SDNL100 - Simple pendulum in great oscillation

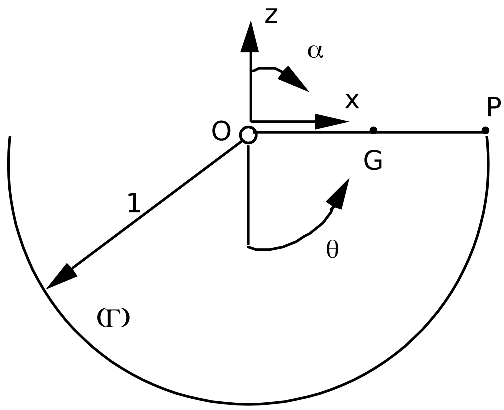
Summary:

The object of this test is to calculate the movement of a heavy bar articulated at a point fixed by one of its ends, free elsewhere and oscillating with great amplitude in a vertical plan.

Interest: to test the element of cable with two nodes - which is in fact an element of bar - under dynamics and its operation in the operator DYNA_NON_LINE.

1 Problem of reference

1.1 Geometry



A pendulum OP rigid length 1 and of centre of gravity G oscillate around the point O .

The angular position of the pendulum is located by: $\alpha=\theta-\pi$

1.2 Material properties

Linear density of the pendulum: $1.kg/m$

Axial rigidity (produced Young modulus by the surface of the cross-section): $1.10^8 N$

1.3 Boundary conditions and loadings

The pendulum is articulated at the fixed point O . Under the action of gravity, its end P oscillate on the half-circle (Γ) of center O and of ray 1 . There is no friction.

1.4 Initial conditions

The pendulum is released without speed of the horizontal position OP .

$$\theta=+\frac{\pi}{2}, \ \dot{\theta}=0$$

2 Reference solution

2.1 Method of calculating used for the reference solution

The period T of a mobile pendulum without friction around the fixed point O, of which the mass is concentrated in the centre of gravity G ($OG=l$) and whose maximum angular amplitude is θ_0 is given by the series [bib1]:

$$T=2\pi\sqrt{\frac{l}{g}}\left[1+\sum_{n=1}^{\infty} a_n^2\left(\sin\frac{\theta_0}{2}\right)^{2n}\right]$$

with

$$a_n=\frac{2n-1}{2n}$$

2.2 Results of reference

For $l=0.5\,m$, $g=9.81\,m/s^2$ and $\theta_0=\pi/2$, one finds: $T=1.6744\,s$

2.3 Uncertainty on the solution

One summoned the terms of the series until $n=12$ inclusively, the last term taken into account being lower than 10^{-5} time the calculated sum.

2.4 Bibliographical references

1) J. HAAG, “movements vibratory”, P.U.F. (1952).

3 Modeling A

3.1 Characteristics of modeling

The pendulum is modelled by an element of cable with 2 nodes, identical to an element of bar of constant section.

Discretizations:

- space: an element of cable MECABL2
- temporal: analysis of the movement over one complete period T by step of times equal to $T/40$.

3.2 Characteristics of the grid

Many nodes:	2
Many meshs and types:	1 mesh SEG2

4 Results of modeling A

4.1 Values tested

Identification	Reference	Tolerance
DX on node P with $t=0{,}4186$	-1,000000	2,5% (relative)
DZ on node P with $t=0{,}4186$	-1,000000	0,05% (relative)
DX on node P with $t=0{,}8372$	-2,000000	0,01% (relative)
DZ on node P with $t=0{,}8372$	0,000000	7,0E- 4% (absolute)
DX on node P with $t=1{,}2558$	-1,000000	7,5% (relative)
DZ on node P with $t=1{,}2558$	-1,000000	0,3% (relative)
DX on node P with $t=1{,}6744$	0,000000	1,0E- 6% (absolute)
DZ on node P with $t=1{,}6744$	0,000000	1,5E- 3% (absolute)

One also tests the structural parameters of data results:

Identification	Reference	Tolerance
INST for NUME_ORDRE= 10	0,418600	0,10%
ITER_GLOB for NUME_ORDRE= 10	9,000000	0,00%
INST for NUME_ORDRE= 15	0,837200	0,10%
ITER_GLOB for NUME_ORDRE= 15	5,000000	0,00%
INST for NUME_ORDRE= 19	1,674400	0,10%
ITER_GLOB for NUME_ORDRE= 19	6,000000	0,00%

4.2 Remarks

- Temporal integration is done by the method of NEWMARK (rule of the trapezoid),
- With each step of time, convergence is reached in less than 9 iterations.

5 Summary of the results

One sees on this CAS-test which temporal integration by the “rule of the trapezoid” of Newmark only modifies very slightly the frequency and does not bring parasitic damping, since at the end of one period one returns to very little close with the initial position.