file using the operator DEFI_FISS_XFEM [U4.82.08]. It is introduced in the middle of the structure by a level set LSN (see Figure 1.1-1) of equation:

$LSN : z$

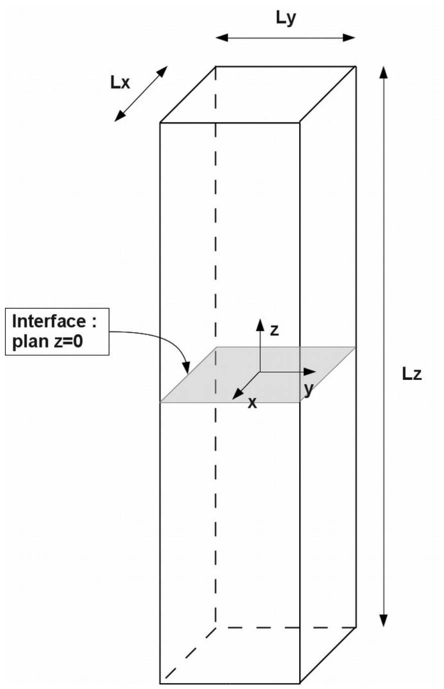

Figure 1.1-1: Geometry of the bar and positioning of the interface

1.2 Geometry 2D

The structure is a rectangle. its dimensions (see Figure 1.2-1) are the following ones:

$Lx=1\text{m}$ and $Ly=5\text{m}$

The interface is defined by functions of level (level sets) directly in the command file using the operator DEFI_FISS_XFEM [U4.82.08]. It is introduced in the middle of the structure by a level set LSN (see Figure 1.2-1) of equation:

$LSN : y$

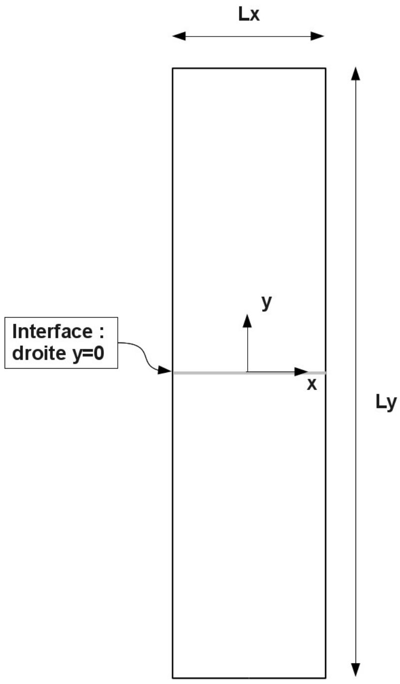

Figure 1.2-1: Geometry of the bar and positioning of the interface

1.3 Properties of material

- $\lambda = 1\, W . m^{-1} \circ C^{-1}$
- $\rho C_p = 2\, J / m^{-3} \circ C^{-1}$

1.4 Boundary conditions and loadings

A temperature is imposed $\bar{T}^{\inf} = 10 \circ C$ on the nodes of the lower edge of the bar, and $\bar{T}^{\sup} = 20 \circ C$ on the nodes of its higher edge.

1.5 Initial conditions

Nothing (the problem is stationary)

2 Reference solution

2.1 Method of calculating

It is about an analytical solution.

The interface being adiabatic, there are on both sides of this one two thermically isolated solids. Each solid sees a temperature imposed on part of its edge, and a null flow on the rest of its edge. The linear and stationary problem being, the temperature is thus constant in each part of the bar (and equalizes at the corresponding), and discontinuous temperature imposed through the interface:

