# TTNL101 - Non-linear thermal source in a bar

**Summary:**

This test checks thermal calculation in the presence of a loading of non-linear source, depend on the temperature.

The reference solution is analytical and variable in time and space. The part considered in three modelings is a symmetrical bar composed of lumpés elements.

The following meshs are checked:

- Modeling a:
  - quadrangles QUAD4, QUAD8, QUAD9 for a modeling AXIS
  - quadrangles QUAD4 for a modeling AXIS_DIAG
- Modeling b:
  - hexahedrons HEXA8 for a modeling 3D_DIAG
- Modeling C:
  - triangles TRIA3 for a modeling AXIS
  - triangles TRIA3 for a modeling AXIS_DIAG

The two ends of the bar are subjected to the conditions of adiabadicity by default. The voluminal source of heat is a linear function of the temperature.

# 1 Problem of reference

## 1.1 Geometry

One considers a unidimensional structure (a bar whose side faces are subjected to adiabatic conditions) length $2\mathrm{L}$ occupying the field $[-L; L]$ .

The temperature being homogeneous in the normal directions with the bar, calculation can be regarded as 1D.

## 1.2 Properties of material

| | |
|---|---|
| $\lambda=2$ | thermal conductivity |
| $\rho C=2$ | voluminal heat |

## 1.3 Boundary conditions and loadings

Loading of voluminal source non-linear, function of the temperature:
$s(T)=2-2\times w\times T$ with $w=2$

The boundary conditions are adiabatic on the side faces and of standard worthless temperature imposed at the end of the bar; a condition of symmetry is put in work as regards symmetry (what is equivalent to an adiabatic boundary condition).

The temporal beach $[0.; 1.]$ is discretized in 100 pas de time (lasted of each step of time equalizes with $0.01$ ).

## 1.4 Initial conditions

An analytical initial state is provided. See the developments in the paragraph detailing the reference solution.

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

The bar is subjected to a source of heat $r(T)=r_0-r_1 T$ , where $r_1>0$ for questions of thermal stability. Its initial temperature is worth $T_0(x)$ and the ends of the bar are maintained at a worthless temperature. The evolution of temperature obeys the equation of heat:

$$\rho c \dot{T}=\lambda \nabla^2 T+r(T) \ ; \ T(x,0)=T_0(x) \ ; \ T(-L,t)=T(L,t)=0$$

By standardisation, one can be reduced without loss of general information to the following equation:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+1-\omega^2 u \ ; \ u(x,0)=u_0(x) \ ; \ u(-1,t)=u(1,t)=0$$

To solve this equation, one is interested initially in the asymptotic solution $u_\infty(x)$ who checks:

$$0=\frac{\partial^2 u_\infty}{\partial x^2}+1-\omega^2 u_\infty \ ; \ u_\infty(-1)=u_\infty(1)=0$$

The solution of this linear differential equation of the second order is worth:

$$u_\infty(x)=\frac{1}{\omega^2}\left(1-\frac{\cosh \omega x}{\cosh \omega}\right)$$

The solution of the transitory equation is then obtained by projection of $v=u-u_\infty$ on the clean functions of the Laplacian on $]-1,1[$ . To simplify the analysis, an initial condition is adopted $u_0$ equalize with the first clean mode, namely:

$$u_0(x)=u_\infty(x)-\cos\frac{\pi x}{2}$$

Only the first mode being activated, one is brought back to the solution of a differential equation in first order time, to obtain the solution finally:

$$u(x,t)=u_\infty(x)-\exp\left(-\omega^2 t-\frac{\pi^2}{4}t\right)\cos\frac{\pi x}{2}$$

Lastly, like previously, one goes up $u$ with $T$ by adopting a specific set of parameters, without taking account of the units, so that $T=u$ . For that, one takes $\lambda=r_0=\rho c$ , $r_1=\omega^2 r_0$ , $L=1$ and $T_0(x)=u_0(x)$ .

## 2.2 Results of reference

CAS-test is carried out with $\omega=\sqrt{2}$ and one examines the temperature with $t=1$ in a node of the symmetry plane $(x=0)$ . The data are the following ones:

| | | |
|---|---|---|
| Thermal conductivity | LAMBDA | 2. |
| Voluminal heat-storage capacity | RHO_CP | 2. |
| Initial temperature | $T_0$ | $u_0(x)=\frac{1}{\omega^2}\left(1-\frac{\cosh\omega x}{\cosh\omega}\right)-\cos\frac{\pi x}{2}$ with $\omega=\sqrt{2}$ |
| Source of heat | $r_0$<br>$r_1$ | 2.<br>4. |

| | |
|---|---|
| Size tested | $T\ (\ x=0, t=1\ )$ |
| Value of reference | $0.258974$ |

# 3 Modeling A

## 3.1 Characteristics of modeling

Modelings are used AXIS then AXIS_DIAG .

Only the half of bar is represented (symmetry).

## 3.2 Characteristics of the grid

The grid is Co nstitué of 40 of the same quadrangles cuts. following types are considered successively :

| Modelings | Meshs |
|---|---|
| AXIS | QUAD4 QUAD8 QUAD9 |
| AXIS_DIAG | QUAD4 |

## 3.3 Largeeurs tested and results

One tests the temperature with $t=1$ in a node of the symmetry plane $(x=0)$

The solution is in conformity with the analytical value with less than 0.1 % for a temporal discretization of 100 pas de time.

# 4 Modeling B

## 4.1 Characteristics of modeling

Modelings are used 3D_DIAG .

Only the half of bar is represented (symmetry).

## 4.2 Characteristics of the grid

The grid is Co nstitué of 40 hexaèdr be of even size of type HEXA8 .

## 4.3 Largeeurs tested and results

One tests the temperature with $t=1$ in a node of the symmetry plane $(x=0)$

The solution is in conformity with the analytical value with less than 0.1 % for a temporal discretization of 100 pas de time.

# 5 Modeling C

## 5.1 Characteristics of modeling

Modelings are used AXIS then AXIS_DIAG .

Only the half of bar is represented (symmetry).

## 5.2 Characteristics of the grid

The grid is Co nstitué of 80 triang of the same size. following types are considered successively :

| Modelings | Meshs |
|---|---|
| AXIS | TRIA3 TRIA6 |
| AXIS_DIAG | TRIA3 |

## 5.3 Large eurs tested and results

One tests the temperature with $t=1$ in a node of the symmetry plane $(x=0)$

The solution is in conformity with the analytical value with less than 0.1 % for modeling AXIS and of 0.13% for modeling AXIS_DIAG, with a temporal discretization of 100 pas de time.

# 6 Summary of the results

The results are in conformity with the analytical solution.