TTNL302 - Infinite wall subjected to a constant flow with variable properties

Summary:

This test is resulting from the validation independent of version 3 in nonlinear transitory thermics.

It is about a linear problem 1D represented by five modelings, one planes, four the other voluminal ones.

The features tested are the following ones:

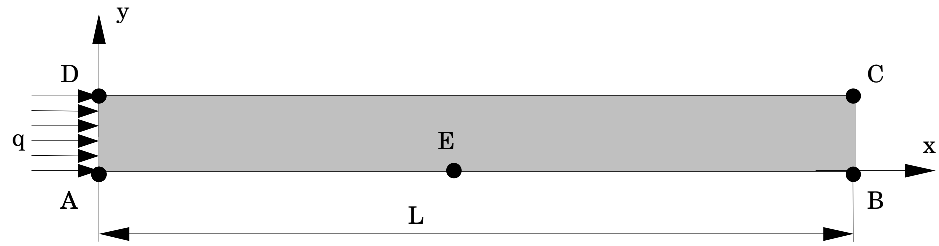
- element of thermal plan,
- voluminal element of thermics,
- algorithm of transitory thermics non-linear,
- variable properties,
- limiting condition: imposed flow.

The interest of the test lies in the taking into account of variable properties (thermal conductivity and voluminal heat).

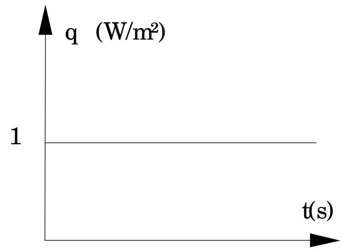
The results are compared with an analytical solution.

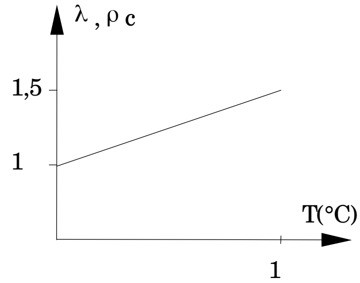
1 Problem of reference

1.1 Geometry



L = 2 m
AE=EB= 1m





1.2 Properties of material

$\lambda = 1.0 + 0.5\,T\ W/m.°C$ Thermal conductivity

$\rho c = 1.0 + 0.5\,T\ J/m^{3.}°C$ Voluminal heat

1.3 Boundary conditions and loadings

- dimensioned $[AD]$: imposed flow $q = 1\ W/m^2$ for $t > 0$,
- dimensioned $[AB]$, $[BC]$, $[CD]$ $\varphi = 0.$

1.4 Initial conditions

$T(x, 0) = 0°C$ for all x

2 Reference solution

2.1 Method of calculating used for the reference solution

Semi-analytical solution utilizing functions of error:

$$T(x,t)=2\left\{\sqrt{\left[1+2\sqrt{(t/\pi)}\exp\left(\frac{-x^2}{4t}\right)+x.erfc\left(\frac{x}{2\sqrt{(t)}}\right)\right]}-1\right\}$$

with $erfc(x)=\frac{2}{\pi}\int_x^\infty e^{-t^2}\,dt$

where $x=$ X-coordinate
$t=$ time

This formula is valid only for $\lambda(T)=\rho c(T)=1.+0.5\mathrm{T}$

2.2 Results of reference

Temperature at the points A ($x=0$) and E ($x=1$) at the moments t following: $t=0.1, 0.3, 0.5, 0.7$ and $1\mathrm{s}$

2.3 Uncertainty on the solution

Unknown factor, due to the evaluation of the functions of error.