in 3D:
$$\begin{cases} T(x,y,z)=\bar{T}^{\inf}=10\,°C\,,\ \forall\,(x,y,z)\in\left[\frac{-Lx}{2},\frac{Lx}{2}\right]\times\left[\frac{-Ly}{2},\frac{Ly}{2}\right]\times\left[\frac{-Lz}{2},0\right[\\ T(x,y,z)=\bar{T}^{\sup}=20\,°C\,,\ \forall\,(x,y,z)\in\left[\frac{-Lx}{2},\frac{Lx}{2}\right]\times\left[\frac{-Ly}{2},\frac{Ly}{2}\right]\times\left]0,\frac{Lz}{2}\right] \end{cases}$$

in 2D:
$$\begin{cases} T(x,y)=\bar{T}^{\inf}=10\,°C\,,\ \forall\,(x,y)\in\left[\frac{-Lx}{2},\frac{Lx}{2}\right]\times\left[\frac{-Ly}{2},0\right[\\ T(x,y)=\bar{T}^{\sup}=20\,°C\,,\ \forall\,(x,y)\in\left[\frac{-Lx}{2},\frac{Lx}{2}\right]\times\left]0,\frac{Ly}{2}\right] \end{cases}$$

In the modelings presented to the following paragraphs, the approximation of the field of temperature is enriched by a function by Heaviside in order to represent introduced discontinuity. The nodes whose support is crossed by the interface carry degrees of freedom “classical” and “Heaviside”: one cannot then directly compare their values with the analytical values obtained above.
In order to be able to test the values of these degrees of freedom, one places oneself if the grid of the bar consists of 5 regular hexahedrons on side $1\,\mathrm{m}$, the central mesh is thus crossed in its medium by the interface. The solution being constant in the directions x and y , one can bring back oneself to consider the unidimensional element are equivalent represented to the Figure 2.1-1.

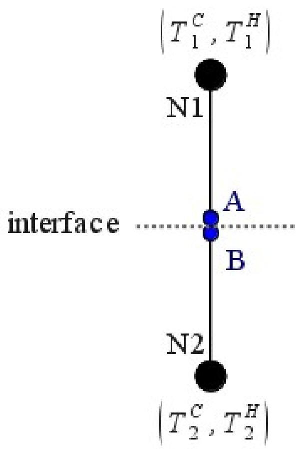

Figure 2.1-1: Linear element 1D equivalent

One notes $\mathrm{N1}$ and $\mathrm{N2}$ two nodes of this element, φ_1 and φ_2 functions of associated forms, $\left(T_1^C, T_1^H\right)$ and $\left(T_2^C, T_2^H\right)$ couples of associated degrees of freedom. One notes moreover x the variable of space, A the point located in $x=0^+$ and B the point located in $x=0^-$. The Heaviside function characterizes that is to say the field $x<0$ (for the node $\mathrm{N1}$), that is to say the field $x>0$ (for the node $\mathrm{N2}$). Let us note then χ_- the function characteristic of the field $x<0$ and χ_+ the function characteristic of the field $x>0$.

In any point x element, the field of temperature is expressed by the following relation:

$$T(x)=\phi_1(x)T_1^C+\phi_2(x)T_2^C-2\chi_-(x)\phi_1(x)T_1^H+2\chi_+(x)\phi_2(x)T_2^H$$

<u>notice</u>: the multiplying coefficients "-2" and "+2" are introduced to respect the convention of the definition of enrichment (even [R7.02.12]).

At the points $\mathrm{N1}$, $\mathrm{N2}$, A and B one a:

$$\begin{cases}\text{En N1}: \chi_+(x)=1 \;,\; \chi_-(x)=0 \;,\; \phi_1(x)=1 \;,\; \phi_2(x)=0\\ \text{En A}\;: \chi_+(x)=1 \;,\; \chi_-(x)=0 \;,\; \phi_1(x)=\phi_2(x)=1/2\\ \text{En B}\;: \chi_+(x)=0 \;,\; \chi_-(x)=1 \;,\; \phi_1(x)=\phi_2(x)=1/2\\ \text{En N2}: \chi_+(x)=0 \;,\; \chi_-(x)=1 \;,\; \phi_1(x)=0 \;,\; \phi_2(x)=1\end{cases}$$

