TTLV301 - Parallelepiped subjected to a temperature imposed on its faces

Summary:

This test is resulting from the validation independent of version 3 in linear transitory thermics.

It is about a voluminal problem represented by only one modeling (3D).

The features tested are the following ones:
- voluminal thermal element,
- transitory algorithm of thermics,
- limiting conditions: imposed temperature.

The results are compared with an analytical solution.

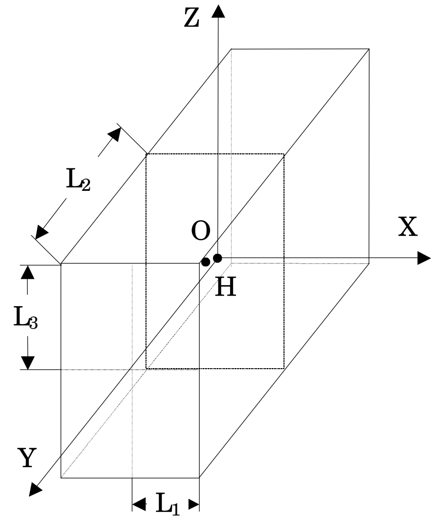
1 Problem of reference

1.1 Geometry

Dimensions du parallélépipède: 2m x 3.2m x 4m

- L_1 = 1.0 m
- L_2 = 1.6 m
- L_3 = 2.0 m

Point O (0.,0.,0.)
Point H (0.5,0.8,1.0)



1.2 Properties of material

$\lambda=1.W/m°C$ thermal conductivity
$c_p=1.J/kg°C$ specific heat
$\rho=1.kg/m^3$ density

1.3 Boundary conditions and loadings

Temperature imposed on the 6 faces $T=2°C=T_w$

1.4 Initial conditions

$T(t=0)=1°C=T_0$

2 Reference solution

2.1 Method of calculating used for the reference solution

$$T_{(x,y,z,t)}=T_w+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty} a_{nml}\exp(-\kappa_{mnl}^2\alpha.t)Tcos_{(x,y,z,m,n,l)}$$

with $Tcos_{(x,y,z,m,n,l)}=\cos\left(\frac{(2m-1)\pi x}{2L_1}\right)\cos\left(\frac{(2n-1)\pi y}{2L_2}\right)\cos\left(\frac{(2l-1)\pi z}{2L_3}\right)$

$$a_{mnl}=\frac{64(T_0-T_w)}{\pi^3(2m-1)(2n-1)(2l-1)}\sin\left(\frac{(2m-1)\pi}{2}\right)\sin\left(\frac{(2n-1)\pi}{2}\right)\sin\left(\frac{(2l-1)\pi}{2}\right)$$

$$\kappa_{mnl}=\left(\frac{(2m-1)\pi}{2L_1}\right)^2+\left(\frac{(2n-1)\pi}{2L_2}\right)^2+\left(\frac{(2l-1)\pi}{2L_3}\right)^2$$

$$\alpha=\frac{\lambda}{\rho c_p}$$

The values of reference are obtained with $m=n=l=100.$

2.2 Results of reference

Temperature at the points: $O(0,0,0)$ and $H(0.5,0.8,1.)$

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

- M.J Chang, L.C Chow, W.S Chang, “Improved alternating direction implicit for solving transient three dimensional heat diffusion problems”, Numerical Heat Transfer, flight 19, pp 69-84, 1991.

3 Modeling A

3.1 Characteristics of modeling

3D (HEXA27)

Modélisation 1/8 du parallélépipède

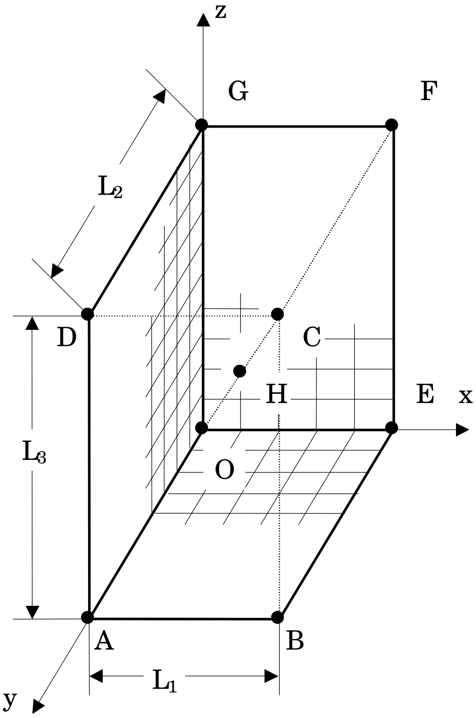
Maillage:

- 5 éléments suivant x
- 8 éléments suivant y
- 10 éléments suivant z

Conditions limites:

- faces [ABCD], [BEFC], [DCFG]: $T = 2°C$
- faces [ABEO], [AOGD], [OEFG]: $\phi = 0.$

Points	x	y	z	Noeud
O	0.00	0.00	0.00	N5
H	0.50	0.80	1.00	N1075



3.2 Characteristics of the grid

Many nodes:	3927
Many meshs and types:	400 HEXA27

3.3 Remarks

The limiting condition $\varphi=0.$ is implicit on the free edges.
Discretization of time: 24 intervals enters $0.$ and $1.2\,s$:

of $t=0.00$	with $t=0.02$	:	4	intervals of 0,005	seconds.
of $t=0.02$	with $t=0.05$	:	3	intervals of 0.01	seconds.
of $t=0.05$	with $t=0.15$	:	4	intervals of 0,025	seconds.
of $t=0.15$	with $t=0.4$	:	5	intervals of 0.05	seconds.
of $t=0.4$	with $t=1.2$	:	8	intervals of 0.1	seconds.

4 Results of modeling A

4.1 Values tested

Identification	**Reference**	***Aster***	**% difference**	**Tolerance**
Not O				
$N5(0.,0.,0.)$				
$t=0.1\,s$	1.05137	1.04934	-0,193	1%
$t=0.2\,s$	1.24768	1.24181	-0,471	1%
$t=0.3\,s$	1.45136	1.44378	-0,522	1%
$t=0.5\,s$	1.73684	1.72955	-0,420	1%
$t=0.7\,s$	1.88010	1.87516	-0,263	1%
$t=1.0\,s$	1.96406	1.96191	-0,110	1%
$t=1.2\,s$	1.98398	1.98282	-0,059	1%

Not H				
$N1075(0.5,0.8,1.0)$				
$t=0.1\,s$	1.33579	1.32490	-0,816	1%
$t=0.2\,s$	1.61081	1.60337	-0,462	1%
$t=0.3\,s$	1.75959	1.75424	-0,304	1%
$t=0.5\,s$	1.90017	1.89718	-0,157	1%
$t=0.7\,s$	1.95657	1.95478	-0,091	1%
$t=1.0\,s$	1.98723	1.98646	-0,039	1%
$t=1.2\,s$	1.99433	1.99391	-0,021	1%

5 Summary of the results

The got results are satisfactory. The maximum change obtained (0,816%), is located at the point H placed halfway enters surface external and the center of the parallelepiped. At the end of $1.2\mathrm{s}$, this variation decreases, the maximum obtained is then of 0,059% (not O : center of the parallelepiped).

This test made it possible to test in linear transient modeling `3D` with meshs `HEXA27`.