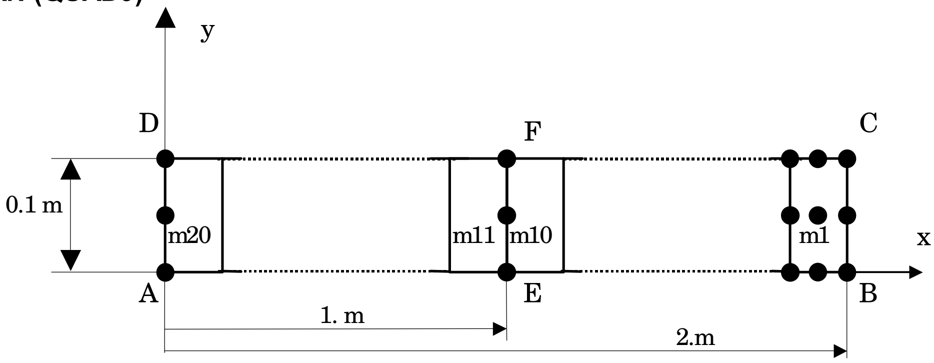
2.4 References bibli ographic

- Segal, NR. Praagman, “with fast implementation of explicit time stepping algorithms with the finite element method for have class of nonlinear evolution problems”, Int. J. num. Meth. Engng, flight 23, pp 155-168, 1986.

3 Modeling A

3.1 Characteristics of modeling

PLAN (QUAD9)



Conditions limites:

- cotés AB, BC, CD ϕ = 0
- coté AD   q = 1 W/m²°C

Point	x	y	Noeud
A	0.0	0.0	N1
D	0.0	0.1	N3
E	1.0	0.0	N61
F	1.0	0.1	N63

3.2 Characteristics of the grid

Many nodes: 123
Many meshs and types: 20 QUAD9

3.3 Remarks

The discretization in step of time is the following one:

10 pas for $[0.,5.D-2]$ that is to say $\Delta t=5.D^{-3}$

19 pas for $[5.D-2,1.D0]$ that is to say $\Delta t=5.D^{-2}$

4 Results of modeling A

4.1 Values tested

Identification	Reference	*Aster*	% difference	tolerance
Temperature $(°C)$				
Node N1 $t=0.1s$	0,330	0,329	˜ 0,204%	1%
“ “ T = 0.3s	0,544	0,544	˜ 0,048%	1%
“ “ T = 0.5s	0,682	0,681	˜ 0,075%	1%
“ “ T = 0.7s	0,789	0,789	˜ 0,036%	1%
“ “ T = 1.0s	0,918	0,920	0,254%	1%
Node N3 $t=0.1s$	0,330	0,329	˜ 0,204%	1%
“ “ T = 0.3s	0,544	0,544	˜ 0,048%	1%
“ “ T = 0.5s	0,682	0,681	˜ 0,075%	1%
“ “ T = 0.7s	0,789	0,789	˜ 0,036%	1%
“ “ T = 1.0s	0,918	0,920	0,254%	1%
Node N61 $t=0.1s$	0,004	0,004	**1,161%**	1%
“ “ T = 0.3s	0,071	0,071	˜ **0,377%**	1%
“ “ T = 0.5s	0,160	0,161	0,573%	1%
“ “ T = 0.7s	0,247	0,251	1,616%	1%
“ “ T = 1.0s	0,366	0,380	**3,951%**	1%
Node N63 $t=0.1s$	0,004	0,004	**1,161%**	1%
“ “ T = 0.3s	0,071	0,071	˜ **0,377%**	1%
“ “ T = 0.5s	0,160	0,161	0,573%	1%
“ “ T = 0.7s	0,247	0,251	1,616%	1%
“ “ T = 1.0s	0,366	0,380	**3,951%**	1%

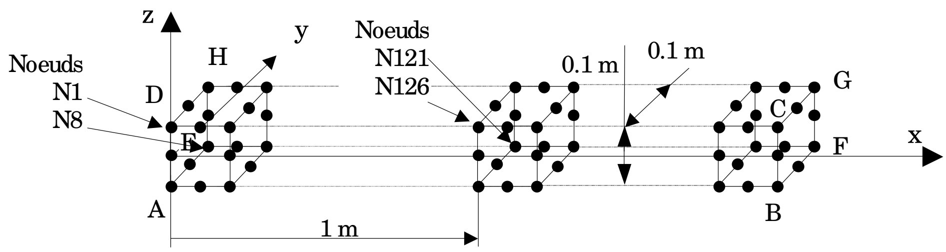
4.2 Remarks

The relative error is to the maximum of 3.9%.