what leads to the linear system:

$$\begin{cases}T_1^C=\bar{T}^{\text{sup}}\ (\mathrm{N1})\\ \frac{1}{2}T_1^C+\frac{1}{2}T_2^C+T_2^H=\bar{T}^{\text{sup}}\ (\mathrm{A})\\ \frac{1}{2}T_1^C-T_1^H+\frac{1}{2}T_2^C=\bar{T}^{\text{inf}}\ (\mathrm{B})\\ T_2^C=\bar{T}^{\text{inf}}\ (\mathrm{N2})\end{cases}$$

admitting the following solution:

$$\begin{cases}T_1^C=\bar{T}^{\text{sup}}\\ T_2^C=\bar{T}^{\text{inf}}\\ T_1^H=T_2^H=\dfrac{\bar{T}^{\text{sup}}-\bar{T}^{\text{inf}}}{2}\end{cases},$$

whose digital application amounts imposing: $\begin{cases}T_1^C=20°C\\ T_2^C=10°C\\ T_1^H=T_2^H=5°C\end{cases}$

2.2 Sizes and results of reference

One tests initially the values of the degrees of classical freedom `TEMP` and Heaviside `H1` (noted respectively T^C and T^H in the paragraph 2.1) field of temperature at exit of the operator `THER_LINEAIRE` [U4.54.01], with the nodes located just in lower part and the top of the interface. One ensures oneself to find the values determined in the paragraph well 2.1.

Identification	Reference
TEMP for all the nodes located just at the top of the interface	$20.°C$
TEMP for all the nodes located just below the interface	$10.°C$
H1 for all the nodes located just in dessous/au above of the interface	$5.°C$

The operator POST_MAIL_XFEM [U4.82.21] allows to net the cracks represented by method X-FEM. The operator POST_CHAM_XFEM [U4.82.22], then allows to export results X-FEM on this new grid. These two operators are to be used only in a posterior way with calculation at sights of postprocessing. They make it possible to generate nodes right in lower part and with the top of the interface and to display the field of the nodal unknown factors there (here the field of temperature). One then tests the value of the field of temperature TEMP (noted T in the paragraph 2.1) at exit of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface. One ensures oneself to find the values determined in the paragraph well 2.1.

Identification	Reference
TEMP for all the nodes located just below the interface	$10.°C$
TEMP for all the nodes located just at the top of the interface	$20.°C$

Lastly, the value of the single component is tested TEMP field TEMP_ELGA (field of temperature by elements at the points of Gauss, only calculated on elements X-FEM by the operator THER_LINEAIRE) on the points of Gauss of the elements nouveau riches.

Identification	Reference
TEMP on the points of Gauss located below the interface	$10.°C$
TEMP on the points of Gauss located at the top of the interface	$20.°C$

Note: *Elements X-FEM contain a significant number of points of integration (which can amount to 480 for the thermal elements being pressed on the mesh HEXA8_) because of cutting in subelements. Moreover, as the position of the crack conditions the result of the procedure of under-cutting, this number is variable for the same type of element. The value of the component is thus not tested TEMP field TEMP_ELGA on all the points of integration of elements X-FEM, and one is satisfied to test it:*

- *in only one point of integration for the elements nouveau riches not being crossed by the interface (the value being constant on the element)*
- *in two points of integration for the elements nouveau riches crossed by the interface (the first located below the interface, the second with the top), and this only for modelings A, F, and I (grid regulated in HEXA8 or QUAD4)*

3 Modeling A

3.1 Characteristics of modeling

Modeling is used 3D phenomenon THERMICS.

3.2 Characteristics of the grid

The grid comprises 5 regular hexahedrons of type HEXA8.

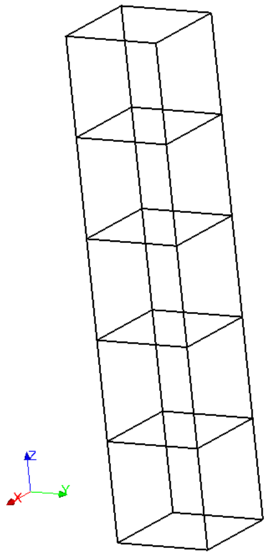

Figure 3.2-1: Grid A

3.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom TEMP and Heaviside H1 field of temperature at exit of the operator THER_LINEAIRE, with the nodes located just in lower part (4 nodes) and at the top of the interface (4 nodes).

Identification	Type of reference	Value of reference	Tolerance
All nodes located just with the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%
All nodes located just in lower part interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just in lower part/at the top of the interface - $H1$	'ANALYTICAL'	5	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Identification	Type of reference	Value of reference	Tolerance
All nodes located just			

below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

4 Modeling B

This modeling is exactly the same one as modeling A. the only difference lies in the grid: HEXA8 grid A are cut out in PENTA6.

4.1 Characteristics of modeling

Modeling is used 3D phenomenon THERMICS.

4.2 Characteristics of the grid

The grid comprises 10 meshs of the type PENTA6.

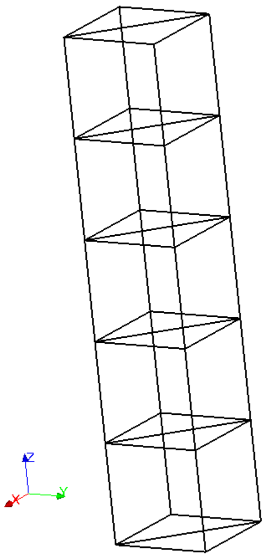

Figure 4.2-1: Grid B

4.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom TEMP and Heaviside H1 field of temperature at exit of the operator THER_LINEAIRE, with the nodes located just in lower part (4 nodes) and at the top of the interface (4 nodes).

Identification	Type of reference	Value of reference	Tolerance
All nodes located just with the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%
All nodes located just in lower part interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just in lower part/at the top of the interface - $H1$	'ANALYTICAL'	5	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Identification	Type of reference	Value of reference	Tolerance
All nodes located just below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

5 Modeling C

This modeling is exactly the same one as modeling A. the only difference lies in the grid: HEXA8 grid A are cut out in TETRA4.

5.1 Characteristics of modeling

Modeling is used 3D phenomenon THERMICS.

5.2 Characteristics of the grid

The grid comprises 30 meshs of the type TETRA4.

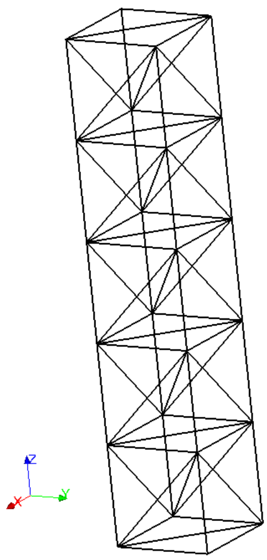

Figure 5.2-1: Grid C

5.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom TEMP and Heaviside H1 field of temperature at exit of the operator THER_LINEAIRE, with the nodes located just in lower part (4 nodes) and at the top of the interface (4 nodes).

Identification	Type of reference	Value of reference	Tolerance
All nodes located just with the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%
All nodes located just in lower part interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just in lower part/at the top of the interface - $H1$	'ANALYTICAL'	5	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Identification	Type of reference	Value of reference	Tolerance
All nodes located just below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

6 Modeling D

This modeling is exactly the same one as modeling A. the only difference lies in the grid: one calls MACR_ADAP_MAIL [U7.03.01] to refine certain meshs HEXA8, which results in to generate meshs PYRA5 (and TETRA4).

6.1 Characteristics of modeling

Modeling is used 3D phenomenon THERMICS.