5 Modeling B

5.1 Characteristics of modeling

3D (HEXA20)



Conditions limites:

- faces ABCD, ABFE $\phi = 0$
- faces EFGH, DCGH $\phi = 0$
- face BFGC $\phi = 0$
- face AEHD $q = 1\ W/m^2$

Noeuds	x	y	z
N1	0.0	0.0	0.05
N8	0.0	0.1	-0.05
N121	1.0	0.0	-0.05
N126	1.0	0.1	0.05

5.2 Characteristics of the grid

Many nodes: 248
Many meshs and types: 20 HEXA20

5.3 Remarks

The discretization in step of time is the following one:

10 pas for $[0.,5.D-2]$ that is to say $\Delta t = 5.D^{-3}$
19 pas for $[5.D-2, 1.D0]$ that is to say $\Delta t = 5.D^{-2}$

6 Results of modeling B

6.1 Values tested

Identification	Reference	*Aster*	% difference	tolerance
Temperature $(°C)$				
Node N1 T = 0.1 S	0,330	0,330	~0,129	1%
" " T = 0.3s	0,544	0,543	~0,149	1%
" " T = 0.5s	0,682	0,681	~0,154	1%
" " T = 0.7s	0,789	0,788	~0,092	1%
" " T = 1.0s	0,918	0,920	0,222	1%
Node N8 T = 0.1s	0,330	0,330	~0,129	1%
" " T = 0.3s	0,544	0,543	~0,149	1%
" " T = 0.5s	0,682	0,681	~0,154	1%
" " T = 0.7s	0,789	0,788	~0,092	1%
" " T = 1.0s	0,918	0,920	0,222	1%
Node N121 T = 0.1s	0,004	0,004	**10,931**	1%
" " T = 0.3s	0,071	0,071	~**0,242**	1%
" " T = 0.5s	0,160	0,161	0,587	1%
" " T = 0.7s	0,247	0,251	1,619	1%
" " T = 1.0s	0,366	0,380	**3.95**	1%
Node N126 T = 0.1s	0,004	0,004	**10,931**	1%
" " T = 0.3s	0,071	0,071	~**0,242**	1%
" " T = 0.5s	0,160	0,161	0,587	1%
" " T = 0.7s	0,247	0,251	1,619	1%
" " T = 1.0s	0,366	0,380	**3.95**	1%

6.2 Remarks

The relative error is to the maximum of 3.95%, except for $x=1$ at the moment $t=0.1s$ the error is of 11%. This error was obtained for the smallest value of the temperature ($T=0.004°C$). This variation is explained by the fact why the function of error in this point is of 0.025347 and that uncertainty on the calculation of the function of error is unknown.

7 Modeling C

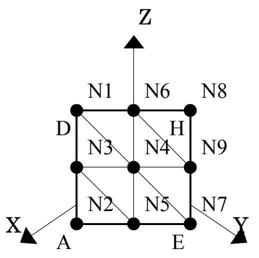
7.1 Characteristics of modeling

3D (TETRA4)

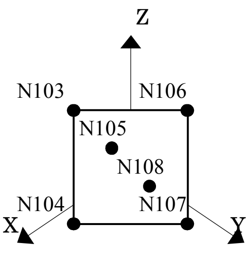
Axe longitudinal x suivant la trisectrice (X=Y=Z)

Conditions limites:

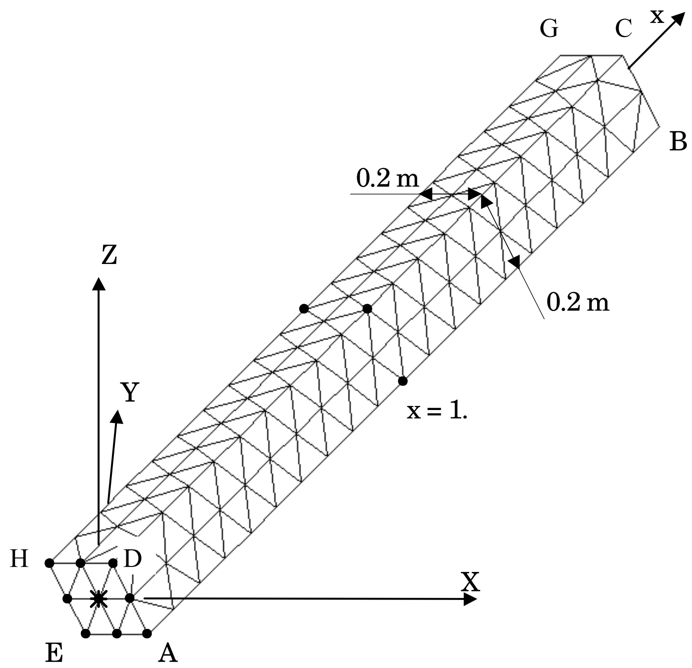
- faces ABCD, ABFE $\phi = 0$
- faces EFGH, DCGH $\phi = 0$
- face BFGC $\phi = 0$
- face AEHD $q = 1 \text{ W/m}^2$



Section x = 0.



Section x = 1.



7.2 Characteristics of the grid

Many nodes:	224
Many meshs and types:	692 TETRA4 (and 8 TRIA3)

7.3 Remarks

The discretization in step of time is the following one:

10 pas for $[0., 5.D-2]$ that is to say $\Delta t = 5.D^{-3}$

19 pas for $[5.D-2, 1.D0]$ that is to say $\Delta t = 5.D^{-2}$

8 Results of modeling C

8.1 Values tested

Identification		**Reference**	***Aster***	**Relative variation (%)**		**Absolute deviation (°C)**	
				difference	**tolerance**	**difference**	**tolerance**
Temperatures in $°C$:							
Face	X = 0.m						
Node N7	T = 0.1s	0,330	0.3295	~ 0,162%	1.%	~ 0.000536	0,005
" "	T = 0.3s	0,544	0.5425	~ 0,273%	1.%	~ 0.00149	0,005
" "	T = 0.5s	0,682	0.6796	~ 0,351%	1.%	~ 0.00239	0,005
" "	T = 0.7s	0,789	0.7861	~ 0,362%	1.%	~ 0.00285	0,005
" "	T = 1.0s	0,918	0.9165	~ 0,159%	1.%	~ 0.00146	0,005
Node N5	T = 0.1s	0,330	0.3279	~ 0,627%	1.%	~ 0.00207	0,005
" "	T = 0.3s	0,544	0.5418	~ 0,406%	1.%	~ 0.00221	0,005
" "	T = 0.5s	0,682	0.6791	~ 0,422%	1.%	~ 0.00288	0,005
" "	T = 0.7s	0,789	0.7858	~ 0,409%	1.%	~ 0.00323	0,005
" "	T = 1.0s	0,918	0.9162	~ 0,192%	1.%	~ 0.00176	0,005
Section	X = 1.m						
Node N107	T = 0.1s	0.00394	0.004140	**5,085%**	1.%	0.000200	0,005
" "	T = 0.3s	0.0706	0.07013	~ 0,665%	1.%	~ 0.000470	0,005
" "	T = 0.5s	0,160	0.1596	~ 0,228%	1.%	~ 0.000364	0,005
" "	T = 0.7s	0,247	0.2488	0,730%	1.%	0.00180	0,005
" "	T = 1.0s	0,366	0.3766	**2,889%**	1.%	**0.0106**	0,005
Node N108	T = 0.1s	0.00394	0.004002	**1,577%**	1.%	~ 0.0000621	0,005
" "	T = 0.3s	0.0706	0.06937	~ **1,742%**	1.%	~ 0.00123	0,005
" "	T = 0.5s	0,160	0.1586	~ 0,895%	1.%	~ 0.00143	0,005
" "	T = 0.7s	0,247	0.2476	0,238%	1.%	0.000587	0,005
" "	T = 1.0s	0,366	0.3753	**2,534%**	1.%	**0.00928**	0,005