6.2 Characteristics of the grid

The grid comprises 3 meshs of the type HEXA8, 8 meshs of the type TETRA4 and 10 meshs of the type PYRA5.

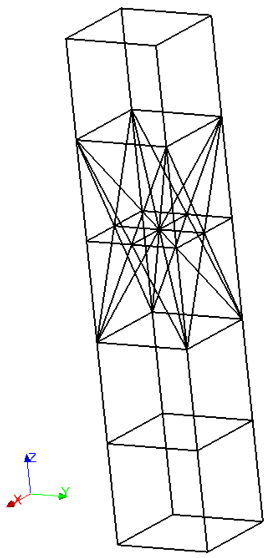

Figure 6.2-1: Grid D

6.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom TEMP and Heaviside H1 field of temperature at exit of the operator THER_LINEAIRE, with the nodes located just in lower part (4 nodes) and at the top of the interface (4 nodes).

Identification	Type of reference	Value of reference	Tolerance
All nodes located just with the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%
All nodes located just in lower part interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just in lower part/at the top of the interface - $H1$	'ANALYTICAL'	5	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Identification	Type of reference	Value of reference	Tolerance
All nodes located just below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

7 Modeling E

For this modeling, one changes the values of dimensions of the bar, the equation of the level-set, as well as the grid. The boundary conditions remain unchanged.
The geometrical position of the interface chosen, generates a configuration a priori unfavourable for method X-FEM. However, with the new formulation of enrichment X-FEM [R7.02.12], one notes that the conditioning of the problem is not degraded. And the precision of the solution is comparable to other modelings. It would be even possible to lower the analytical tolerances until the precision machine.

7.1 Characteristics of modeling

For this modeling, the values dimensions of the bar are:

$Lx=1\text{m}$, $Ly=1\text{m}$ and $Lz=7\text{m}$

as well as the equation of the level-set, one considers here a plane interface of normal $n=(-1,1,1)^T$ and passing by the point A coordinates $(0.5,\ -0.5(1-\delta),\ 0.5)$.

The interface is introduced by a level set LSN (see Figure 7.2-1) of equation:

$$LSN\ :\ -x+y+z-(-0.5+\delta)$$

Lastly, modeling is used 3D phenomenon THERMICS.

7.2 Characteristics of the grid

The grid comprises 7 meshs of the type HEXA8. This choice makes it possible to be ensured to have “classical” elements (not X-FEM) at the two ends of the bar (see Figure 7.2-1), and thus to impose the boundary conditions on nodes not carrying degrees of freedom nouveau riches.

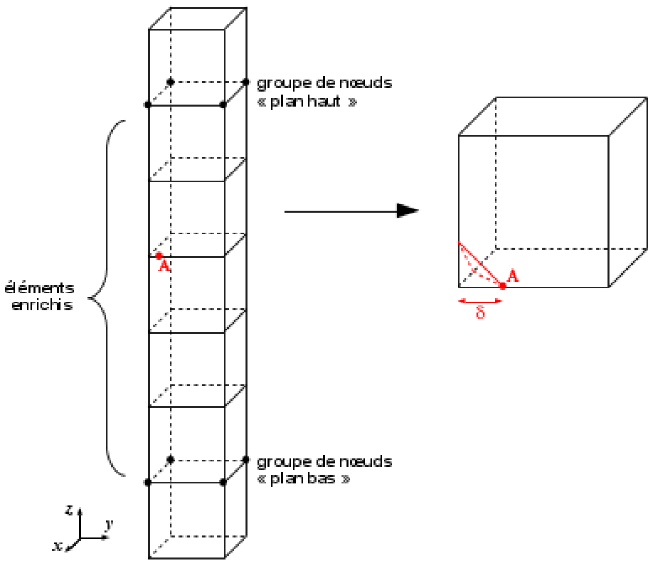

Figure 7.2-1: Grid E and position of the level-set

7.3 Sizes tested and results

One tests initially the values of the classical degrees of freedom TEMP and Heaviside H1 field of temperature at exit of the operator THER_LINEAIRE, on the four nodes which set up the group “plan low” and on the four nodes which set up the group “plan high” (see Figure 7.2-1). These nodes being divided by classical elements and nouveau riches, the degrees of freedom Heaviside H1 must be worthless and the classical degrees of freedom TEMP must correspond to the physical temperature ($\bar{T}^{\text{inf}}$ or $\bar{T}^{\text{sup}}$).

Identification	Type of reference	Value of reference	Tolerance
All nodes of the group “low plan” - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes of the group “low plan” - $H1$	'ANALYTICAL'	0	0.1%
All nodes of the group “high plan” - $TEMP$	'ANALYTICAL'	20	0.1%
All nodes of the group “high plan” - $H1$	'ANALYTICAL'	0	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, on the four nodes which set up the group “plan low” and on the four nodes which set up the group “plan high” (see Figure 7.2-1).

Identification	Type of reference	Value of reference	Tolerance
All nodes of the group “low plan” - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes of the group “high plan” - $TEMP$	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

8 Modeling F

This modeling corresponds to the problem of reference 2D describes in the page 2.

8.1 Characteristics of modeling

Modeling is used `PLAN` phenomenon `THERMICS`.

8.2 Characteristics of the grid

The grid comprises 5 meshs of the type `QUAD4`.

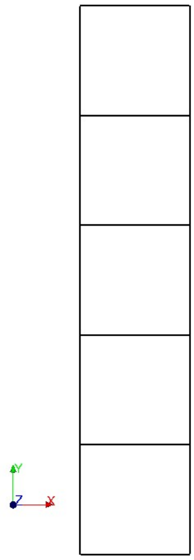

Figure 8.2-1: Grid F

8.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom `TEMP` and Heaviside `H1` field of temperature at exit of the operator `THER_LINEAIRE`, with the nodes located just in lower part (2 nodes) and at the top of the interface (2 nodes).

Identification	Type of reference	Value of reference	Tolerance
All nodes located just with the top of the interface - $TEMP$	`'ANALYTICAL'`	20	0.1%
All nodes located just in lower part interface - $TEMP$	`'ANALYTICAL'`	10	0.1%
All nodes located just in lower part/at the top of the interface - $H1$	`'ANALYTICAL'`	5	0.1%

One tests then the value of the degree of freedom `TEMP` field of temperature at exit of `POST_CHAM_XFEM`, with the nodes located just in lower part and the top of the interface.

Identification	Type of reference	Value of reference	Tolerance
All nodes located just below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just at the top of the interface $TEMP$ -	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

9 Modeling G

This modeling is exactly the same one as modeling F. the only difference lies in the grid: QUAD4 grid F are cut out in TRIA3.

9.1 Characteristics of modeling

Modeling is used PLAN phenomenon THERMICS.

9.2 Characteristics of the grid

The grid comprises 10 meshs of the type TRIA3.

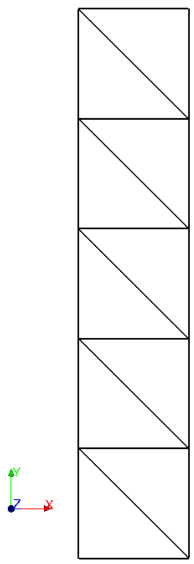

Figure 9.2-1: Grid G

9.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom TEMP and Heaviside H1 field of temperature at exit of the operator THER_LINEAIRE, with the nodes located just in lower part (2 nodes) and at the top of the interface (2 nodes).