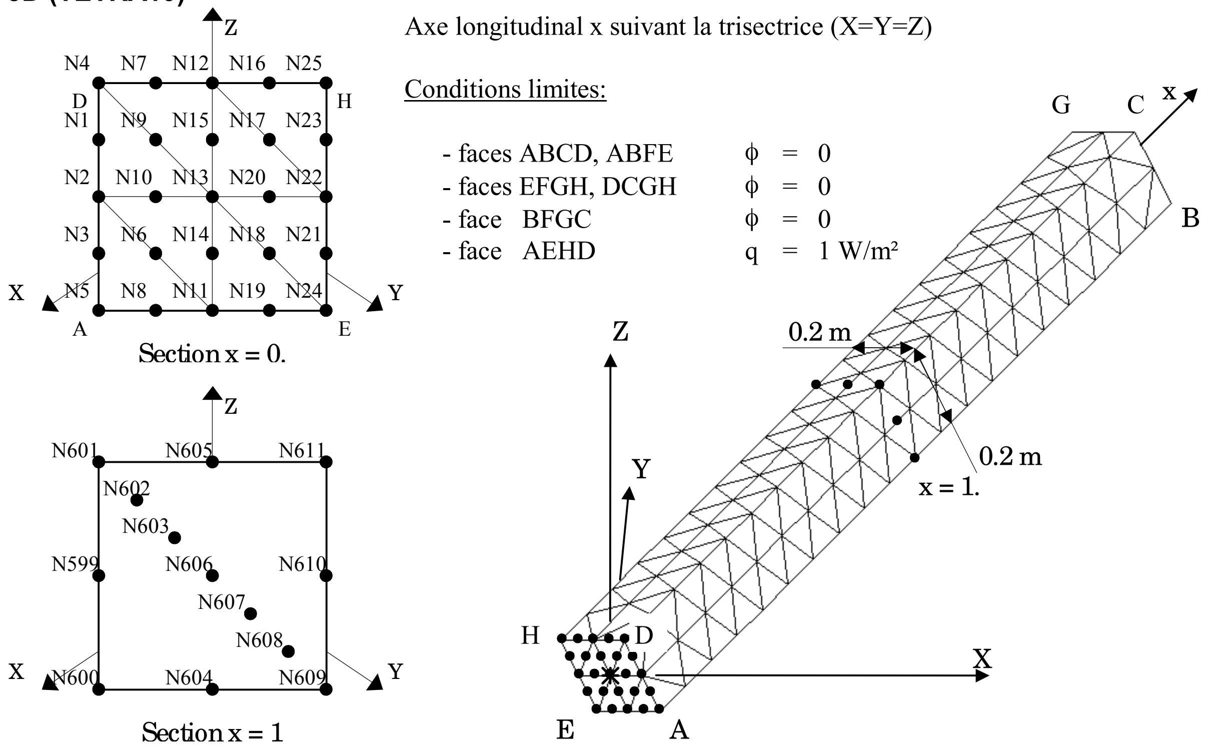
8.2 Remarks

The selected nodes correspond to the extreme results on the same section.