Identification	Type of reference	Value of reference	Tolerance
All nodes located just with the top of the interface - *TEMP*	'ANALYTICAL'	20	0.1%
All nodes located just in lower part interface - *TEMP*	'ANALYTICAL'	10	0.1%
All nodes located just in lower part/at the top of the interface - *H1*	'ANALYTICAL'	5	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Identification	Type of reference	Value of reference	Tolerance
All nodes located just below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just at the top of the interface $TEMP$ -	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

10 Modeling H

For this modeling, one changes the values of dimensions of the bar, the equation of the level-set, as well as the grid. The boundary conditions remain unchanged.
The geometrical position of the interface chosen, generates a configuration a priori unfavourable for method X-FEM. However, with the new formulation of enrichment X-FEM [R7.02.12], one notes that the conditioning of the problem is not degraded. And the precision of the solution is comparable to other modelings. It would be even possible to lower the analytical tolerances until the precision machine.

10.1 Characteristics of modeling

For this modeling, the values dimensions of the bar are:

$Lx=1\text{m}$ and $Ly=7\text{m}$

as well as the equation of the level-set, one considers here a plane interface of normal $n=(1,1)^T$ and passing by the point A coordinates $(-0.5(1-\delta),\ 0.5)$.

The interface is introduced by a level set LSN (see Figure 10.2-1) of equation:

$$LSN\ :\ x+y-\delta$$

Lastly, modeling is used PLAN phenomenon THERMICS.

10.2 Characteristics of the grid

The grid comprises 7 meshs of the type QUAD4. This choice makes it possible to be ensured to have "classical" elements (not X-FEM) at the two ends of the bar (see Figure 10.2-1), and thus to impose the boundary conditions on nodes not carrying degrees of freedom nouveau riches.

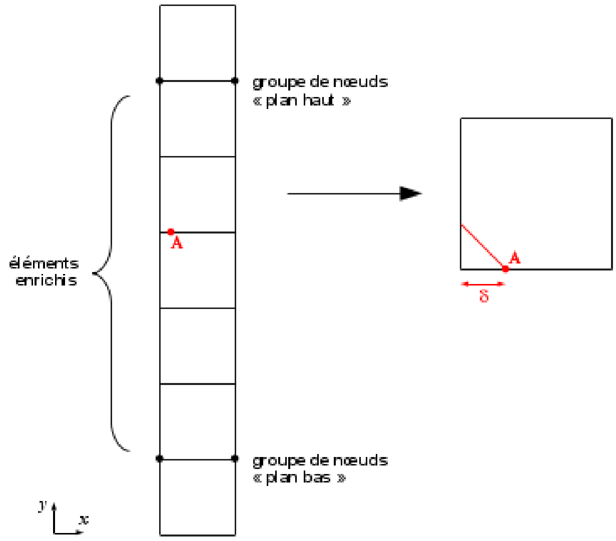

Figure 10.2-1: grid H and position of the level-set

10.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom TEMP and Heaviside H1 field of temperature at exit of the operator THER_LINEAIRE, on the two nodes which set up the group "plan low" and on the two nodes which set up the group "plan high" (see Figure 10.2-1). These nodes being divided by classical elements and nouveau riches, the degrees of freedom Heaviside H1 must be worthless and the classical degrees of freedom TEMP must correspond to the physical temperature ($\bar{T}^{\text{inf}}$ or $\bar{T}^{\text{sup}}$).

Identification	Type of reference	Value of reference	Tolerance
All nodes of the group "low plan" - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes of the group "low plan" - $H1$	'ANALYTICAL'	0	0.1%
All nodes of the group "high plan" - $TEMP$	'ANALYTICAL'	20	0.1%
All nodes of the group "high plan" - $H1$	'ANALYTICAL'	0	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, on the two nodes which set up the group "plan low" and on the two nodes which set up the group "plan high" (see Figure 10.2-1).

Identification	Type of reference	Value of reference	Tolerance
All nodes of the group "low plan" - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes of the group "high plan" - $TEMP$	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

11 Modeling I

This modeling is based on the whole of the characteristics of the problem of reference 2D describes in the page 2. , except that an axisymmetric modeling is considered. It is thus about a cylindrical bar, but the analytical solution described in the page 4 remain valid for this problem.