9 Modeling D

9.1 Characteristics of modeling

3D (TETRA10)



9.2 Characteristics of the grid

Many nodes: 1310
Many meshs and types: 697 TETRA10 (and 8 TRIA6)

9.3 Remarks

The discretization in step of time is the following one:

10 pas for $[0., 5.D-2]$ that is to say $\Delta t = 5.D^{-3}$

19 pas for $[5.D-2, 1.D0]$ that is to say $\Delta t = 5.D^{-2}$

10 Results of modeling D

10.1 Values tested

Identification		**Reference**	***Aster***	**Relative variation (%)**		**Absolute deviation (°C)**	
				difference	**tolerance**	**difference**	**tolerance**
Temperatures in $°C$:							
Face	X = 0.m						
Node N4	T = 0.1s	0,330	0.3291	˜ 0,281%	1.%	˜ 0.000926	0,005
“ “	T = 0.3s	0,544	0.5423	˜ 0,318%	1.%	˜ 0.00173	0,005
“ “	T = 0.5s	0,682	0.6794	˜ 0,383%	1.%	˜ 0.00261	0,005
“ “	T = 0.7s	0,789	0.7860	˜ 0,384%	1.%	˜ 0.00303	0,005
“ “	T = 1.0s	0,918	0.9164	˜ 0,180%	1.%	˜ 0.00165	0,005
Node N25	T = 0.1s	0,330	0.3292	˜ 0,255%	1.%	˜ 0.000843	0,005
“ “	T = 0.3s	0,544	0.5423	˜ 0,314%	1.%	˜ 0.00171	0,005
“ “	T = 0.5s	0,682	0.6794	˜ 0,382%	1.%	˜ 0.00261	0,005
“ “	T = 0.7s	0,789	0.7860	˜ 0,383%	1.%	˜ 0.00303	0,005
“ “	T = 1.0s	0,918	0.9163	˜ 0,180%	1.%	˜ 0.00165	0,005
Section	X = 1.m						
Node N606	T = 0.1s	0.00394	0.004331	**9,913%**	1.%	0.000391	0,005
“ “	T = 0.3s	0.0706	0.07021	˜ 0,551%	1.%	˜ 0.000389	0,005
“ “	T = 0.5s	0,160	0.1596	˜ 0,251%	1.%	˜ 0.000402	0,005
“ “	T = 0.7s	0,247	0.2488	0,710%	1.%	0.00175	0,005
“ “	T = 1.0s	0,366	0.3764	**2,855%**	1.%	**0.0104**	0,005
Node N611	T = 0.1s	0.00394	0.004332	**9,944%**	1.%	0.000392	0,005
“ “	T = 0.3s	0.0706	0.07021	˜ 0,550%	1.%	˜ 0.000388	0,005
“ “	T = 0.5s	0,160	0.1596	˜ 0,251%	1.%	˜ 0.000402	0,005
“ “	T = 0.7s	0,247	0.2488	0,710%	1.%	0.00175	0,005
“ “	T = 1.0s	0,366	0.3764	**2,855%**	1.%	**0.0104**	0,005

10.2 Remarks

The calculated results are almost identical on the nodes of the same section.

11 Modeling E

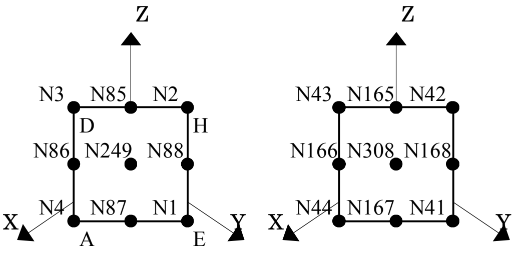
11.1 Characteristics of modeling

3D (HEXA27)

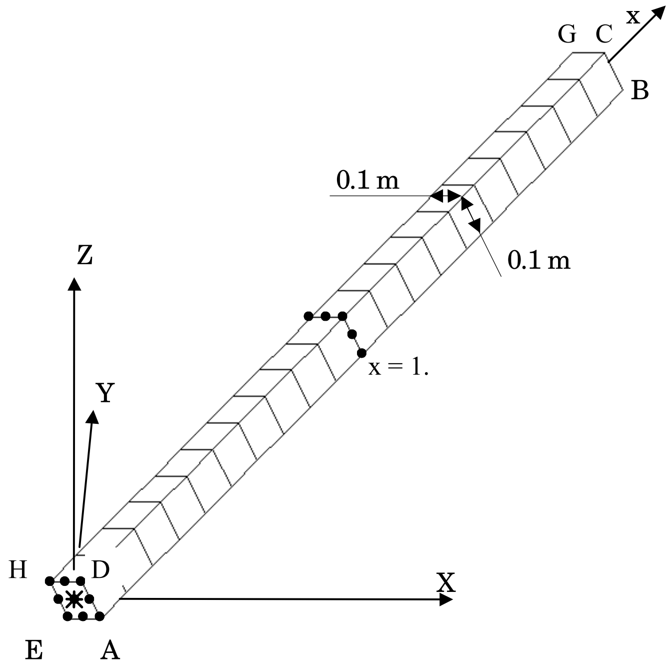
Axe longitudinal x suivant la trisectrice (X=Y=Z)

<u>Conditions limites:</u>

- faces ABCD, ABFE $\phi = 0$
- faces EFGH, DCGH $\phi = 0$
- face BFGC $\phi = 0$
- face AEHD $q = 1\ W/m^2$



Section x = 0. Section x = 1.



11.2 Characteristics of the grid

Many nodes:	369
Many meshs and types:	20 HEXA27 (and 1 QUAD9)

11.3 Remarks

The discretization in step of time is the following one:

10 pas for $[0., 5.D-2]$ that is to say $\Delta t = 5.D^{-3}$

19 pas for $[5.D-2, 1.D0]$ that is to say $\Delta t = 5.D^{-2}$

12 Results of modeling E

12.1 Values tested

Identification		Reference	*Aster*	Relative variation (%)		Absolute deviation (°C)	
				difference	**tolerance**	**difference**	**tolerance**
Temperatures in $°C$:							
Face	X = 0.m						
Node N249	T = 0.1s	0,330	0.3291	˜ 0,283%	1.%	˜ 0.000933	0,005
" "	T = 0.3s	0,544	0.5423	˜ 0,317%	1.%	˜ 0.00173	0,005
" "	T = 0.5s	0,682	0.6794	˜ 0,376%	1.%	˜ 0.00256	0,005
" "	T = 0.7s	0,789	0.7860	˜ 0,378%	1.%	˜ 0.00298	0,005
" "	T = 1.0s	0,918	0.9165	˜ 0,168%	1.%	˜ 0.00154	0,005
Section	X = 1.m						
Node N308	T = 0.1s	0.00394	0.004331	**9,926%**	1.%	0.000391	0,005
" "	T = 0.3s	0.0706	0.07021	˜ 0,554%	1.%	˜ 0.000391	0,005
" "	T = 0.5s	0,160	0.1596	˜ 0,227%	1.%	˜ 0.000363	0,005
" "	T = 0.7s	0,247	0.2488	0,726%	1.%	0.00179	0,005
" "	T = 1.0s	0,366	0.3766	**2,886%**	1.%	**0.0106**	0,005

12.2 Remarks

The calculated results are identical (to 10^{-7} near) on the nodes of the same section.

13 Summary of the results

Five modelings carried out, have same cutting in the direction of propagation of the temperature, they are different only by their type of meshs.

Five modelings give results whose certain values exceed the tolerance fixed initially (1%). The maximum change is of 9.9%. It appears for the smallest value of reference located in the middle of the wall and at the beginning of the transient.

A grid finer associate with a finer temporal discretization should improve quality of the results.

Moreover, the reference solution utilizes a function of error whose precision is unknown.

The results are regarded as acceptable taking into account the points evoked above.

This test made it possible to test hexahedral and tetrahedral meshs in transitory non-linear thermics as well as the principal following thermal orders:

- DEFI_MATERIAU associated with the keyword THER_NL, allowing to define the characteristics of a material whose characteristics vary according to the temperature (conductivity and enthalpy),
- THER_NON_LINE order allowing the resolution of a stationary thermal nonlinear problem or not.