11.1 Characteristics of modeling

For this modeling, the bar is cylindrical and its dimensions are the following ones:

- ray $Lx=0,5\,m$
- height $Ly=5\mathrm{m}$

The interface is introduced with middle height of the bar by a level set LSN of equation:

$$LSN \; : \; y$$

Modeling is used AXIS phenomenon THERMICS.

11.2 Characteristics of the grid

The grid comprises 5 meshs of the type QUAD4.

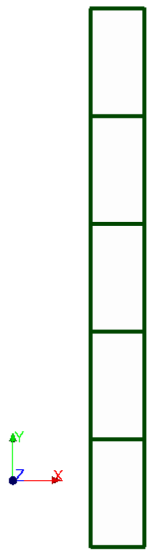

Figure 11.2-1: Grid I

11.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom TEMP and Heaviside H1 field of temperature at exit of the operator THER_LINEAIRE, with the nodes located just in lower part (2 nodes) and at the top of the interface (2 nodes).

Identification	Type of reference	Value of reference	Tolerance
All nodes located just with the top of the	'ANALYTICAL'	20	0.1%

interface - $TEMP$			
All nodes located just in lower part interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just in lower part/at the top of the interface - $H1$	'ANALYTICAL'	5	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Identification	Type of reference	Value of reference	Tolerance
All nodes located just below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just at the top of the interface $TEMP$ -	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

12 Modeling J

This modeling is based on the whole of the characteristics of the problem of reference 2D describes in the page 2. , except that an axisymmetric modeling is considered. It is thus about a cylindrical bar, but the analytical solution described in the page 4 remain valid for this problem.

12.1 Characteristics of modeling

For this modeling, the bar is cylindrical and its dimensions are the following ones:

- ray $Lx=0,5\,m$
- height $Ly=5\mathrm{m}$

The interface is introduced with middle height of the bar by a level set LSN of equation:

$$LSN\ :\ y$$

Modeling is used `AXIS` phenomenon `THERMICS`.

12.2 Characteristics of the grid

The grid comprises 5 meshs of the type `TRIA3`.

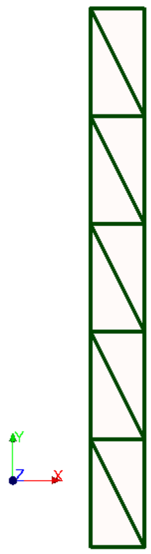

Figure 12.2-1: Grid J

12.3 Sizes tested and results

One tests initially the values of the degrees of classical freedom `TEMP` and Heaviside `H1` field of temperature at exit of the operator `THER_LINEAIRE`, with the nodes located just in lower part (2 nodes) and at the top of the interface (2 nodes).

Identification	Type of reference	Value of reference	Tolerance
All nodes located just with the top of the	'ANALYTICAL'	20	0.1%

interface - $TEMP$			
All nodes located just in lower part interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just in lower part/at the top of the interface - $H1$	'ANALYTICAL'	5	0.1%

One tests then the value of the degree of freedom TEMP field of temperature at exit of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Identification	Type of reference	Value of reference	Tolerance
All nodes located just below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
All nodes located just at the top of the interface $TEMP$ -	'ANALYTICAL'	20	0.1%

One tests finally the value of the component TEMP field TEMP_ELGA on the points of Gauss located in lower part and at the top of the interface (*cf.* notice page 6).

Identification	Type of reference	Value of reference	Tolerance
On the points of Gauss located below the interface - $TEMP$	'ANALYTICAL'	10	0.1%
On the points of Gauss located at the top of the interface - $TEMP$	'ANALYTICAL'	20	0.1%

13 Summary of the results

The goals of this test are achieved:

- To validate the taking into account of enrichment by the Heaviside function of the classical functions of form.
- Moreover, modeling E and H make it possible to validate the contribution of the new enrichment X-FEM [R7.02.12]: the position of the interface does not affect any more the precision of the results and conditioning.